An Optimization of Gaussian UWB Pulses

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***Abstract* — UWB is a new interesting technology for wireless communications. It can replace traditionally carrier-based radio transmission by pulse-based transmission using ultrawide band frequency but at a very low energy. An important aspect of research in this domain is to find a pulse with an optimal shape, whose power spectral density respects and best fits emission limitation mask imposed by FCC.**

In this paper we review common used Gaussian pulses and its derivatives and the influence of shape factor, finding an optimal specific value for each derivative. Next, we search to obtain possible better pulse shapes as linear combinations of Gaussian derivatives. Older studies refer in one case to the same shape factor for all derivatives and in other case to higher factor for first derivative and smaller shape factors for subsequent derivatives.

Our new idea is to use Gaussian derivatives, each with its specific optimal shape factor and to use a "trial and error" algorithm to obtain a linear combination pulse with better performance.

***Index Terms* — Gaussian Monocycle; Shape factor; Trial and error; UWB; Wireless LAN**

I. INTRODUCTION

UWB (Ultra-Wide-Band) wireless transmission is based on impulse radio and can provide very high data rates over short distances. Its traditional application was in non-cooperative radar. UWB device by definition has a bandwith equal or greater than 20% of the center frequency or a bandwidth equal or more than 500MHz. Since FCC (Federal Communications Commission) authorized in 2002 the unlicensed use in the domain 3.1– 10. 6 GHz, UWB became very interesting for commercial development. High data rate UWB can enable wireless monitors, efficient transfer of data between computer in a Personal Wireless Area Network, from digital camcorders, transfer of files among cell phone and home multimedia devices, and other radio data communication over short distances [7].

Traditional wireless technologies use radio sine waves that provide "continual" transmission at a specific frequency. UWB radio system is associated with its impulse-based, carrier-free, time-domain radio system format that transmits very short UWB pulse signal trains (sub-nanosecond pulse width) without using any continuous sinusoidal wave carriers. For pulse-based UWB system, an extremely short pulse spreads its signal power over a very wide frequency spectrum (3.1-10.6 GHz) where the duty cycle of UWB pulse train can be as low as 1% [3]. The pulses are emitted in a rhythm unique to each transmitter. The receiver must know the transmitter's rhythm signature or pulse sequence to "know how to listen" for the data being transmitted [5].

The study of the shape of base pulse is fundamental because on it depend the performances of an UWB system, like efficiently use of permitted emission power, coexistence with other radio communication systems and a simple circuit implementation.

One of the fundamental challenges is to maximize the radiated energy of the pulse while the power spectral density complies with the spectral FCC mask.

The FCC regulated the use of UWB devices respecting emission limit values as depicted in Fig.1 [4]. Due to the extremely low emission level currently allowed, comparable with unintended emission (FCC Part 15), UWB systems are suited for short-range and indoor applications. UWB operates best over short distance of about 2-3 meters delivering data speeds of 480 Mbps. As distance increases, speed decreases, but at 10 meters still reaches or exceeds 100 Mbps.

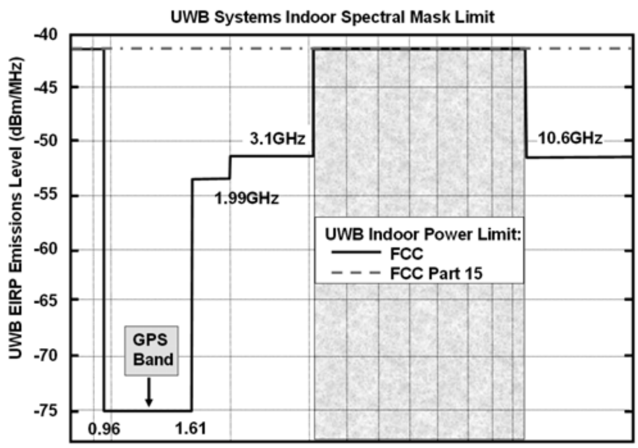

Figure 1 UWB indoor emission mask reglemented by FCC.

