# Spectral Analysis of Three Miller-like Codes

Nicolae Dumitru ALEXANDRU
*"Gheorghe Asachi" University of Iaşi*
*Bd.Carol I No.11, RO-700506 Iaşi, Romania*
*nalex@etti.tuiasi.ro*

***Abstract* — There is a variety of codes that derive from the delay modulation or Miller code. Unlike Miller code, some are D.C.-constrained. They are also RLL (*Run-Length Limited*). A thorough spectral analysis of three DC-free Miller-like codes was performed. The derived coding factor formulae and the p.s.d. representations confirm the D.C.-free properties of this code.**

***Index Terms* — Codes, Digital filters, Markov processes, Spectral analysis**

## I. INTRODUCTION

There is a variety of codes that derive from the delay modulation or Miller code [1-5]. Unlike Miller code, some are D.C.-constrained. They are also RLL (*Run-Length Limited*) codes. Generally speaking, a RLL code is described by the tuple (*d, k, C*), where

*d* - minimum runlength

*k* - maximum runlength

*C* - maximum instantaneous value of the accumulated charge.

The parameters *d* and *k* represent the lower bound and the upper bound of the number of consecutive zeros between two adjacent ones (or transitions), respectively.

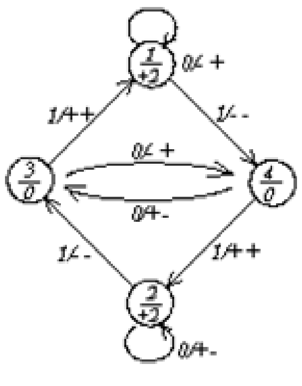

Figure 1 FSTD of Ferreira (0,3,3) code

In digital magnetic signal recording the data are stored as flux transitions and this implies a NRZ-M or differential encoding.

If the transition takes place at the mid-point of the bit interval, the code is known as NRZI (*Non Return to Zero Inverted*). Here a bit '*1*' represents a transition and '*0*' represents no transition in the binary level output of the coder.

So, there are two possible meanings of the tuple (*d, k, C*), depending on the presence or absence of differential encoding. In NRZI notation, the values of *d* and *k* are diminished by *–1*.

Miller code is also known under the names of delay modulation, *DM*-code, *High Density Digital Recording*, HDDRII or MFM (*Modified Frequency Modulation*) code.

The Miller code was one of the first RLL codes to be used in magnetic data recording.

It can be considered as a particular 1B2B code, as each input data bit is mapped into two output symbols.

In DM coding a mark is represented by a transition at the midpoint of the symbol period, while a space is represented by a transition at the end of the symbol period, excepting the single spaces or the last zeros in a string of spaces, which are coded as no transition.

There is a small D.C. component that is introduced by the sequences of the type 101, which have non-zero RDS and introduce intervals without ttransitions of length 2T, T being the bit duration.

In order to make the code D.C.-free or improve its synchronization features, some constraints were imposed and several new codes derived from Miller codes were devised. The DC-free are known as Miller squared or $M^2$, Howells Woodman Miller and Ferreira code. Jordan and Radev codes are not D.C –free but have better properties.

## II. FERREIRA CODE

The Ferreira code [1] is a DC-free RLL code described as (0,3,3) with a minimum Hamming distance $d_{\min}$ equal to 4.

$$C(f,p)=\frac{8p(1-p)(a+b\cos x-c\cos 2x-2(1-2p)\cos 3x}{e-f\cdot\cos 2x-(2-8p+12p^2-8p^3)\cos 4x+(1-2p)^2\cos 6x}$$

$$-\frac{d\cos 4x+2(1-2p)\cos 5x+p(1-2p)\cos 6x)\sin^2 x/2}{e-f\cdot\cos 2x-(2-8p+12p^2-8p^3)\cos 4x+(1-2p)^2\cos 6x} \qquad (1)$$

$$a=6-18p+28p^2-20p^3+8p^4 \qquad b=4-8p+8p^2$$

$$c=2-9p+12p^2-8p^3 \qquad d=4-10p+6p^2-4p^3$$

$$e=2-8p+16p^2-16p^3+8p^4 \qquad f=1-4p+8p^2-8p^3$$

The Ferreira code (0, 3, 3) was introduced by Ferreira in 1983. The finite state transition diagram (FSTD) associated to this code is represented in figure 1.

