Analysis of Coupled Oscillators through a Series RLC Network

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***Abstract* —Voltage controlled oscillators are present in almost every digital communication system. Thus, coupled microwave oscillators are the subject of intense research activities. Recently, they are used to control the phase in microwave antenna arrays as an alternative to electronic beam steering methods. Researches are made so that a particular phase shift can be obtained by choosing the free-running frequencies of the oscillators in the array. In this paper, we have analyzed, in different ways, in time domain and also in frequency domain, the phase shift between output voltages of each pair of coupled oscillators and also, the behavior of multiple coupled oscillators**

***Index Terms* — phase shift, synchronization, oscillator, microwave, VCO.**

I. INTRODUCTION

Voltage controlled oscillators are an important component in almost every digital communication system. Recently microwave oscillators are used to control the phase in microwave antenna arrays as an alternative to electronic beam steering methods. This new approach is based on the synchronization property of coupled oscillators. All oscillators' networks must satisfy two conditions. Elementary oscillators must be able to synchronize stably at a common frequency and the phase shift must have a constant value. The first requirement may be accomplished by coupling the oscillators (injection-locking phenomenon) [2, 8]. The second requirement, controlling the phase shift and assuring an appropriate value, is very difficult to put into practice. It's necessary to know the influence of various parameters like the free running frequencies or the coupling force. If the free running frequencies have certain values oscillators synchronize spontaneously with a phase relationship related to the original distribution of these free running frequencies [4].Thus, coupled microwave oscillators are the subject of intense research activities. A voltage controlled oscillator (VC)) is a circuit that produces an oscillatory output. The frequency of the output signal depends on the level of an input voltage signal supplied to the VCO. The range of the frequency is chosen according to our purpose, but mainly VCOs produce high frequency signals. In this paper, we have analyzed, in different ways, the phase shift between output voltages of each oscillator and also, the behavior of multiple coupled oscillators.

II. TWO COUPLED VAN DER POL OSCILLATORS

The base of this analysis is represented by VCO's that have different free-running frequencies and are able to lock at a common frequency thanks to coupling circuits. Two oscillators coupled through a resonant network can be synchronizing at the same frequency. But the synchronization is highly dependent on the coupling network. Coupled microwave oscillators have been modeled as simple Van der Pol oscillators [6]. This model provides satisfactory results for many applications. Also the simplicity of the equations is very helpful.

In figure 1 are represented two oscillators coupled through a series resonant circuit. These oscillators are considered identical, except for their free-running frequencies. Thus, this work aims to analyze coupling oscillators and determine the phase shift, through different methods and compare our results with those obtained in [5]. The two voltage-controlled nonlinear resistors in figure 1 are identically, and their characteristics are approximated by piecewise linear continuous curves as in figure 2.

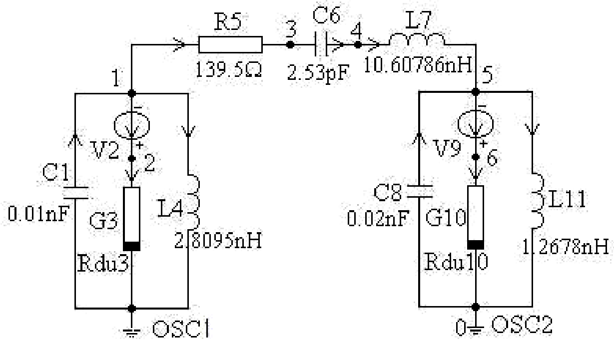

Fig.1. Two parallel resonant circuits coupled through a series RLC network.

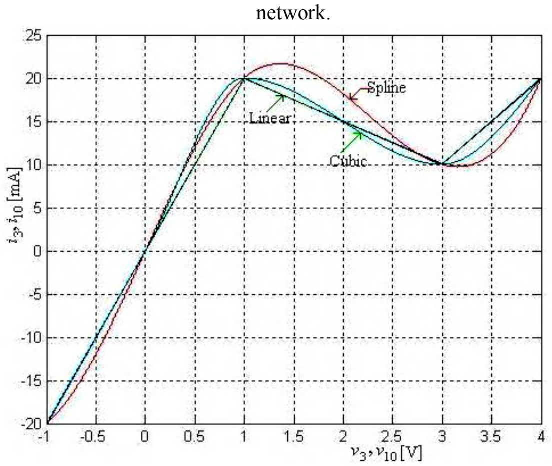

Fig.2. Piecewise linear approximations of the nonlinear characteristics for the nonlinear resistors.