Since the ultra-short pulses are relatively easy to generate only with analog components, the Gaussian Monocycle and his derivatives are commonly used for UWB.

In this paper, we study the spectral properties of UWB pulses. Section II analyzes the power spectral density of the Gaussian monocycle and then in Section III one extends the study to higher-order derivatives of the Gaussian pulse and the influence of shape factor σ and his optimal values.

In Section IV, we study the pulses obtained by linear combinations of Gaussian derivatives and an algorithm to find optimal coefficients. We propose a new set of basic pulses, having a shape factor specific for each derivative.

The conclusion regarding the performance of the pulse combination we obtained is presented in Section V.

II. GAUSSIAN MONOCYCLE

By far the most popular pulse shapes discussed in UWB communication literature are the Gaussian pulse and its derivatives, as they are easy to describe and work with.

The basic Gaussian pulse is described analytically as:

$$x(t) = \frac{A}{\sqrt{2\pi}\,\sigma} e^{-\frac{t^2}{2\sigma^2}} \tag{1}$$

If a Gaussian pulse is transmitted, due to the derivative characteristics of the antenna, the output of the transmitter antenna can be modeled by the first derivative of the Gaussian pulse [1]. Therefore, the pulse radiated is given by the first derivative of the Gaussian pulse, called a monocycle :

$$x^{(1)}(t) = \frac{At}{\sqrt{2\pi}\,\sigma^3} e^{-\frac{t^2}{2\sigma^2}} \tag{2}$$

The waveform of the pulse is presented in Fig.2 for a value of shape factor σ = 0.1ns, and its corresponding spectrum in Fig. 3.

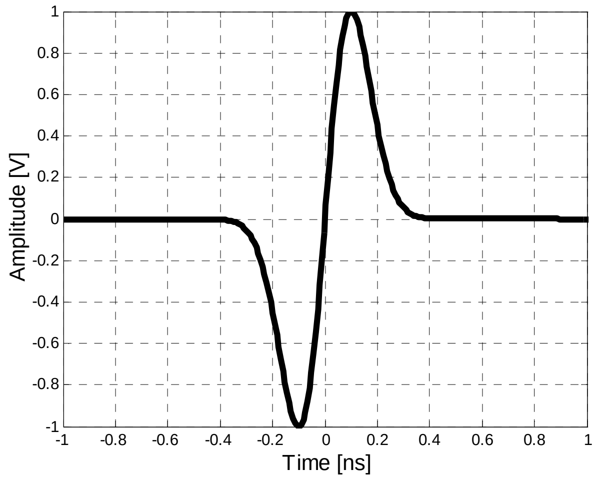

Figure 2 The waveform of a Gaussian monocycle

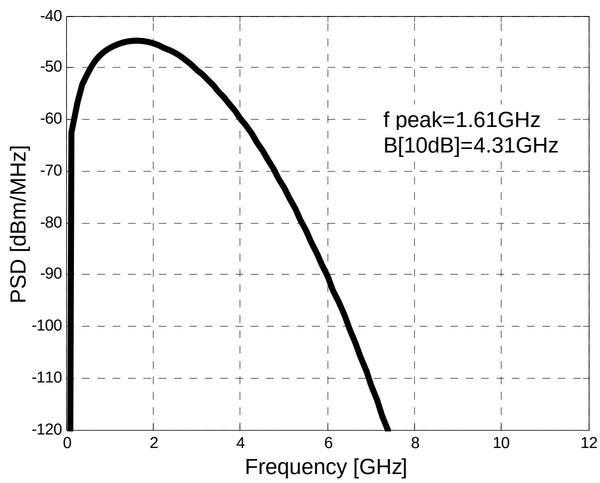

Figure 3 Power spectral density of Gaussian monocycle

Strictly speaking, the duration of the Gaussian pulse and all of its derivatives is infinite. Here, we define the pulse width, Tp, as the interval in which 99.99% of the energy of the pulse is contained. Using this definition, it can be shown that Tp ≈ 7σ for the first derivative of the Gaussian pulse.[2]

The possibility for tuning PSD spectrum in order to respect and fit the mask FCC is the choice of the shape factor σ. We observe that decreasing the value of σ in time domain the duration of pulse Tp is shorter, and leads in frequency domain the spectrum to migrate to higher frequencies.