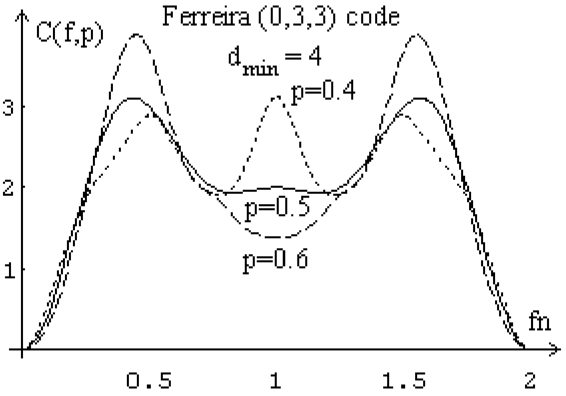


Figure 2 Coding factors of Ferreira (0, 3, 3) code

The patterns $++$ and $--$ increase or diminish the charge with 2 units, while the patterns $+-$ and $-+$ keep the charge unchanged. So, loops are possible only if the code words $+-$ and $-+$ are involved.

The coding factor was obtained as shown in eq.(1), where $x = \pi f T$ .

It is represented in figure 2 for three values of the probability p.

Decreasing the probability *p* makes the patterns $+-$ and $-+$ predominant and the code becomes more similar to bi-phase L, which has maximum energy at $fn = 1$ .

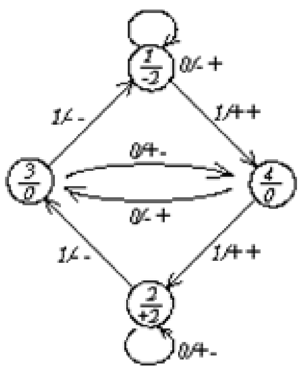

Figure 3 FSTD of Ferreira (0,5,5) code

Here $fn$ is the normalized frequency with respect to the bit rate.

An increase of probability *p* results in more energy being transferred at low frequency, as the patterns $++$ and $--$ become predominant.

The finite state transition diagram (FSTD) represented in figure 1 can be modified in order to alter the coding factor, as shown in figure 3.

The code keeps it D.C.-free character but becomes a (0, 5, 5) code with $d_{\min}$ equal to 2.

Its coding factor was determined as illustrated in eq.(2) and is represented in figure 4 for three values of the probability *p*.

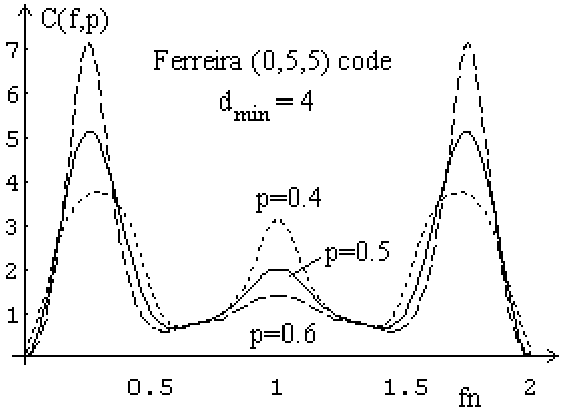


Figure 4 Coding factors of (0,5,5) code

A comparison of the coding factors of (0, 3, 3) and (0, 5, 5) codes is illustrated in figure 5.