The phase shift between the two oscillators can be computed by three procedures: simulation in time domain with Spice, using the complex representation, using the equations proposed by R. York [5].

Simulation with Spice

The resonant frequencies for the two oscillators are:

$$f_{01} = \frac{1}{2\pi\sqrt{C_1 L_4}} = 950\,\text{MHz},\ f_{02} = \frac{1}{2\pi\sqrt{C_8 L_{11}}} = 1000\,\text{MHz} \tag{1}$$

Resonant frequency of the RLC coupling circuit is:

$$f_{0c} = \frac{1}{2\pi\sqrt{C_6 L_7}} = 972\ \text{MHz} \tag{2}$$

The circuit of figure 1 simulated with Spice, has lead to two sinusoidal waves at 982.209 MHz (Fig. 4), with amplitudes of: Vos1m = 1.4503 V, Vos2m=1.4375 (Fig. 3).

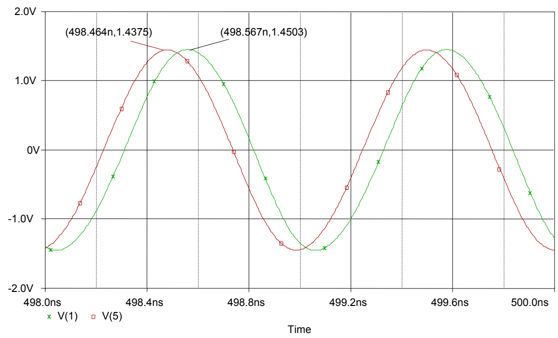

Fig.3. Waveforms present at the output of each oscillator.

The frequency characteristics for the output voltages corresponding to two oscillators are shown in figure 4. In order to compute the phase shift we can use the following relation:

$$\varphi_{2\_1} = f \cdot \left(t_{os2\_m} - t_{os1\_m}\right) \cdot 360 \tag{3}$$

Replacing the numeric values the result is:

$$\varphi_{2\_1} = -29.31° \tag{4}$$

Taking into account the amplitude of the output voltages, the ratio of these amplitudes has the value:

$$\frac{V_{0s1m}}{V_{os2m}} = \frac{1.4503}{1.4375} = 1.0089. \tag{5}$$

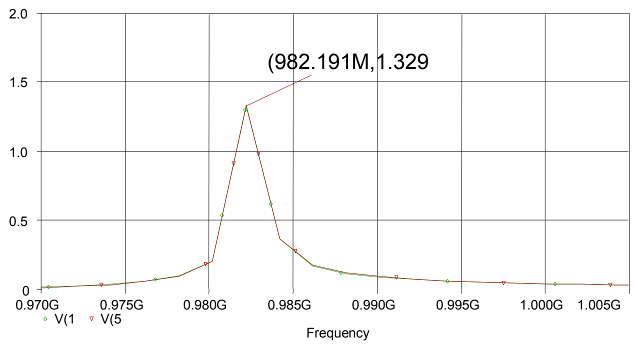

Fig.4. Synchronization frequency.

The voltages and the currents for the circuit in figure 1, in steady-state are sinusoidal and have the synchronization frequency f = 982.191 MHz.

Using complex representation:

In this case, the two nonlinear resistors can be substituted by two linear resistors with the same negative conductance G_0. Therefore, we can use to analyze this circuit the complex representation [7 - 9].