For example, when value of σ is 0.12 ns, Tp=0.84 ns and frequency at maxim PSD is f_peak=1.24 GHz (continuous curve in Fig.4); when σ decreases at 0.080 ns, Tp become shorter, 0.56 ns and f_peak = 2GHz, higher (dashed curve) ; for σ=0.04 ns, Tp=0.28ns and f_peak moves to higher frequencies, f_peak=4GHz (dotted curve).

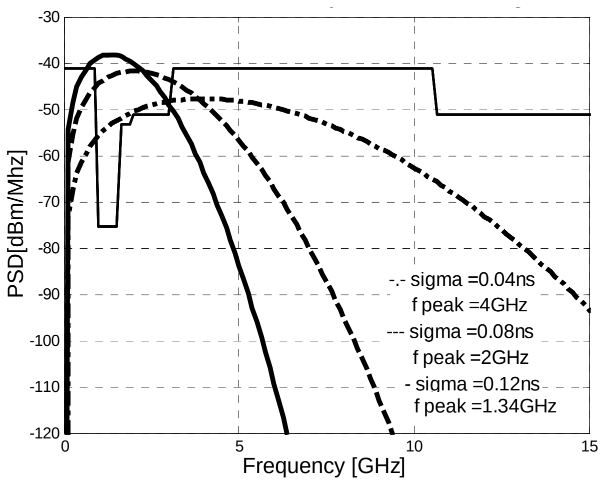

Figure 4 PSD of Gaussian monocycle for three values of σ

As one can see, it is clear that the PSD of the first derivative pulse does not meet the FCC requirements, no matter what value of the pulse width is used [9].

Therefore, a new pulse shape must be found that satisfies the FCC emission requirements. One possibility is to shift the center frequency and adjust the bandwidth, so that the requirements are met.

This could be done by modulating the monocycle with a sinusoid to shift the center frequency and by varying the values of σ. For example, for a pulse width Tp = 0.3 ns, by shifting the center frequency of the monocycles by 3 GHz, the PSD will fall completely within the spectral mask.

Impulse UWB, however, is a carrierless system; modulation will increase the cost and complexity. Therefore, alternative approaches are required for obtaining a pulse shape which satisfies the FCC mask [2].

In the time domain, the high-order derivatives of the Gaussian pulse resemble sinusoids modulated by a Gaussian pulse-shaped envelope. As the order of the derivative increases, the number of zero crossings in time also increases; more zero crossings in the same pulse width correspond to a higher “carrier” frequency sinusoid modulated by an equivalent Gaussian envelope. A Gaussian monocycle has a single zero crossing. Further derivatives yield additional zero crossings, one additional zero crossing for each additional derivative [9].

These observations lead to considering higher-order derivatives of the Gaussian pulse as candidates for UWB transmission . Specifically, by choosing the order of the derivative and a suitable pulse width, we can find a pulse that satisfies the FCC’s mask [2].

III. HIGHER ORDER DERIVATIVES

We investigate in the sequel the pulses obtained as derivatives of the basic Gaussian pulse.

The equation of n-th derivative pulse is given by:

$$x^{(n)}(t) = \frac{d^n}{dt^n}\left(\frac{A}{\sqrt{2\pi}\,\sigma} e^{-\frac{t^2}{2\sigma^2}}\right) \tag{3}$$

In time domain, we observe that the duration of pulse remains the same for various high-order derivatives, but one can consider Tp = 10σ [2] for all pulses. The waveforms illustrating several of these pulses and their PSD are presented in Fig.5 and 6, respectively.

