Analysis and Simulation of Unbalanced Systems

[1]Virgil MAIER, [2]Antoniu TURCU, [3]Sorin PAVEL, [4]Iulian BIROU
Technical University of Cluj-Napoca
15, C-tin Daicoviciu Str., RO-400020 Cluj-Napoca
[1]Virgil.Maier@eps.utcluj.ro, [2]Antoniu.Turcu@eps.utcluj.ro, [3]Sorin.Pavel@eps.utcluj.ro,
[4]iulian.birou@edr.utcluj.ro

***Abstract* — The correct appreciation of the unbalanced states requires the establishment of the ranges and availability situations for the multitude of known calculus relations. Their verification, as well as the calibration and testing of the virtual instruments, dedicated to the unbalanced states, are demanding the unbalanced phasors system simulation.**

The method of the equal modulus and equal, consecutive, phases, developed in the paper, presents the next advantage: the phasors system is defined in comparison with only one variable, this one being the phase between two consecutive phasors. The analytically study on a definition range of this variable (2π), emphasizes the complete covering of the unbalanced state indicators range, so any type of asymmetries may be described.

The corresponding functions for the symmetrical components, as well as for the unbalanced state indicators, as the dissymmetry and asymmetry coefficients, are analytically and graphically presented.

The approaching of the inverse problem affords the unbalanced states identification, which have to fulfill certain unbalance degrees, expressed through the precised dissymmetry and asymmetry coefficients.

***Index Terms* — calculus relations of the unbalanced states, phasors system, dissymmetry and asymmetry coefficients, Stokvis-Fortescue theorem, iterative calculus method**

I. INTRODUCTION

The proliferation of the unbalanced states characterization methods leads to the necessity of verifying and comparing them in order to distinguish their applicability area. The complexity of the proposed relations [1,2] does not represent an impediment for their comparative analysis while it may be done using CAD, but problems may appear in practice when power-meters are used.

Six scalars variables are corresponding to a three phasors system ($\underline{Y_1}$, $\underline{Y_2}$, $\underline{Y_3}$): three amplitudes (or effective values) and three phases. If one of these phasors is considered as reference (for instance $\underline{Y_1}$), the amplitudes of the other two can be expressed in comparison with the reference phasor amplitude (ex.: Y_2/Y_1 and Y_3/Y_1), and if its phase will be considered equal with zero, that is the reference phasor will be placed in the system axis (ex.: φ_1=0), only four variables will remain. In conclusion, a homologous system of three phasors has four degree of freedom corresponding to the two amplitudes ratios, Y_2/Y_1 and Y_3/Y_1 and two phases, φ_{12} and φ_{23}, between the reference phasor and the second one, respectively between the second and the third phasor. [4].

Firstly, the problem is to determine the ranges of the four independent variables which may generate any type of unbalanced running among all possibilities. On the second hand, the range of both the characteristic variables and the unbalanced running indicators must be estimated. Finally, the third aspect of the research is represented by the identification of a phazors system which has to correspond to certain values of the unbalanced state indicators.

The utility of the proposed objectives consists in verifying the methods and the evaluation relations for the unbalanced state and in adjusting the virtual instruments and any type of aparatus (counter, power-meter) dedicated to emphasize the unbalanced state characteristic variables.

II. THE UNBALANCED STATE ANALITICAL BASIS

A. The Stokvis-Fortescue theorem and terminology

A multi phase system of voltages or currents may have different characteristics on the phases, regarding the effective values or phases of the periodical, sinusoidal quantities, which compose the respective systems. Consequently, for an unbalanced system of alternative, sinusoidal variables ($\underline{Y_1}$, $\underline{Y_2}$, $\underline{Y_3}$), the symmetrical components set, formed by the direct (positive) succession (sequence) quantity Y_d, inverse (negative) Y_i and homopolar (zero succession) Y_h, may be determined in accordance with the Stokvis-Fortescue theorem:

$$\begin{aligned} \underline{Y_d} &= \frac{1}{3}\cdot\left(\underline{Y_1} + a\,\underline{Y_2} + a^2\,\underline{Y_3}\right); \\ \underline{Y_i} &= \frac{1}{3}\cdot\left(\underline{Y_1} + a^2\,\underline{Y_2} + a\,\underline{Y_3}\right); \\ \underline{Y_h} &= \frac{1}{3}\cdot\left(\underline{Y_1} + \underline{Y_2} + \underline{Y_3}\right), \end{aligned} \tag{1}$$

where $a = e^{j2\pi/3}$ is the rotation operator and the three phased system ($\underline{Y_1}$, $\underline{Y_2}$, $\underline{Y_3}$) can be relative to voltages or currents. Referring to the terminology, the use of both families of notions is preferred: the classical one, naming the symmetrical components as of direct, inverse and homopolar sequence and the new one, influenced by the anglo-saxone scientific literature, naming the same components, respectively, as of positive, negative and zero sequence. The omopolar therm is preferred to the homopolar one, even the index "*h*" is used in order to avoid the confusion with the initial values (with the index "*zero*"). Also, for the direct (positive) and inverse (negative) sequence quantities, the annotation with the index *d* and *i* has been chosen instead the annotation with the polarity signs (+, respectively -) to the exponent in order to facilitate the identification. These options are justified, on the hand, by the existence of a big number of specialists familiar with the established terms and, on the other hand, by the confusions which can be created using the polarity terms to the exponent and to avoid the following expressions: "the zero sequence component is zero" or "the case of zero null component".[4]

In the context of calculus relations proliferation, for which the availability domain remains unspecified, is important to underline that the symmetrical component systems are strictly calculated according to the relations (1), which presents the disadvantage to be in complex and less software are working in this plan.

B. The iterative calculus method

Developing (1) through the explaining of the *a* and a^2 operators and identifying the arguments of the trigonometric functions such as the sums may be written in an iterative form, based on the same summing index, the following set of calculus relations for the symmetrical components is proposed:

$$Y_d = \frac{1}{3}\cdot\left\{\left[\sum_{k=1}^{3} Y_k \cdot cos\left(\varphi_k + \frac{2\pi}{3}\cdot(k-1)\right)\right]^2 + \left[\sum_{k=1}^{3} Y_k \cdot sin\left(\varphi_k + \frac{2\pi}{3}\cdot(k-1)\right)\right]^2\right\}^{\frac{1}{2}} \qquad (2)$$

$$Y_i = \frac{1}{3}\cdot\left\{\left[\sum_{k=1}^{3} Y_k \cdot cos\left(\varphi_k + \frac{2\pi}{3}\cdot(4-k)\right)\right]^2 + \left[\sum_{k=1}^{3} Y_k \cdot sin\left(\varphi_k + \frac{2\pi}{3}\cdot(4-k)\right)\right]^2\right\}^{\frac{1}{2}} \qquad (3)$$

$$Y_h = \frac{1}{3}\cdot\left[\left(\sum_{k=1}^{3} Y_k \cdot cos\,\varphi_k\right)^2 + \left(\sum_{k=1}^{3} Y_k \cdot sin\,\varphi_k\right)^2\right]^{\frac{1}{2}} \qquad (4)$$

corresponding to the direct (positive sequence) Y_d, inverse (negative sequence) Y_i and homopolar (zero sequence) Y_h components. If the variables Y_k, for k€ {1,2,3}, are amplitudes or effective values, the variables (Y_d, Y_i, Y_h) are resulting like amplitudes, respectively like effective values. It can be remarked that according to (2)-(4), the phases of he symmetrical components may be determined as well; the real and imaginary parts of the expressions appear in this order and are comprised between square brackets, in the relations (2) and (3), respectively between round brackets, in (4).

The relations set (2)-(4) represents a scalar, iterative, calculus basis for the symmetrical components, which guide to identical results as the Stokvis-Fortescue theorem [4].