The complex admittance, in the sinusoidal behavior, corresponding to the coupled RLC circuit has the following expression:

$$\underline{Y}_c = \frac{\omega^2 C_6}{R_5 C_6 \omega + j\left(C_6 L_7 \omega^2 - 1\right)} \tag{6}$$

The total complex current of the first oscillator is:

$$\underline{I}_{o1} = j\omega C_1 \underline{U}_{o1} - G_0\left(\underline{U}_{o1} + 2.0\right) - \frac{j}{\omega L_4}\underline{U}_{o1} \tag{7}$$

where: G_0 is the differential conductance of the first voltage-controlled nonlinear resistor in the sinusoidal behavior, and the current through the second oscillator has the following expression:

$$\underline{I}_{o2} = j\omega C_8 \underline{U}_{o2} - G_0\left(\underline{U}_{o2} + 2.0\right) - \frac{j}{\omega L_{11}}\underline{U}_{o2} \tag{8}$$

The complex current $\underline{I}_c$ has the expression:

$$\underline{I}_c = \underline{Y}_c\left(\underline{U}_{o1} - \underline{U}_{o2}\right) \tag{9}$$

Solving the equations:

$$-\underline{I}_{o1} = \underline{I}_c = \underline{I}_{o2} \tag{10}$$

We obtain the following results:

$$\underline{V}_{os1} / \underline{V}_{os2} = 0.8424 - 0.4731j \Rightarrow$$

$$\begin{cases} \varphi_{2\_1} = \alpha_{v_{os1}} - \alpha_{v_{os2}} = \arg\left(\underline{V}_{os1} / \underline{V}_{os2}\right) = -29.38° \\ V_{os1} / V_{os2} = 0.966 \end{cases} \tag{11}$$

Equations proposed by R. York:

R. York proves that the ability of two oscillators to lock to a common frequency is affected by the following parameters [1, 5]:

$\lambda_0 = \frac{1}{G_0 R_c}$ the coupling constant, with G_0 being the first-order term of the Van Der Pol nonlinear conductance;

$\omega_a = \frac{G_0}{2 \cdot C_i}$ - is the bandwidth of the oscillator i;

$\omega_{ac} = \frac{G_0}{2 \cdot C_c}$ - represents the bandwidth of the coupling circuit;

$\omega_c = \frac{R_c}{2 \cdot L_c}$ - is the bandwidth of the unloaded coupling circuit.

In the following, the oscillators free-running frequencies ω_{01} and ω_{02}, and the synchronization frequency of the system ω, are referred to the frequency of the coupling circuit, of the oscillators, ω_{0c}, using the substitutions below:

$$\Delta\omega_{01} = \omega_{01} - \omega_{0c},\ \Delta\omega_{02} = \omega_{02} - \omega_{0c},$$

$$\Delta\omega_c = \omega - \omega_{0c} \tag{12}$$

The formula proposed by R. York to compute the phase shift between the two oscillators has the following expression:

$$\varphi_{2\_1} = \arctan\left(\frac{\dfrac{\Delta\omega_{02}}{\omega_a} + \dfrac{\Delta\omega_c}{\omega_{ac}}\left(1 - A_2^2 - \dfrac{\omega_{ac}}{\omega_a}\right)}{1 - \lambda_0 - A_2^2 - \dfrac{\Delta\omega_c \Delta\omega_{02}}{\omega_{ac}\omega_a} + \dfrac{\Delta\omega_c{}^2}{\omega_{ac}\omega_a}} \right) \quad (13)$$

where A_2 is the magnitude of the output voltage corresponding to the second oscillator.

Using the numeric values of the circuit parameters corresponding to the circuit in figure 1, the result is:

$$\varphi_{2\_1} = -37.81° \quad (14)$$

III. FOUR PARALLEL RESONANT CIRCUITS COUPLED THROUGH A SERIES RLC NETWORK

Figure 5 represents four oscillators coupled through a series resonant circuit. These oscillators are considered identical, except for their free-running frequencies. The four voltage-controlled nonlinear resistors from figure 5 are identical, and their characteristics are approximated by piecewise linear continuous curves as in Fig. 9.

