Some Useful Properties of STBC codes for Residential Applications

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Abstract — **Main drawbacks of Wi-Fi home networks are inadequate range and coverage. A way to improve both of them is the use of transmit diversity, which also increases network throughput by finding paths with highest data rates and avoiding signal paths that induces packet errors and retransmissions. The main idea of the transmit diversity is the use of space-time coding (STC) techniques that spread the same information across different antennas at the transmitter in order to obtain a transmit diversity. This paper aims to study coherent space-time block codes to estimate the channel coefficients in the presence of flat fading. It focuses on the popular transmit diversity scheme of Alamouti assuming a flat fading Rayleigh multipath channel and BPSK and QPSK modulation schemes.**

Index Terms — **Space Time Coding, Transmit Diversity, Zero Forcing algorithm, Alamouti code, channel state information at the receiver**

I. INTRODUCTION

In the never-ending search for higher data rates and better reliabilities it has been shown that the use of multiple antennas in a wireless communication system represents the solution that improves bandwidth efficiency and system reliability. Information theory shows that MIMO (multiple-input multiple output) systems can yield substantially capacity gains compared with the conventional systems (SISO, SIMO, and MISO). Usually, fading induces a lot of problems in wireless communication, but MIMO systems take advantage of random fading to increase the capacity.

Most of the work on wireless communication had focused on having antenna arrays at one or both ends of a wireless link, significant capacity gains being enabled by the rich scattering wireless environments [1]-[3].

The main obstacle towards reliable wireless transmissions is the time-varying multipath fading. Increasing the quality under these conditions is extremely difficult compared to channels with only additive noise. Reducing the error rate from 10^{-2} to 10^{-3} with a additive white Gaussian noise (AWGN) may require only a 1 to 2 dB signal-to-noise ratio (SNR) increase. Achieving the same improvement in a fading channel requires up to 10 dB SNR increase [4]. This increase in SNR may not be possible with higher transmit power or additional bandwidth because of the constraints of these parameters in the next generation communication. Besides transmitter power control techniques, diversity methods are the most effective solutions for increased capacity in a multipath fading environment.

II. DIVERSITY

MIMO systems send the same information on different signals transmitted by each of array element, so that the receiving antenna array receives a superposition of all of the transmitted signals. The capacity gain increases when the channels between pairs of transmit and receive antennas are independent of each other. Independence of channels means that the receiver has more than one independent copy of the transmitted signal. This phenomenon is known as diversity. All signals are transmitted simultaneously from all elements and the receiver solves a linear equation system to demodulate the message. Since the receiver detects the same signal several times at different positions in space it is important that at least one position be not in a "deep" fade.

In a wireless system signal power fluctuates permanently, this meaning that the signal power drops significantly for short time intervals; the channel is said to be in fade. Antenna diversity is used to mitigate fading and to improve link quality. The receive antenna collects independent faded copies of the same signal and combines these signals so that the resultant signal shows considerably reduced amplitude fading.

In most scattering environments, antenna diversity is an effective and widely applied technique for reducing the ill-effects of multipath fading. The classical approach is to use a multi-element antenna with channels fully utilized by each piece of transmitted information. The maximum diversity available to a space-time system is $N_t N_r$, which is the total number of channels between the transmitter and the receiver. When adding new antennas to a system, the receiver can use the extra channels to improve the probability of correct recovery of the true transmitted signal. The other channels can be viewed as redundancy or backup in case other channels fail. For example, suppose we have one transmit and two receive antennas and that one of the channels goes into a deep fade and becomes useless.

In this case the other channel may still be able to recover the data; but if both channels fail simultaneously we arrive at an unpleasant situation, comparable to the event of a single channel failure. This is demonstrated by the following

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property of independent events [5]:

P("channel 1 fails" and "channel 2 fails")
= P("channel 1 fails")P("channel 2 fails")

This way the error probability is reduced. The traditional diversity techniques use multiple antennas at the receiver, but the problem is that the receive antennas have to be sufficiently separated by some distance interval, so that the signals received at each antenna undergo independent fade. It is very costly and inconvenient to have more than one antenna in the remote unit. For this reason, transmit diversity schemes appear to be a more attractive solution.