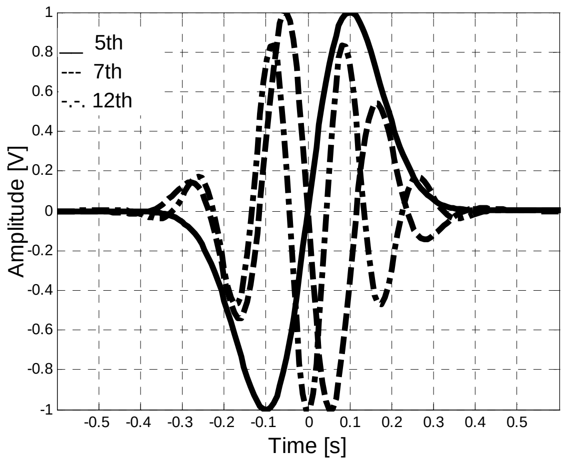

Figure 5 Waveforms for three derivatives of Gaussian pulse (σ=0.1ns)

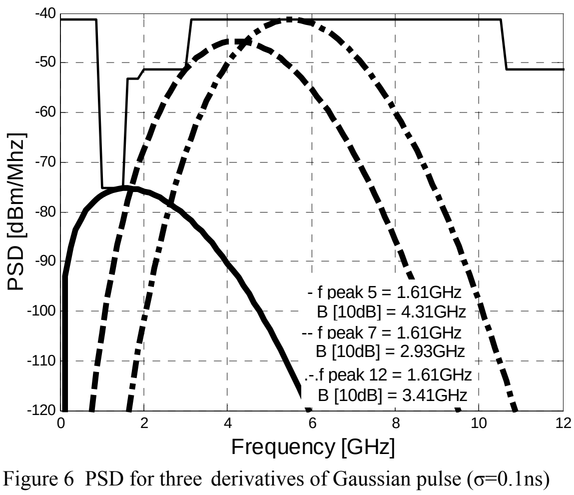

Figure 6 PSD for three derivatives of Gaussian pulse (σ=0.1ns)

Interesting, in frequency domain the Fourier transforms of those pulses have a relatively simple expression, and their spectrum has a amplitude given by:

$$\left|X^{(n)}(f)\right| = A(2\pi f)^n e^{-\frac{(2\pi f\sigma)^2}{2}} \tag{4}$$

So, one can study the spectrum by easily computing the frequency peak and the bandwidth. The frequency peak of spectrum is given by eq.(5)

$$\left(\left|X^{(n)}(f)\right|\right)' = 0 \quad \Rightarrow f_M = \frac{\sqrt{n}}{2\pi\sigma} \tag{5}$$

For every derivative, we chose a particular value for σ in order to obtain a pulse that matches the FCC's PSD mask as closely as possible.

For example, Fig.7 illustrates the results of simulation for the 5th derivative of the Gaussian pulse for several values of σ. The plotted curve with a thicker line is the result for the optimal value σ_{5opt} = 0.051 ns.