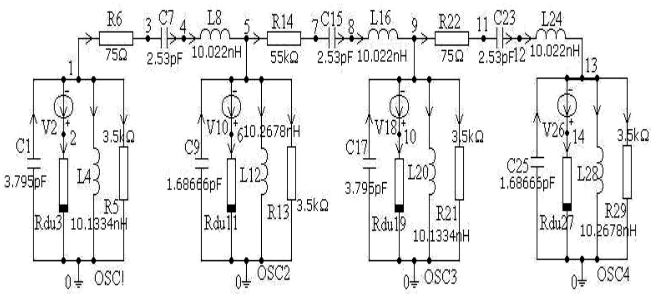

Fig.5. Four parallel resonant circuits coupled through a series RLC network.

Simulation with Spice

The coupled system in figure 5 was simulated with Spice and we obtained the four sinusoidal waves at 962.148 MHz and with amplitudes of: 657.879 mV, 691.56 mV, 658.604 mV, and 693.495 mV at the output of each oscillator (Fig. 6 and 7). The average phase shift between adjacent oscillators was found to be equal to 29°

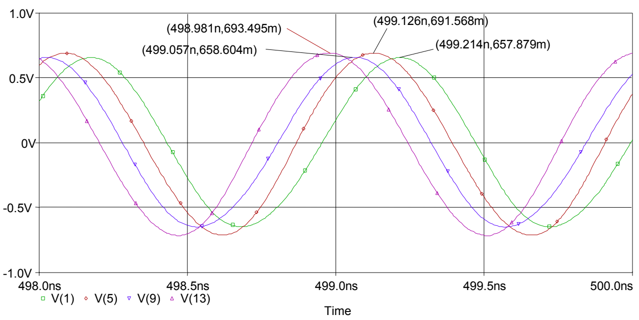

Fig.6. Waveforms present at the output of each oscillator.

The frequency characteristics for the output voltages corresponding to four oscillators are shown in Fig. 7.

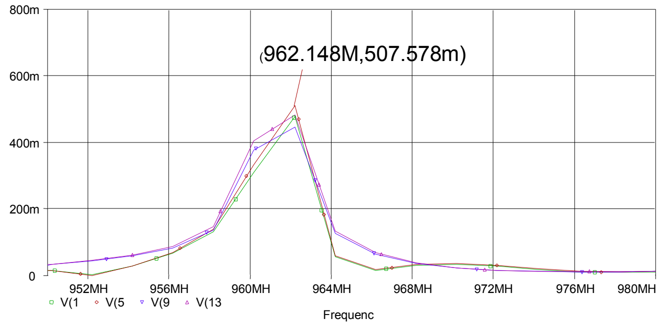

Fig.7. Synchronization frequency.

Using complex representation:

Due to the fact that self-sustained oscillation is possible only in nonlinear systems, in order to be able to analyze our circuit using complex representation we have substituted the nonlinear resistors by linear resistors that have the same negative conductance G_0. Therefore, we can apply to analyze this circuit the complex representation.

We have considered as independent variables the complex output oscillators' voltages. After solving the equations in sinusoidal behavior and processing our results, the medium value obtained for the phase shift is 28.74°.

Equations proposed by R. York:

R.York has resumed and generalized previous methods in order to analyze any number of coupled oscillators both narrow band and broadband. Analyzing our circuit with the equation used by R. York (13) and comparing with previous results, we conclude they are very similar. In this last case the medium phase-shift is 28.73°.

IV. SIX PARALLEL RESONANT CIRCUITS COUPLED THROUGH A SERIES RLC NETWORK

Figure 5 represents six oscillators coupled through a series resonant circuit. These oscillators are considered identical, except for their free-running frequencies. The six voltage-controlled nonlinear resistors in figure 8 are identically, and their characteristics are approximated by piecewise linear continuous curves as in figure 9.

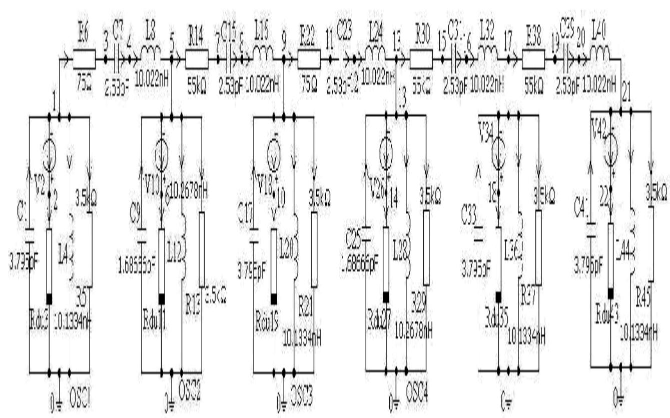

Fig.8. Six parallel resonant circuits coupled through a series RLC network.

