A New Family of CSK Signals

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***Abstract* — A new family of CSK pulses with good spectral properties defined on the time interval [-T, T] is proposed and its spectral properties are investigated. Imposing the condition that the signals of this family should exhibit a large number of derivatives which are continuous and equal to zero at the ends of the time interval [-T, T], the expressions of Fourier transforms of the component signals are deduced. So, exact analytical formulae for the power spectral densities (p.s.d.) of these signals can be derived.**

The asymptotic decay rate of the envelope of power spectral density can be made small enough. These pulses are of interest in digital communications and in particular in wireless transmissions, as the intercarrier interference is diminished.

***Index Terms* — Digital communication, digital modulation Fourier transforms, frequency domain analysis, pulse analysis**

I. INTRODUCTION

A technique used in digital communications for spectral shaping CPM (Continuous Phase Modulation) [1, 6, 7, 9]. It prevents signals from having large spectral side lobes and makes them spectrum compact.

The CPM signal is obtained starting from a PAM (Pulse Amplitude Modulation) signal:

$$d(t) = \sum_{m=-\infty}^{n} A_m \, h \, g(t - kT) \qquad (1)$$

where $\{A_m\}$ denotes the sequence of amplitudes obtained by mapping k-bit blocks of binary data from the information sequence $\{i_n\}$ into the antipodal amplitude levels $\pm 1, \pm 3, \cdots, \pm(2^{k-1} - 1)$, *h(t)* is a modulation index and *g(t)* is the signaling waveform.

A frequently used signaling waveform is the rectangular pulse, as it can be produced easily, even at high speeds. However, the rectangular pulse shows a power spectrum that decays slowly.

The p.s.d. of a polar NRZ-L transmission using equi-probable data bits ($p = 1 - p = 0.5$), where p is the probability of a bit 1, is given [9] by

$$W(f) = \frac{1}{T}\left|G(f)\right|^2 \qquad (2)$$

where *G(f)* denotes the Fourier transform of the signaling pulse g(t) and *T* is the duration of the bit interval.

The rectangular pulse of amplitude *A* and duration *T* has a Fourier transform

$$G(f) = AT\frac{\sin \pi f T}{\pi f T} = AT\frac{\sin(\pi f / f_0)}{\pi f / f_0} \qquad (3)$$

where $f_0 = 1/T$ is the signaling frequency (data rate) and f / f_0 is a normalized frequency with respect to the data rate. The Fourier transform decays rather slowly as 1/f taking into account the discontinuous character of the signaling waveform (rectangular pulse). As a consequence, the p.s.d. will decay as f^{-2}.

In order to obtain better spectral properties the rectangular pulse is being replaced by other pulses [3]. If they are generated by filters, a necessary condition is the even frequency symmetry of the transfer characteristic, which implies that the time pulse is real valued. The fact that the frequency characteristic is real implies even symmetry in the time characteristic.

A well-known theorem in the theory of Fourier transform states that if the signaling waveform *g(t)* is continuous and equal to zero at the ends of the signaling interval $(\pm T/2)$ and has a number of *k - 1* derivatives that are continuous and equal to zero at the ends of the signaling interval, then the Fourier transform will decay as $f^{-(k+1)}$. Accordingly, the p.s.d. will decay as $f^{-2(k+1)}$. We will denote this as the continuity feature of (k-1) - *th* order [8].

N. C. Beaulieu and M. O. Damen proved a theorem [5] that says: If the first m-1 derivatives of *S(f)* are continuous and the *m*-th derivative of *S(f)* has one or more finite amplitude discontinuities, then the impulse response associated to *S(f)* decays as $1/\left|t\right|^{m+1}$ when $\left|t\right|$ is large.

To exhibit better spectral properties the signaling waveform *g(t)* should be continuous and equal to zero at the ends of the signaling interval $(\pm T)$ and possess as large as possible number of derivatives that are continuous and equal to zero at the ends of the signaling interval.

II. CSK SIGNALS

A quadrature amplitude modulation (QAM) scheme, such as QPSK, QASK, etc. uses two sinusoidal carriers that are out of phase with each other by $\pi/2$. The amplitudes of the carrier waves are varied by two information-bearing signals.