III. SIMULATION AND ANALYSIS OF THE UNBALANCED STATE

A. The method of the equal modulus and equal, consecutive, phases

The verification of the symmetrical components calculus relations on as large as possible range of the dissymmetry coefficient, was made in [4] through the variation of the phase φ between the successive phasors of the three phase system inside the interval $\varphi \in [-2\pi/3,\ 2\pi/3]$. This fact permitted the scalling of a large domain of unbalanced states, starting from the direct sequence system, established for $\varphi = -2\pi/3$, passing through the omopolar sequence one when $\varphi = 0$ and arriving to the inverse sequence (negative) one, for which $\varphi = 2\pi/3$, even if the phasors modulus was mentained equals.

Applying the same simulation method of the unbalanced states, the analytically identifying of the unbalanced state quantities and indicators is made further on together with the graphical representations of these ones. In addition, the range of the variable φ will be extended to a complete interval (2π), in order to cover all possible unbalanced states.

Consequently, the unbalanced state simulation method consists in the following steps:

- the phasor modulus are considered equals,

$$Y_1 = Y_2 = Y_3 \qquad (5)$$

- the phases of the three phasors, expressed in comparison with the variable $\varphi \in [-2\pi/3,\ 4\pi/3]$ and considering the first phasor in the axis of the reference system, are given by the relations:

$$\varphi_1 = 0;\ \ \varphi_2 = \varphi;\ \varphi_3 = 2\varphi \qquad (6)$$

so that the three-phase system will be symmetrical for $\varphi = \pm 2\pi/3$ and omopolar for $\varphi = 0$.

Introducing in (2) the phase quantities corresponding to the hypothesis mathematically expressed through (5) and (6), the modulus of the direct component is obtained as follows:

$$Y_d = \frac{Y_1}{3}\sqrt{4cos^2 x + 4cos\,x + 1} \qquad (7)$$

where the notation $x = \varphi + 2\pi/3$ has been used.

Making the possible restriction and explaining the modulus function, a parts defined function is obtained as following:

$$Y_d = \begin{cases} \frac{Y_1}{3}\left[2cos\left(\varphi + \frac{2\pi}{3}\right) + 1\right],\ pt.\ \varphi \in \left[-\frac{4\pi}{3}, 0\right]; \\ -\frac{Y_1}{3}\left[2cos\left(\varphi + \frac{2\pi}{3}\right) + 1\right], pt.\ \varphi \in \left(0, \frac{2\pi}{3}\right). \end{cases} \qquad (8)$$

The method is analogously used for the inverse succession component, for which the initial relation is (3) and resulting a similar parts defined function, like the next one:

$$Y_i = \begin{cases} \frac{Y_1}{3}\left[2cos\left(\varphi - \frac{2\pi}{3}\right) + 1\right],\ pt.\ \varphi \in \left[0, \frac{4\pi}{3}\right]; \\ -\frac{Y_1}{3}\left[2cos\left(\varphi - \frac{2\pi}{3}\right) + 1\right], pt.\ \varphi \in \left(\frac{4\pi}{3}, 2\pi\right). \end{cases} \qquad (9)$$

Finally, the next function was identified for the omopolar component:

$$Y_h = \begin{cases} \frac{Y_1}{3}(2cos\varphi + 1),\ pt.\ \varphi \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]; \\ -\frac{Y_1}{3}(2cos\varphi + 1),\ pt.\ \varphi \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right). \end{cases} \qquad (10)$$

The three functions, expressed by the relations (8)-(10), are periodical with the period (2π); the graphical representations of these ones are given in figure 1 for the range of the independent variable $\varphi \in [-2\pi/3,\ 4\pi/3]$ and considering the phasor modulus $Y_1 = 100$.

It may be noticed from both the expressions (8)-(10) and the graphical representation that the functions $Y_i(\varphi)$ şi $Y_h(\varphi)$ result through the function $Y_d(\varphi)$ translation to the right side of the axis (0φ) with $4\pi/3$, respectively with $2\pi/3$.