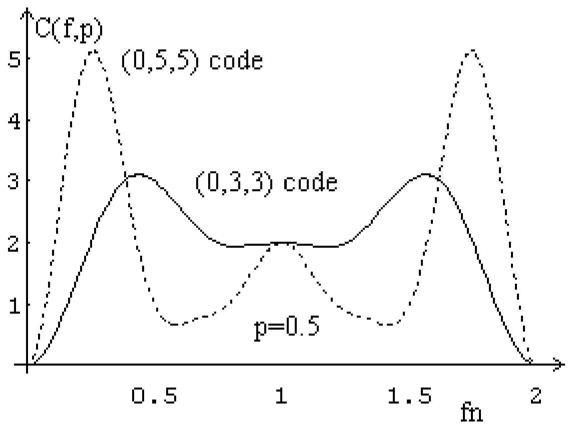


Figure 5 A Comparison of p.s.d. for (0,5,5) and (0,3,3) codes

The (0, 5, 5) code has more energy in the low-frequency area, as its DSV (*Digital Sum Variation*) is bigger.

To generate a D.C.-free code the encoder must be a FRDS (*Finite Running Digital Sum*) automaton. This results in certain rigid relationships between the individual states and also in the interconnection of states.

As each state owns a certain charge attribute, when the encoder is in that state the disparity or accumulated charge can take only the value of the charge attribute. It was assumed that the encoder starts from a zero charge state.

One should consider the case of binary codes. Suppose that the encoder maps *m* information bits into *n* code bits (*mBnB* code).

If *n* is even, then only states with even charge attributes can exist.

If *n* = 2, which corresponds to many usual codes currently used (CMI, bi-phase, Hedeman, Miller, etc.), the interconnection of states can be done only for states having charge attributes that differ either by zero or two units.

$$C(f,p) = \frac{8p(1-p)(7-5p+2p^2+(10-4p)\cos x+(6-2p)\cos 2x+2\cos 3x+p\cos 4x)\sin^2 x/2}{1-2p+2p^2-(1-2p)\cos 4x} \qquad (2)$$

Searching for a D.C.-free code is made possible and also easier by observing several rules:

1. Look for symmetry and make use of it.
2. Assign a suitable set of waveform patterns *O* to data bits *0* and *1*. For the non-zero waveforms there should be dual isomorphic waveform patterns in the set, each expecting to compensate the charge of the other. This requires that the total number of assigned waveforms should be even.

## I. JORDAN CODE

In Jordan code [2] a binary '1' is represented either by a transition at the mid-point of the bit interval, or by transitions at the beginning and at the end of the bit interval.

This happens in such a way that, when the distance between two successive transitions is one bit interval, the second transition is associated to a binary '*1*' in the input data.

A binary '*0*' is represented by a transition that takes place either at the beginning or at the end of the bit interval.

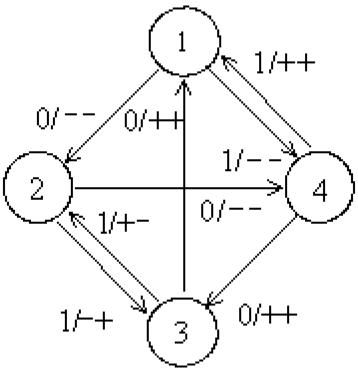


Figure 6 FSTD of Jordan code

So, a string composed of two zeros determines the appearance of a interval of length *2T* without transitions (the same level *L* or *H*), if preceded either by a bit'*1*' coded by transitions at the beginning and at the end of the bit interval, or by a '*0*' preceded by a bit'*1*', coded by a transition at the mid-point of the bit interval.

The Jordan code contains mainly low frequency components when coding repetitive strings composed of marks ('*1*') or spaces ('*0*'), in opposition with Miller code that exhibits the same property for the input sequence composed of alternating mark and spaces (*0101…*).

The Jordan code is described by the state transition diagram in figure 6.