For instance, the QPSK signal can be written as

$$y(t) = \frac{1}{\sqrt{2T}} s_1(t)\cos(\omega_0 t + \varphi) + \frac{1}{\sqrt{2T}} s_2(t)\sin(\omega_0 t + \varphi) \qquad (4)$$

where ω_0 is the angular frequency of the carrier and φ its phase.

The signals $s_1(t)$ and $s_2(t)$ take only the values +1 or -1, so eq.(4) becomes

$$y(t) = \frac{1}{\sqrt{T}}\cos[\omega_0 t + \psi(t)] \qquad (5)$$

where $\psi(t)$ takes the values $\{0, \pi/2, \pi, 3\pi/2\}$ if $\varphi = \pi/4$, or $\{\pm\pi/4, \ \pm 3\pi/4\}$, if $\varphi = 0$, depending on the values of s_1 and s_2.

For $\varphi = 0$ and

$$s_1(t) = \frac{1}{\sqrt{T}} \cos \Phi(t)$$

and

$$s_2(t) = -\frac{1}{\sqrt{T}} \sin \Phi(t)$$

eq.(4) becomes

$$y(t) = \frac{1}{\sqrt{T}} \cos[\omega_0 t + \psi(t)] \tag{6}$$

The signals $s_1(t)$ and $s_2(t)$ can be multiplied by the binary values +1 and -1, so *y(t)* can convey information.

The spectral shaping of *y(t)* can be done choosing an appropriate *s(t)* signal

$$s(t) = \frac{1}{\sqrt{T}} \cos \Phi(t) \tag{7}$$

If *s(t)* possesses a bigger number of derivatives that are continuous and equal to zero at the ends of the definition interval [-*T, T*] , then the attenuation of spectral components is greater and as a consequence the inter-carrier interference (ICI) in the adjacent channel is smaller.

The first derivative of *s(t)* is

$$\frac{ds(t)}{dt} = \frac{1}{\sqrt{T}} \frac{d\Phi(t)}{dt} \sin \Phi(t) \tag{8}$$

Imposing that *s(t)* = 0 at the ends of the [- *T* , *T*]interval, one gets:

$$\Phi(t)\big|_{t=\pm T} = \frac{\pi}{2} \tag{9}$$

Imposing $\frac{ds(t)}{dt}\big|_{t=\pm T} = 0$, the necessary and sufficient condition for the signal to have the first derivative equal to zero at the ends of the definition interval is

$$\frac{d\Phi(t)}{dt}\big|_{t=\pm T} = 0 \tag{10}$$

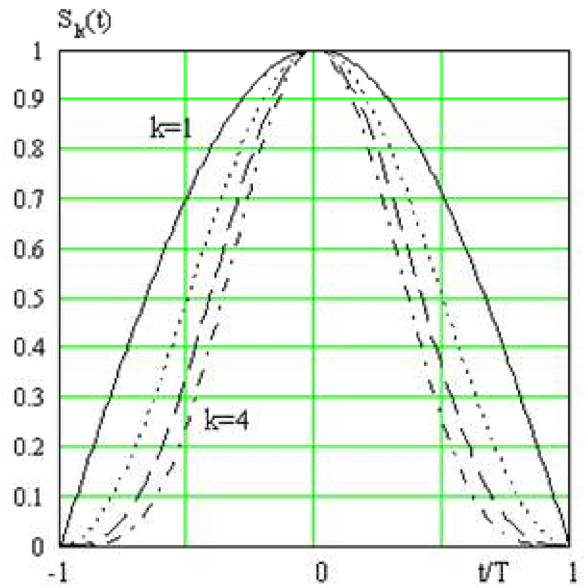

Figure 1 Time variation of CSK pulses (*k* = 1 to 4)

Generalizing for the first *n* consecutive derivatives to be continuous and equal to zero at the ends of the definition interval [- *T* , *T*], the necessary and sufficient condition is

$$\frac{d^n\Phi(t)}{dt^n}\big|_{t=\pm T} = 0 \tag{11}$$

According to the definition of Reiffen & White [2], the signals with $n \geq 3$ are denoted as CSK (*Continuous Shift Keying*) signals.