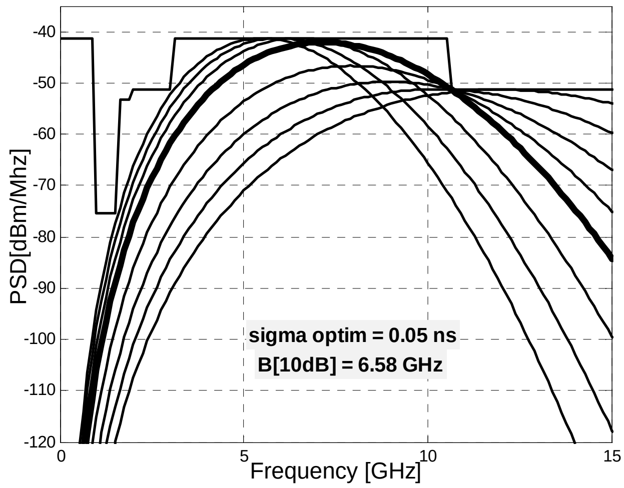

Figure 7 Optimization of σ for the 5th derivative pulse

In Table I, we summarize the values of the optimal parameter σ_n, the peak emission frequency, and the 10dB bandwidth obtained for first 15 order derivatives of Gaussian pulses.

TABLE I. OPTIMAL VALUES
FOR DERIVATIVES GAUSSIAN PULSE

n-order	σ_{nopt} [ns]	f_M [Ghz]	B_{10dB} [Ghz]
1	0.033	4.79	7.50
2	0.039	5.78	7.50
3	0.044	6.34	7.40
4	0.047	6.72	7.07
5	0.051	7.01	6.64
6	0.053	7.23	6.19
7	0.057	7.42	5.59
8	0.060	7.57	5.67
9	0.062	7.70	5.48
10	0.064	7.81	5.24
11	0.067	7.90	5.08
12	0.069	8.01	4.94
13	0.071	8.10	4.79
14	0.073	8.18	4.66
15	0.075	8.25	4.54

These results show that the pulse width will be less than 1 nanosecond for all cases, and the 10dB bandwidth is 4.5 GHz or greater; peak frequency increases with square root of order of derivatives.The maximum PSD can be controlled by changing the value of the amplitude A of the pulse [8].

Note that the derivative operation could be implemented as highpass filtering; to transmit the fifth derivative over the air, the Gaussian pulse must be filtered to the fourth-order derivative [2].

IV. OPTIMAL COMBINATION OF PULSES

In order to obtain UWB pulse with a better performance, we investigated linear combinations of Gaussian derivatives pulse. Let us consider a base pulses composed of first 15 derivatives of Gaussian pulse, each with an individual shape factor.

The combination pulse has the expression given by eq.(6).

$$x(t) = \sum_{n=1}^{15} c_n \cdot x^{(n)}(\sigma_n, t) \tag{6}$$

The problem is to find the optimal set of coefficients S={c_n },so that the resulting combination pulse respects and best fits FCC requirements [1]. We propose a computer-based method by means of starting with random sets of coefficients and testing with a 'trial-and-error' procedure described as follows:

Step 1. Initialize the random number generator.

Step 2. Generate a random set of coefficients S.

Step 3.Check if the PSD of the linear combination obtained with coefficients and base pulses satisfies the emission limits.

Step 4. If the emission limits are not met, go to step 2 and generate another combination.

Step 5. If the emission mask are met and this is the first set verifying the limits, initiate the optimal set BS = S

Step 6. If the emission mask is met and already exists an optimal set, compare actual valid set S with optimal set BS. If S have a better fit of a mask, i.e. the sum of PSD (expressed in mW/Hz) at all frequencies is greater, redefine BS=S.

Step 7. Repeat this procedure going to step 1 for some number of "trial" cycles, in order to obtain a new possible improved pulse.

Step 8. After that number of independent random searches (because in step 1 the random generator is reinitialized), algorithm stops and the current BS is the optimal found.

If one runs this algorithm for a sufficiently big number of "trials", the results converge and one obtains a best result possible. Our results are obtained by running the algorithm with a 1000 number of independent "trials".

We run this algorithm for three cases, as a function of choice of the set of shape factors σ_n of each based pulse for combination.

A. The case of the same shape factor σ for all derivatives.

In this case we consider the pulses derivatives having same shape factor σ. The result for a value of σ_n = 0.2ns is presented in Fig.8.