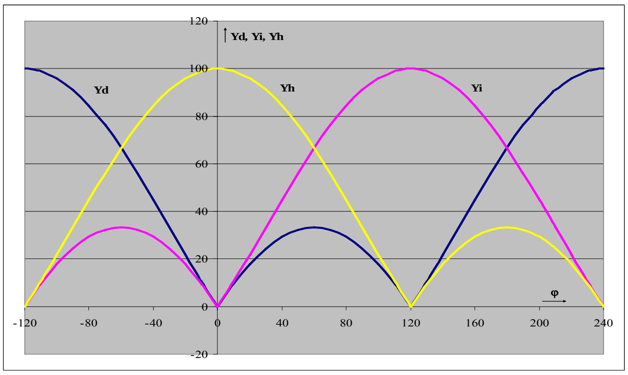

Figure 1. The symmetrical components Yd, Yi, Yh graphical representations for a phasors system generated through the method of the equal modulus and equal, consecutive, phases.

The $Y_d(\varphi)$ function, given by (8), is continuous in the points where will be null:

$$\varphi \in \{2k\pi ; 2\pi/3 + 2k\pi\},\ k \in Z \tag{11}$$

these ones representing minimum and angular points of the function.

The maximum of the function $Y_d(\varphi)$, $Y_{dM}=Y_1$ is given for

$$\varphi \in \{-2\pi/3 + 2k\pi\},\ k \in Z \tag{12}$$

a local maximum point is existing as well, given by the relation:

$$Y_{dMl}(\varphi \in \{\pi/3 + 2k\pi\}) = Y_1/3,\quad k \in Z \tag{13}$$

Similar considerations can be made for the function $Y_i(\varphi)$, correspondent to the inverse (negative) succession:

- the function will be annulled and presents minimum points (and angular) to the abscissa:

$$\varphi \in \{2k\pi ; 4\pi/3 + 2k\pi\},\ k \in Z \tag{14}$$

- the maximum of the function $Y_i(\varphi)$, $Y_{iM}=Y_1$ is given for the abscissa:

$$\varphi \in \{2\pi/3 + 2k\pi\},\ k \in Z \tag{15}$$

- the local maximum points are given by the relation:

$$Y_{dMl}(\varphi \in \{-\pi/3 + 2k\pi\}) = Y_1/3,\quad k \in Z \tag{16}$$

In addition, the same characteristics for the $Y_h(\varphi)$ function, corresponding to the homopolar (zero succession) component are succinctly presented:

- the function is cancelled and presents minimum (and angular) points to the abscissa:

$$\varphi \in \{-2\pi/3 + 2k\pi ; 2\pi/3 + 2k\pi\},\ k \in Z \tag{17}$$

- the maximum of $Y_i(\varphi)$, $Y_{iM}=Y_1$ is given at the abscissa:

$$\varphi \in \{2k\pi\},\ k \in Z \tag{18}$$

- the local maximum points are given by the relation:

$$Y_{hMl}(\varphi \in \{(2k+1)\pi\}) = Y_1/3,\quad k \in Z \tag{19}$$

B. The dissymmetry and asymmetry coefficients

The dissymmetry coefficient, named as well as negative unbalance factor (proposed notation - k_Y^-), is defined through the percentage ratio between the inverse (negative) succession Y_i and direct (positive) succession Y_d components, given by the relation:

$$K_{id\%} = \frac{Y_i}{Y_d} \cdot 100,\ \%. \tag{20}$$

The asymmetry coefficient, named as well as zero unbalance factor (proposed notation - k_Y^0), is defined through the ratio between the omopolar (zero succession) and direct (positive) succession components, in percent:

$$K_{hd\%} = \frac{Y_h}{Y_d} \cdot 100,\ \% \tag{21}$$

If, in the relation (20), which defines the dissymmetry coefficient, the determined expressions for the inverse succession (9) and direct succession (8) components are replaced according to the ranges of the corresponding functions and renouncing to percentage expression, the following relation for this factor is obtained:

$$K_{id} = \begin{cases} \dfrac{\cos(\varphi - 2\pi/3) + 0{,}5}{\cos(\varphi + 2\pi/3) + 0{,}5},\ pt.\ \varphi \in \left(0, \dfrac{2\pi}{3}\right) \cup \left(\dfrac{4\pi}{3}, 2\pi\right); \\ -\dfrac{\cos(\varphi - 2\pi/3) + 0{,}5}{\cos(\varphi + 2\pi/3) + 0{,}5},\quad pt.\ \varphi \in \left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right]. \end{cases} \tag{22}$$