III. SPACE TIME CODING

Transmit diversity is an effective technique for achieving spatial diversity in fading channels with an antenna array at the transmitter. In the design and analysis of such schemes it is generally assumed that the channel is static across two consecutive symbol transmission periods (one space-time codeword). The technique proposed by Alamouti is an algorithm for improving the signal quality at the receiver and is based on the use of two transmit antennas. The good thing is that the obtained diversity gain is the same as maximal ratio combining with two receive antennas. The new scheme may be easily generalized to multiple transmit and receive antennas. The beauty in Alamouti code is the fact that the error performance, data rate and capacity of the wireless system are simultaneously improved. The transmit diversity scheme may also be adopted to increase the range or the coverage area of wireless system, meaning it is effective in all of the applications where system capacity is limited by multipath fading.

A key concept in smart antennas is that of beamforming by which the instantaneous SNR can be increased through focusing the energy into desired directions, in either transmitter or receiver. To estimate the response of each antenna element for a given desired signal and possibly for interference signals, the elements may be combined with weights chosen as a function of each element response. After that, one can maximize the average desired signal level or minimize the level of other components whether noise or co-channel interference.

A. Maximum ratio receive combining

In figure 1 a baseband model of a two branch maximum ratio receive combining (MRRC) is depicted. At a given time instant, a signal $s_0 \in \{-1,1\}$ is propagated from the transmitter. The baseband representation of the complex channel from transmit antenna zero to receive antenna zero is denoted by h_0 and corresponding channel for receive antenna one is denoted by h_1, where:

$$h_0 = \alpha_0 e^{j\theta_0} \quad h_1 = \alpha_1 e^{j\theta_1} \qquad (1)$$

After adding the noise introduced by the channel the resulting signals can be written as:

$$r_0 = s_0 h_0 + n_0 \quad r_1 = s_0 h_1 + n_1 \qquad (2)$$

where n_0 and n_1 are independent complex noise. It is assumed that n_0 and n_1 are Gaussian distributed.

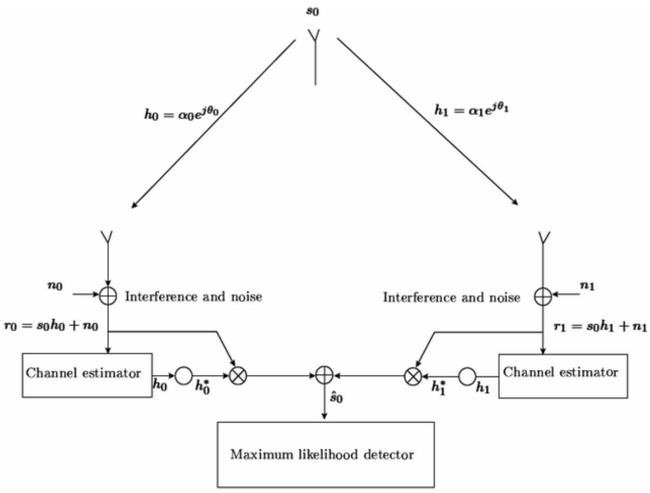

Fig. 1 Maximum ratio receive combining

To minimize the noise for these received signals a maximum likelihood decision algorithm is employed, which finds the signal s_i that is at the minimum Euclidean distance from the received sequence:

$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) \le d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \qquad (3)$$

where $d^2(x, y)$ is the squared Euclidian distance between two signals x and y, and is calculated by the following formula:

$$d^2(x, y) = (x - y)(x^* - y^*) \qquad (4)$$

and the * denotes the complex conjugate.

The combining scheme for the MRRC system depicted in figure 1 is as follows:

$$\hat{s}_0 = h_0^* r_0 + h_1^* r_1 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1 \qquad (5)$$

The cophasing and weighting factors discussed in previous section are represented is in equation (5) by h_0^* for path (antenna) zero, and h_1^* for path (antenna) one, respectively.

Using equations (3), (4), and (5) and the fact the PSK signals have an equal energy constellation, the decision rule can be simplified to:

$$d^2(\hat{s}_0, s_i) \le d^2(\hat{s}_0, s_k), \quad \forall i \ne k \qquad (1) \quad (6)$$

where $\hat{s}_0$ is the received signal and s_i and s_k are the symbols between which to choose.