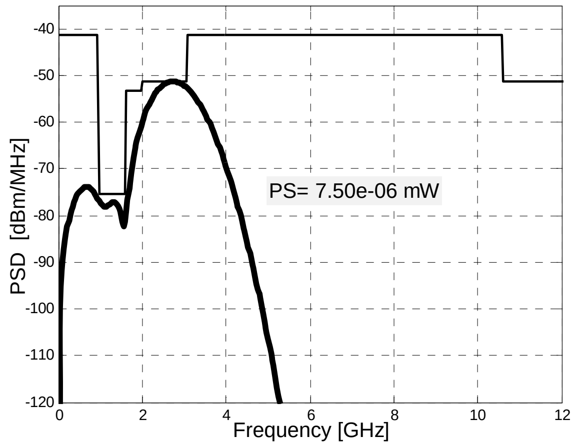

Figure 8 PSD of optimal combination for same σ_n=0.2ns

As we observe, FCC limits are respected at all frequencies, but the fit of the mask is not better, except about 4 GHz, therefore outside this band power is not efficiently used .

The compliance with mask guaranteed by the trial-and-error procedure is obtained at the price of a reduced efficiency[1].

B. The case of different shape factors.

Improved performance can be achieved by adopting different values for component derivative pulses.

Consider a second set of a values characterized by a higher value of σ (0.42 ns) for the first derivative and smaller values (0.08 ns) for the higher derivatives [1].

The new values improve the performance of the trial-and-error procedure, leading to a PSD that is quite close to the target for frequencies up to about 8 GHz, as is depicted in Fig.9.

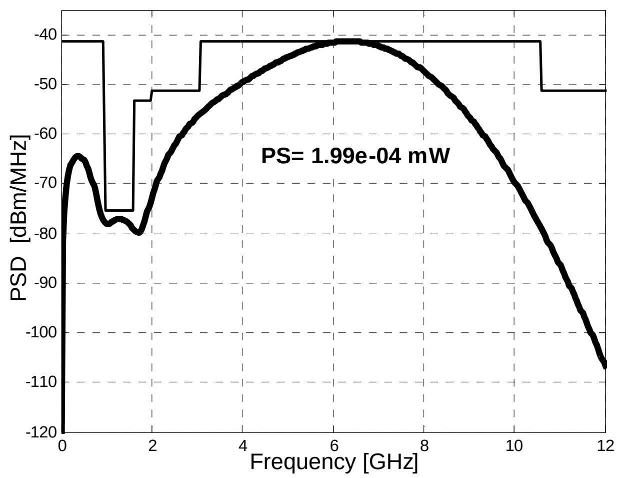

Figure 9 PSD of combination σ1=0.42ns and σ2-15=0.08ns

This PSD achieves a better approximation of the mask at higher frequencies than the continuous one, thanks to the larger bandwidth of higher derivatives.

Note that the selection of a relatively high a for the first derivative improves efficiency in power utilization at low frequencies.

An upper bound for a is given, however, by waveform duration, as determined by the chip time [1].

C. The case of optimized shape factors

Regarding the discussion in Section 2, we propose the idea to consider as base pulses for the combination of Gaussian derivatives, pulses having its specific optimized shape factor σ_{nopt} (cf. Table 1).

The PSD for those optimized pulses is depicted below in Fig.10.

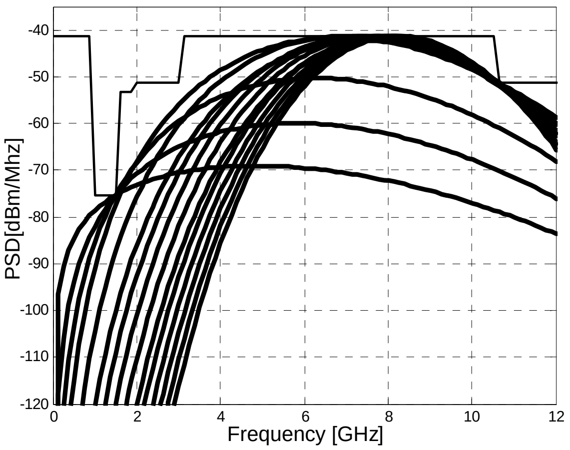

Figure 10 PSD of σ-optimal Gaussian derivatives pulses

With those based pulses, we consider a combination pulse having the expression determined by eq.(7).