Analogously, is proceeded for the asymmetry coefficient (21) for which is determined the dimensionless expression:

$$K_{hd} = \begin{cases} \dfrac{\cos\varphi + 0{,}5}{\cos(\varphi + 2\pi/3) + 0{,}5},\ pt.\ \varphi \in \left(-\dfrac{2\pi}{3}, 0\right); \\ -\dfrac{\cos\varphi + 0{,}5}{\cos(\varphi + 2\pi/3) + 0{,}5},\quad pt.\ \varphi \in \left[0, \dfrac{2\pi}{3}\right]. \end{cases} \tag{23}$$

The graphical representations for both coefficients are presented in figure 2, for the same defining domain of the independent variable $\varphi \in [-2\pi/3,\ 4\pi/3]$. The both functions, $K_{id}(\varphi)$ şi $K_{hd}(\varphi)$, are not defined for the values $\varphi \in \{2k\pi ; 2\pi/3 + 2k\pi\}, k \in Z$.

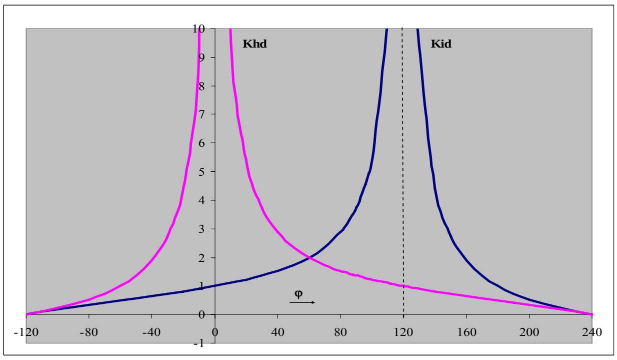

Figure 2. The dissymmetry and asymmetry coefficients Kid, Khd graphical representations for a phasors system generated by the equal amplitudes and equal consecutive phases method.

Restricting the definition domain to $\varphi \in [-2\pi/3,\ 4\pi/3]$, for which the graphical representations are made, it can be demonstrated that the function $K_{id}(\varphi)$ presents equal limits to the left and to the right, even it is not defined in origin:

$$\lim_{\varphi \to 0, \varphi < 0} K_{id} = \lim_{\varphi \to 0, \varphi > 0} K_{id} = 1 \tag{24}$$

In the $\varphi=2\pi/3$ abscissa point, the function $K_{id}(\varphi)$ presents a vertical asymptote. The range of the function is $K_{id}(\varphi) \in [0, \infty)$, totally covered by the branch of the function from the right side of the asymptote, that is for $\varphi \in [2\pi/3,\ 4\pi/3]$, while the branch from the left side of the asymptote, for which the argument is placed in the range $\varphi \in [-2\pi/3,\ 2\pi/3]$, covers the range $K_{id}(\varphi) \in [0,1) \cup (1, \infty)$.

Regarding the function $K_{hd}(\varphi)$, expressed by (23), it has as a vertical asymptote the Y axis, with the equation $\varphi=0$ equation, i.e. at the abscissa for which the function $K_{id}(\varphi)$ is not defined, and for the abscissa where $K_{id}(\varphi)$ has the vertical asymptote $\varphi=2\pi/3$, where is not defined, it presents equal limits, to the left and to the right:

$$\lim_{\varphi\to 2\pi/3,\varphi<2\pi/3} K_{hd} = \lim_{\varphi\to 2\pi/3,\varphi>2\pi/3} K_{hd} = 1 \qquad (25)$$

The range of the function is $K_{hd}(\varphi) \in [0, \infty)$, totally covered by the branch from the left side of the asymptote, that is for $\varphi \in [-2\pi/3, 0)$, while the function branch from the right side of the asymptote, for which the argument is placed in the interval $\varphi \in (0, 4\pi/3]$, covers the interval $K_{hd}(\varphi) \in [0,1)\cup(1, \infty)$.