Their phase $\phi(t)$ should exhibit an odd-symmetry variation around *t* = 0, such as:

$$\Phi(t) = \begin{cases} \pm\pi/2 & t = -T \\ arbitrary & -T < t < 0 \\ 0 & t = 0 \\ \pm\pi/2 \pm \Phi(t-T) & 0 < t < T \\ \pi/2 & t = T \end{cases} \tag{12}$$

III. A NEW FAMILY OF CSK SIGNALS

A new family of CSK signals is proposed, defined by

$$s_k(t) = \cos^k[\pi t/(2T)] \tag{13}$$

An illustration of the first four components of this family is given in figure 1.

One denotes

$$\Phi(t) = \left(\frac{\pi t}{2T}\right) \tag{14}$$

(5.105)

TABLE I. THE FIRST FOUR PULSE COMPONENTS

Order k	Fourier transform $S_k(f)$
1	$\frac{4\sqrt{T}}{\pi} \frac{\cos(2\pi f T)}{1-16f^2T^2}$
2	$\frac{1}{2\pi f\sqrt{T}} \frac{\sin(2\pi f T)}{1-4f^2T^2}$
3	$\frac{24\sqrt{T}}{\pi} \frac{\cos(2\pi f T)}{(1-16f^2T^2)(9-16f^2T^2)}$
4	$\frac{3}{8\pi f\sqrt{T}} \frac{\sin(2\pi f T)}{(1-4f^2T^2)(1-f^2T^2)}$
5	$\frac{480\sqrt{T}}{\pi} \frac{\cos(2\pi f T)}{(1-16f^2T^2)(9-16f^2T^2)(25-16f^2T^2)}$
6	$\frac{45}{16\pi f\sqrt{T}} \frac{\sin(2\pi f T)}{(1-4f^2T^2)(9-4f^2T^2)(1-f^2T^2)}$
7	$\frac{20160\sqrt{T}\cos(2\pi f T)}{\pi(1-x^2)(9-x^2)(25-x^2)(49-x^2)}$
8	$\frac{315}{32\pi f\sqrt{T}} \frac{\sin(2\pi f T)}{(1-y^2)(4-y^2)(1-4y^2)(9-4y^2)}$
9	$\frac{1451520\sqrt{T}\cos(2\pi f T)}{\pi(1-x^2)(9-x^2)(25-x^2)(49-x^2)(81-x^2)}$
10	$\frac{(14175/64\pi f\sqrt{T})\sin(2\pi f T)}{(1-y^2)(4-y^2)(25-4y^2)(9-4y^2)(1-4y^2)}$

where $x^2 = 16f^2T^2$ and $y = f^2T^2$.

The Fourier transforms of the component signals are given by

$$S_k(f) = \frac{1}{\sqrt{T}} \int_{-T}^{T} \cos[\Phi_k(t)] \cos(2\pi f t)\, dt \tag{15}$$

taking into account the even-symmetry of the signal.

The first ten pulses of this family are listed in Table I.

A recurrence relation is found, connecting two consecutive odd values of k, expressed as

$$S_{2i+1}(f) = \frac{2i(2i+1)}{(2i+1)^2 - 16 f^2 T^2} S_{2i-1}(f) \tag{16}$$

For even values of k, a similar relation exists

$$S_{2i}(f) = \frac{2i(2i-1)}{i^2 - 4 f^2 T^2} S_{2i-2}(f) \tag{17}$$

The p.s.d. of a signal modulated by a binary, antipodal and equiprobable random data sequence [6] using the pulse $s_k(t)$ is given by

$$W_k(f) = \frac{\left|S_k(f)\right|^2}{T} \tag{18}$$

In view of the even symmetry in the time characteristic the frequency characteristic is real and so,

$$W_k(f) = \frac{\left[S_k(f)\right]^2}{T} \tag{19}$$

The p.s.d. defined by eq.(18) for k=2 and k=4 is represented in figure 2. The p.s.d. were normalized in order to have 0dB at D.C. Here, *fn* is the normalized frequency with respect to the symbol rate 1/2T.