Its coding factor is given by eq.(3) where $x = \pi f T$ and is represented in figure 7 for three values of the probability *p* of a mark (bit "1") at the coder input.

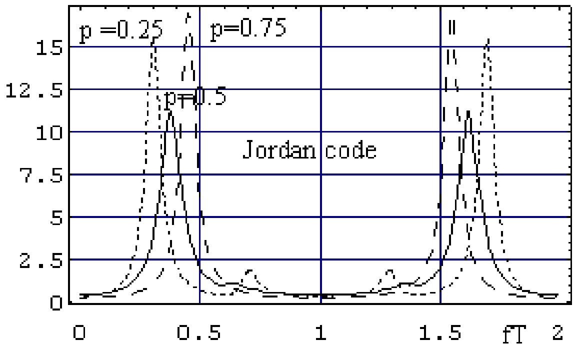


Figure 7 Coding factor of Jordan code

The coding factor of Jordan code is identical with that of DM code for $p = 0.5$, but differs from it for other values of the probability *p* of a mark.

## II. RADEV CODE

This code was introduced by Radev and Stoyanov [3] in 1984 and was denoted as new 1B2B code.

The Radev code preserves the narrow power spectral density of the DM code and provides 10 % level transitions more and also, a better balancing.

It offers also an error multiplication factor of 0.75 and error monitoring capabilities, which make it a better choice, as compared with Miller code.

This code was designed with the purpose of offering increased transition density for a string composed of alternating marks and spaces, which corresponds to an idle digital transmission channel.

This increases the timing content of the line coded signal and eases bit synchronization.

The Radev code is described by the state transition diagram in figure 8. Its coding factor is given by eq. (4).

$$C(f,p) = \frac{2(1-p)\,p\,(2-2p+2p^2+(1-p)\cdot 2\cos x+2p\cdot\cos 2x-(1-p)\cdot\cos 3x)}{1-2p+6p^2-4p^3+2p^4+4p(1-p+p^2)\cdot\cos 2x+(1-2p+2p^2)\cdot\cos 4x} \tag{3}$$

$$C(f,p) = \frac{4(1-p)\,p(a+4p(1-p)^2\cdot\cos x+b\cdot\cos 2x-c\cdot\cos 3x+2(1-5p+9p^2-8p^3+4p^4)\cdot\cos 4x)}{(3-2p+2p^2)(3-4p+8p^2-8p^3+4p^4+4(1-p+p^2)\cdot\cos 2x+2(1-2p+2p^2)\cdot\cos 4x)}$$

$$a = 7-19p+45p^2-60p^3+50p^4-24p^5+8p^6 \qquad c = 4(1-3p+4p^2-2p^3+p^4) \tag{4}$$

$$b = 4(1-2p+4p^2-4p^3+2p^4)$$

$$C(f,x) = \frac{4(0.5-x)(0.5+x)(a+b\cdot\cos y+c\cdot\cos 2y-d\cdot\cos 3y+e\cdot\cos 4y)}{(1.25+x^2)(0.5625+0.5x^2+x^4+(0.75+x^2)\cdot\cos 2y+2(0.25+x^2)\cdot\cos 4y)}$$

$$a = 0.46875+).9375x^2+2.5x^4+x^6 \qquad c = 0.3125+0.5x^2+x^4 \qquad e = 0.75x^2+x^4 \tag{5}$$

$$b = 0.15625+0.75x^2+0.5x^4 \qquad d = 0.15625+1.25x^2+0.5x^4$$

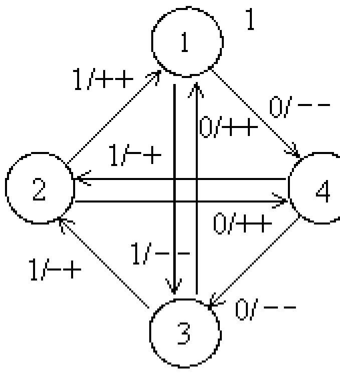


Figure 8 FSTD of Radev code

The p.s.d. of Radev code is identical with that of DM and Jordan code for $p = 0.5$, but differs from it for other values of probability *p*. Denoting $p = 0.5 - x$, so $x$ takes values from $-0.5$ to $+0.5$, we get eq.(5), where $y = \pi f T$.