C. The inverse problem

In many situations, the determination of an unsymmetrical phasors system that presents unbalanced state indicators with précised values can be useful. The generation method of the unbalanced state, proposed in [4] and developed in the present paper, has the advantage of using only one variable, that is φ, the angle between the consecutive phasors. The determination of this angle for certain values of the dissymmetry and asymmetry factors can completely and correctly identify the initial phasors system, according to the premises of the equal modulus and equal, consecutive phases method.

Returning to the $K_{id}(\varphi)$ function graphic, which has the range $K_{id}(\varphi)\in[0, \infty)$, it can be noticed that for any function value from the range of the function, excepting the $K_{id}(\varphi)=1$ case, exist two distinguished solutions: one for the independent value from the domain $\varphi \in [-2\pi/3,0)\cup(0,2\pi/3)$ and the other one for the domain $\varphi \in (2\pi/3, 4\pi/3)$.

For the value $K_{id}(\varphi)=1$ it exists one solution, in the interval $\varphi \in (2\pi/3, 4\pi/3)$, determinable directly from (22); this is $\varphi=\pi$, for which the phasors system is a pulsating type one, so it can be decomposed in two rotating systems, with equal modules, but in opposition as sense (the direct one and the inverse one).

Concretely, the inverse function of $K_{id}(\varphi)$, given by the (22) relation is searched. Keeping the part definition for this function, the φ angle identification, which will determine the phasors system, corresponding to a set factor K_{id} , is given by the relation:

$$\varphi = \begin{cases} 2\cdot arctg\dfrac{\sqrt{3}(K_{id}+1)}{K_{id}-1} + 2k\pi, & pt.\ \varphi \in \left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right]; \\ 2\cdot arctg\dfrac{\sqrt{3}(K_{id}-1)}{K_{id}+1} + 2k\pi, & pt.\ \varphi \in \left(0, \dfrac{2\pi}{3}\right)\cup\left(\dfrac{4\pi}{3}, 2\pi\right). \end{cases} \qquad (26)$$

For instance, for $K_{id}=2$, the solutions $\varphi\in\{60°; 158,2°\}$ are corresponding and for $K_{id}=0,2$, the solutions $\varphi\in\{-98,2°; 222,1°\}$ are found.

Proceeding in the same way for the asymmetry coefficient, given by (23), its inverse function is determined as follows:

$$\varphi = \begin{cases} 2\cdot arctg\dfrac{-\sqrt{3}}{2K_{hd}+1} + 2k\pi, & pt.\ \varphi \in \left[-\dfrac{2\pi}{3}, 0\right); \\ 2\cdot arctg\dfrac{\sqrt{3}}{2K_{hd}-1} + 2k\pi, & pt.\ \varphi \in \left(0, \dfrac{2\pi}{3}\right)\cup\left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right). \end{cases} \qquad (27)$$

Some examples can be, in this case also, probative: the case $K_{hd}=1$ presents only one solution, $\varphi=-\pi/3$; for $K_{hd}=0,2$ two solutions are identified as $\varphi\in\{-102,1°; 218,2°\}$, which can be followed on the corresponding graphic (figure 2).

IV. IV. CONCLUSIONS

The utilization of the Stokvis-Fortescue theorem is essentially to characterize and analyze the unbalanced states. The derived scalar relations, like the iterative calculus ones, are very useful and practical for the analytical approach of the phasors unbalance systems.

The phasors unbalanced systems generation is simple and efficient through the proposed method that is the method of the equal modulus and equal, consecutive, phases. The simplicity derives from the use of only one variable, namely the φ angle between two consecutive phasors and the efficiency is sustained by covering of a complete range of a function, from zero to plus infinite, for both unbalance state indicators, as the dissymmetry and asymmetry coefficients.

The analytical study of the symmetrical components and of the dissymmetry and asymmetry coefficients, as well as the graphical representation of the corresponding functions of these variables, on the hole interest domain, facilitates the apprehension and handling of the phenomena related to the unbalanced states.

The inverse problem solving gives the possibility to identify the unbalanced systems with imposed asymmetry and dissymmetry coefficients, which may be useful to verify some calculus relations or for the calibration of some dedicated apparatus.

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