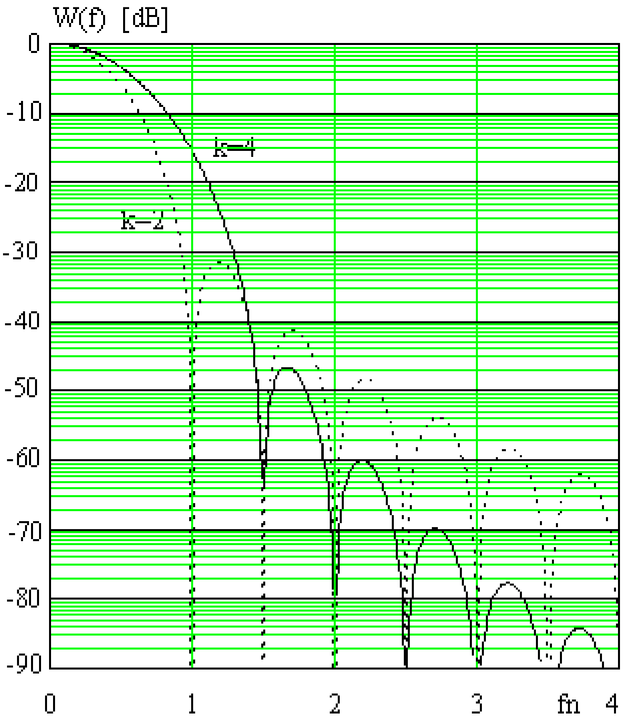

Figure 2 P.s.d. of CSK signals for k=2 and 4

A comparison of the p.s.d. for k=3 and k=5 is illustrated in figure 3, under the same assumptions as above.

It can be easily shown that a component signal has *k-1* consecutive derivatives that are continuous and equal to zero at the ends of the definition interval [- *T* , *T*].

So, for $k \geq 4$, all components qualify as CSK signals. The third derivative of $S_k(f)$ is given by

$$\frac{ds_k(t)}{dt^3} = \frac{k\pi^3 \left[\cos m\right]^{k-3} \left(k^2 \cos 2m \cdot \sin m - k^2 + 6k - 4\right)}{16 T^3} \tag{20}$$

where $m = \pi t / 2T$

The $\cos m$ factor is always zero at the ends of the definition interval, so, in order to have the third derivative zero at the ends of the definition interval [-*T, T*], k should be greater than 3.

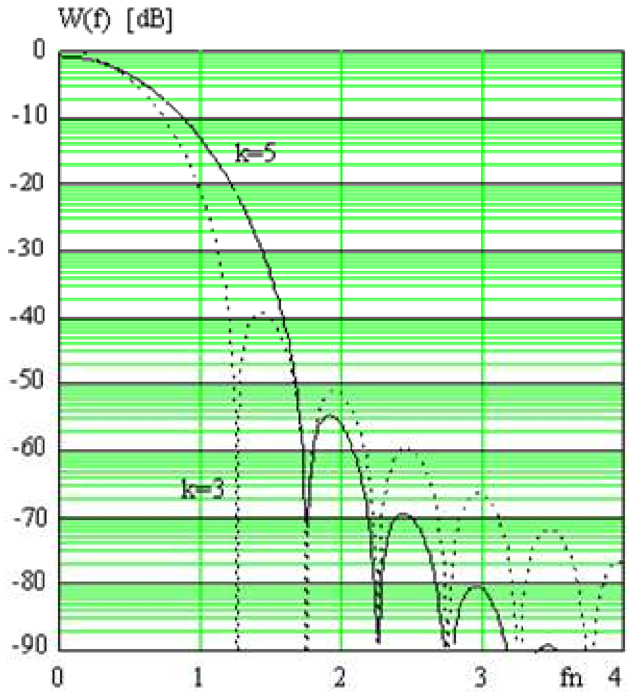

Figure 3 P.s.d. of CSK signals for k = 3 and 5

IV. CONCLUSION

A time-limited family of signals on [-*T, T*] interval was introduced. For k = 1 and CPM modulation this is the MSK signal. For k = 2 the well-known raised cosine signal is obtained [4]. The asymptotic decay rate of the envelope of p.s.d. (spectral roll-off) can be made small enough by choosing a larger value of k.

The pulses are of interest in digital communications and in particular in wireless transmissions, as they lead to diminished intercarrier interference (ICI).

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