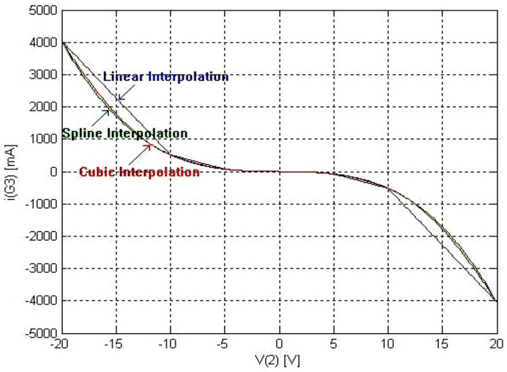

Fig.9. Piecewise linear approximations of the nonlinear characteristics for the nonlinear resistors.

The coupled system in figure 8 was simulated with Spice and we obtained the six sinusoidal waves at 962.097 MHz and with amplitudes of: 657.778 mV, 691.890 mV, 678.882 mV, 709.933 mV, 696.693 MV, and 736.491 mV at the output of each oscillator (Fig. 10 and 11). The average phase shift between adjacent oscillators was found to be equal to 29°.

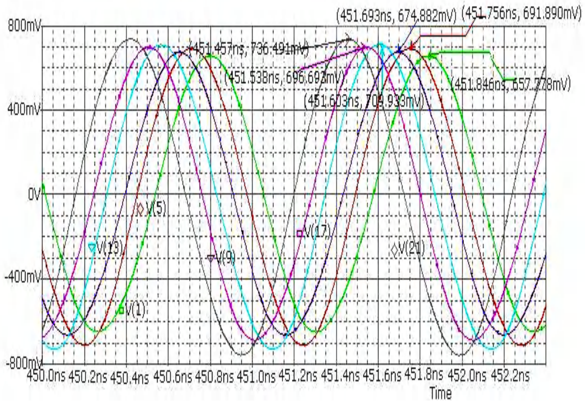

Fig.10. Waveforms present at the output of each oscillator in the case of six oscillators array.

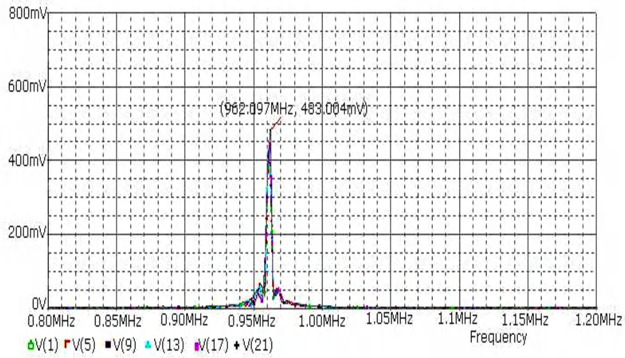

Fig.11. Frequency characteristics for the output voltages corresponding to six oscillators.

These results show that it is possible to adjust, with a high accuracy; the free-running frequencies required in achieve the desired phase shift in an n–element array and also, the validation of the results with the three methods is realized.

V. CONCLUSION

The way oscillators work and the phase shift are very important in orienting the radiation pattern, in a phased antenna array, in a certain direction. Researches are made so that a particular phase shift can be obtained by choosing the free-running frequencies of the oscillators in the array. But a big problem with autonomous circuits is our limited control of solution characteristics. This lack of control is determined by their nonlinear behavior and by their dependence of parameters values. In this paper different types of analysis were applied in order to compare the results obtained and also, for a better understanding of the influence the parameters have in the oscillators' synchronization. The main limit we encounter is that the analysis is made around the synchronization frequency. Thus, there is a risk of malfunction outside this region.

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