B. Transmit diversity scheme proposed by Alamouti

In [4] Alamouti developed a simple scheme (displayed in figure 2) to achieve transmit diversity for an array of two transmit antennas. With this algorithm two symbols, s_0 and s_1 are simultaneously send at every given time instant from antenna zero and antenna one, respectively. At the next symbol period, s_0^* is transmitted from antenna one and $-s_1^*$ is transmitted from antenna zero.

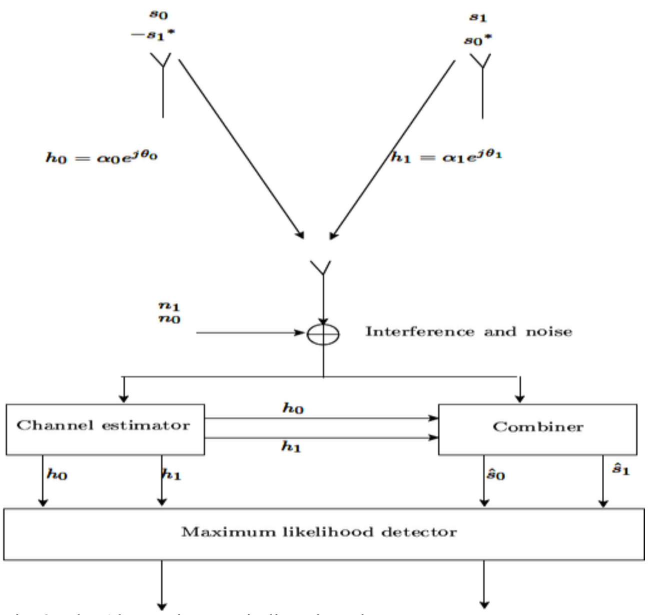

Fig. 2 The Alamouti transmit diversity scheme

The encoding and mapping of this scheme can be summarized in Table I.

TABLE I: THE ENCODING AND MAPPING FOR THE TWO-RAY TRANSMIT DIVERSITY SCHEME

Time	Antenna 0	Antenna 1
t	s_0	s_1
t+T	$-s_1^*$	s_0^*

The transmit sequence from antenna zero and antenna one denoted by s^0 and s^1, respectively, and are encoded in both space and time domains:

$$s^0 = \begin{bmatrix} s_0 & -s_1^* \end{bmatrix}$$
$$s^1 = \begin{bmatrix} s_1 & s_0^* \end{bmatrix} \tag{7}$$

The inner product of s^0 and s^1 is equal to zero. This confirms the orthogonality of the Alamouti space-time scheme.

The fading coefficients denoted by $h_0(t)$ and $h_1(t)$ are assumed constant over two consecutive symbol transmission periods and may be defined as:

$$h_0(t) = h_0(t+T) = \alpha_0 e^{j\theta_0}$$
$$h_1(t) = h_1(t+T) = \alpha_1 e^{j\theta_1} \tag{8}$$

where $h_0(t)$ and $h_1(t)$ are the fading coefficients from the first and the second antennas, respectively, to the receive antenna at time t, α_i and θ_i, i=0,1, are the amplitude gain and the phase shift, respectively. T is the symbol period. Further, the received signal can be written as:

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \tag{9}$$

where the additive white Gaussian noise samples at time t and $t+T$ are represented as in MRRC by the independent complex variables n_0 and n_1, with zero mean and power spectral density $N_0/2$. The received signals r_0 and r_1 can be expressed in vector-matrix notation as:

$$\begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix}}_{H} \begin{bmatrix} s_0 \\ s_1^* \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix} \tag{10}$$

or with obvious notation:

$$r = Hs + n \tag{11}$$

In most space-time codes it is always assumed that the receiver has a perfect knowledge of the channel coefficients, which in this case are h_0 and h_1. This implies that the Alamouti space-time decoder will use these coefficients as the channel state information (CSI).