$$x(t) = \sum_{n=1}^{15} c_n \cdot x^{(n)}(\sigma_{nopt}, t) \qquad (7)$$

Applying optimization algorithm will result an optimal pulse combination with PSD presented in Fig.11.

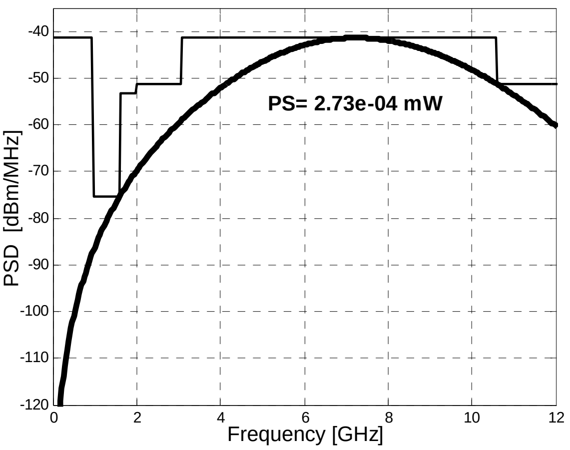

Figure 11 Optimal Pulse Combination

As we see, this combination pulse has very good properties in fitting the mask and therefore using maximum allowed emission power.

For an objective comparison and correct performance evaluation of this pulse, in each figure is displayed the parameter used in algorithm "trial and error", PS, what is "Spectral Power" obtained summarizing numerical values of PSD (expressed in mW/Hz), at all sampled frequencies .

The value obtained here is the best, PS $=2.73\cdot10^{-4}$ mW. In the case of using the same values for σ, the obtained value is poor PS$=7.5\cdot10^{-6}$ mW and in case of higher σ for first derivative and smaller for higher order derivatives, PS is better, PS$=1.99\cdot10^{-4}$mW.

V. CONCLUSION

UWB technology is new and subject to more improvements. Signal pulses with duration of the order of ns are easy to generate using an analogue circuit. Gaussian monocycle and his derivatives is a commonly used waveform for pulses in UWB.

It is easy to generate, but does not have enough good performance. One of the problems is to obtain a PSD that better fits limited emission mask. Higher order derivatives have been studied and proved to bring better performance, but not enough.

For indoor application the fifth-order derivative is currently a choice for implementation.

Now, we try an algorithm to obtain a pulse from a linear combination of first 15 derivatives of the Gaussian pulse. Using this method with same shape factor for all derivatives, and next an old implementation with small values for monocycle and high values for higher derivatives leads to UWB pulses with PSD that respect and show improved fitting of the mask, but not enough good above 4, respectively 7 GHz.

We proposed the idea to use as base pulses for combination the Gaussian derivatives with σ values individually optimized by the criterion to obtain maximum bandwidth. With these pulses, the optimal combination obtained by a "trial and error" algorithm leads to a pulse with better performance that respects and better fit the mask. The diagram illustrates than theses pulses have a PSD well approximating the mask at all frequencies, therefore they efficiently use the available bandwidth and power.

This result from the diagram is argued by the computed parameter PS (Spectral Power), as the sum of values of power spectral density, the combination pulse having in this case the best value.

Numerical results and diagrams are obtained by means of a simulation program written by us in MATLAB. For already discussed pulses in technical literature we obtain the same values and diagrams as for those found in the literature. This proves the accuracy of our simulation program.

The results for the new proposed pulses are interesting and subject to further theoretical analysis and practical implementations.

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