As only even powers of *x* are found in the formula of the coding factor and the cosine functions possess even

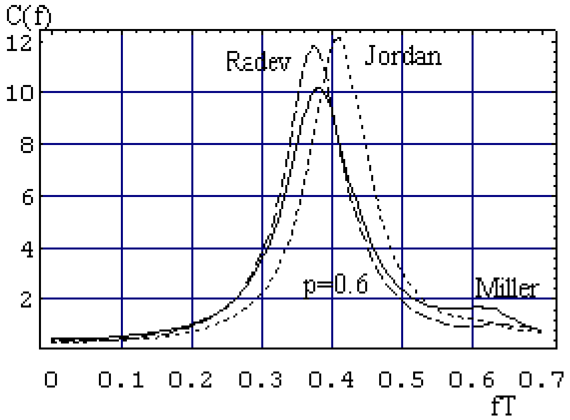


Figure 9 P.s.d. of Jordan, Miller and Radev codes

symmetry, we can conclude that the coding factor shows even symmetry defined by,

$$C(f, p - x) = C(f, p + x) \qquad -0.5 \le x \le 0.5 \qquad (6)$$

For comparison purposes figure 9 illustrates the coding factors of Miller, Jordan and Radev codes for $p = 0.6$.

The calculations of the coding factors were performed in MATHEMATICA, using the methods in [5], [6] and [8].

## III. CONCLUSION

A thorough spectral analysis of several DC-free Miller-like codes was performed, expressing their power spectral density both as a function of normalized frequency fn with respect to the bit rate and the probability *p* of a mark at the coder input.

The derived coding factor formulae and the p.s.d. representations confirm the D.C.-free properties of the Ferreira codes.

An even symmetry around $p = 0.5$ was evidenced for the first time in the formula of power spectral density of Radev code.

## REFERENCES


[1] Ferreira, H., "On DC free Magnetic Recording Codes Generated by Finite State Machines", IEEE Trans. Magnetics, vol. MAG-19, No 6, pp.2691-2693, Nov.1983

[2] Isailovic,"Realisation of a decoder for two MFM codes", Electronics Lett., vol.17, No 3, pp.117-118, Febr.1981

[3] P Radev, G. Stoianov, "New 1B2B line code for digital fibre optic transmission systems", Electronics Lett., vol. 20, No 8, pp.355-356, April 1984

[4] K.A. Schouhammer.Immink, Coding Techniques for Digital Recorders. Prentice Hall,1991

[5] N.D. Alexandru, and G. Morgenstern, *Digital Line Codes and Spectral Shaping*. Matrix ROM, Bucharest, 1998.

[6] N.D. Alexandru, and Dae Young Kim, *Spectral Shaping via Coding*, CERMI, Iaşi, 2003.

[7] J. Justesen, "Calculation Of Power Spectra for Block Coded Signals", IEEE Trans. Commun. Vol 49, No 3, pp.389-392, March 2001.

[8] N.D. Alexandru, and C. Chatellier, "Extending the Calculation of Correlation Function for 1I – nO Block Coded Signals", in *Proc. SCS2003*, Iaşi, Romania, pp. 629-632, 2003

[9] G. S. Poo, "Computer aids for code spectra calculations", *Proc. IEE*, vol. 128, pt. F, No. 5, pp. 323, 1981.

[10] G.L. Cariolaro, G.L. Pierobon, and G.P. Tronca, "Analysis of codes and spectra calculations", Internat. Jnal of Electronics, vol.55, No.1, pp.35-79, 1983