The task of the combiner in figure 2 is to extract the original signal from the received signal, which is a combination of the signals sent from the transmitter antennas. Since the signal is encoded in space and time according to the Alamouti algorithm the combiner will need the signal at time t and $t+T$. This combiner is different compared to one in the MRRC, and builds the following two combined signals that are sent to the maximum likelihood (ML) detector:

$$\begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} h_0^* & h_1 \\ h_1^* & -h_0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \begin{bmatrix} h_0^* r_0 + h_1 r_1^* \\ h_1^* r_0 - h_0 r_1^* \end{bmatrix} \tag{12}$$

(7)

The maximum likelihood (ML) decoder chooses a pair of signals $(\hat{s}_0, \hat{s}_1)$ from the signal constellation to minimize the following decision metric over all possible values of $\hat{s}_0$ and $\hat{s}_1$:

$$d^2(r_0, h_0\hat{s}_0 + h_1\hat{s}_1) + d^2(r_1, -h_0\hat{s}_1^* + h_1\hat{s}_0^*) \tag{13}$$

For phase-shift keying (PSK) signals the decision rule simplifies to:

$$d^2(\hat{s}_0, s_i) \le d^2(\hat{s}_0, s_k) \quad \forall i \ne k \qquad (14)$$
$$d^2(\hat{s}_1, s_i) \le d^2(\hat{s}_1, s_k) \quad \forall i \ne k$$

Substituting r_0 and r_1 from equation (9) into equation (12) the combined signals can be written as:

$$\hat{s}_0 = (\alpha_0^2 + \alpha_1^2)s_0 + h_0^* n_0 + h_1 n_1^* \qquad (15)$$
$$\hat{s}_1 = (\alpha_0^2 + \alpha_1^2)s_1 - h_0 n_1^* + h_1^* n$$

The combined signals in equation (15) are equivalent to those of the two branch MRRC, equation (5). Unlike MRRC, Alamouti presents a phase rotation of the noise but this will not influence the performance. Thus the diversity gain is the same as for MRRC.

The maximum likelihood decoding rule in equation (13) can be split using equation (15) into two independent expressions which minimize the decision metric for detecting s_0 and s_1 :

$$s_0 = \arg \min_{(\hat{s}_0, \hat{s}_1) \in C} (|h_0|^2 + |h_1|^2 - 1)|s_0|^2 + d^2(\hat{s}_0, s_0) \qquad (16)$$
$$s_1 = \arg \min_{(\hat{s}_0, \hat{s}_1) \in C} (|h_0|^2 + |h_1|^2 - 1)|s_1|^2 + d^2(\hat{s}_1, s_1)$$

IV. CHANNEL MODEL AND ASSUMPTIONS

Consider a transmission sequence $\{s_0, s_1, s_2 ... s_n\}$. In normal transmission s_0 is sent in the first time slot, s_1 – in the second time slot, and so on. Employing Alamouti scheme these symbols are encoded into groups of two. In the first time slot, the first and second antenna send s_0 and s_1, respectively; in second time slot, the above antennas send $-s_1^*$ and s_0^*, respectively; in the third time slot the antennas send s_2 and s_3, respectively; in the fourth time slot, the antennas send $-s_3^*$ and s_2^*, respectively, and so on. Even if the symbols are grouped into groups of two, we still use two time slots to send two symbols. Hence, there is no change in data rate. Other assumptions are that the receiver has one antenna and the channel is flat fading and the amplitude of fading is Rayleigh distributed. The channel experience between each transmit to the receive antenna is randomly varying in time. However the fading is assumed to be slow enough to be constant across two time slots. This way the channel can be modeled by a complex multiplicative distortion $h(t)$ which can be written as:

$$h(t) = h = \alpha\, e^{j\theta} \qquad (17)$$

Besides this distortion the additive white Gaussian noise is also added to the transmitted signal. On the receive antenna, the noise n has the Gaussian probability density function: $p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(n-\mu)^2}{2\sigma^2}}$ with $\mu = 0$ and $\sigma^2 = N_0 / 2$

It is also assumed that the receiver has perfect knowledge of the channel coefficients.

V. TECHNIQUE FOR RECEIVER DESIGN

In this section zero-forcing (ZF) method for detecting the Alamouti space-time code is described, when the channel is randomly varying in time. Employing a linear receiver means to decouple the detection of the two symbols by inverting the effect of channel and to detect each of the symbols separately. This way, the zero-forcing receiver nulls out the effect of the other user (interferer), while demodulating the symbol of one user.

VI. PERFORMANCE OF ALAMOUTI SCHEME

In a one-tap channel with coefficients h_0 and h_1 the receiver vector is :

$$r = \begin{bmatrix} s_0 h_0 + s_1 h_1 \\ s_0^* h_1 - s_1^* h_0 \end{bmatrix} + noise \qquad (18)$$

It is well known this construction is orthogonal and the decoding matrix is given by:

$$H = \begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix} \qquad (19)$$

To solve for $\begin{bmatrix} s_0 \\ s_1 \end{bmatrix}$ we need to find the inverse of H.

Assuming perfect channel estimation, $\hat{H} = H$ and neglecting the noise term in (18), the estimate of the transmitted symbol is obtained as:

$$\begin{bmatrix} \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + (H^H H)^{-1} H^H \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix} \qquad (20)$$

Compared with the estimated symbol following equalization in MRC the above equation is identical. Since the estimate of the transmitted symbol with Alamouti STBC scheme is identical to that obtained from MRC the BER should be same as that for MRC[4]. Still it is a small catch.

In figure 3 the Alamouti scheme BER vs Eb/No performance with coherent BPSK modulation is plotted. From the simulation results we can see that BER of Alamouti case has a roughly 3dB poorer performance than MRC scheme and this is because the energy radiated from two antennas is twice of that used from a single antenna in the MRC.

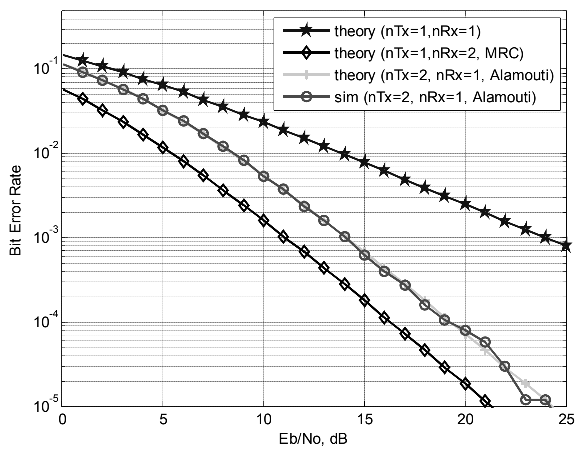

Fig. 3 BER vs Eb/No for BPSK modulation with Alamouti STBC(Rayleigh channel)

Figure 4 shows BER performance vs Eb/No of Alamouti scheme on a flat Rayleigh fading channel with QPSK modulation. From both figures it can be easily seen that the BER for the Alamouti scheme using BPSK modulation is better than when QPSK modulation is used. This performance is due the fact that higher order modulation schemes carry more bits per symbol than lower order modulation schemes. In this case, QPSK scheme modulates two bits per symbol and BPSK scheme modulates one bit per symbol.

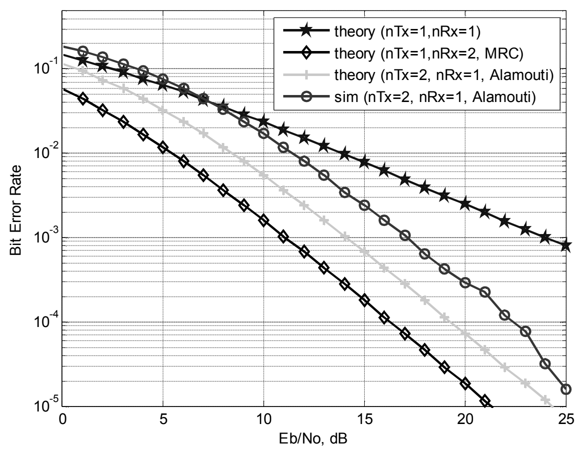

Fig. 4 BER vs Eb/No for QPSK modulation with Alamouti STBC(Rayleigh channel)

VII. CONCLUSIONS

In this paper was studied the performance of Alamouti scheme with perfect channel knowledge at the receiver. The encoding, decoding and the zero-forcing detection method of the Alamouti code were also explored. BPSK and QPSK modulation scheme were employed with one type of implementation (two transmit antenna and one receive antenna).

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