Tuning of the PID Controller to the System with Maximum Stability Degree using Genetic Algorithm

Irina Cojuhari, Ion Fiodorov, Bartolomeu Izvoreanu, Dumitru Moraru
Technical University of Moldova
Chisinau, Republic of Moldova
irina.cojuhari@ati.utm.md, ion.fiodorov@ati.utm.md, izvor@mail.utm.md, dumitru.moraru@ati.utm.md

Abstract—**In this paper is proposed a tuning algorithm of PID controller that offers the maximum stability degree of the control system, based on the genetic algorithm. The tuning algorithm was designed based on the maximum stability degree method with iterations, where the tuning parameters depend on maximum stability degree which is varied. Based on its values, it was proposed to implement genetic algorithm to find the tuning parameters. The maximum stability degree method permits to obtain the high stability and high performance of the system, but this method has some limitations in case when control object is described by the model of object with inertia low order. In this case to find the best tuning parameters was proposed to use the genetic algorithm. For efficacy analysis of the proposed algorithm, there are presented some case studies and practical applications.**

Keywords—PID controller; genetic algorithm; automatic control system; maximum stability degree method

I. Introduction

Proportional (P), Proportional Integral (PI) and Proportional Integral and Derivative (PID) controllers have been the heart of control engineering practice for seven decades [3], but the practice of the automation of the technological processes demonstrates that PI and PID controllers remain poorly understood and poorly tuned in many industrial applications. In case when controller is tuned incorrectly, it can lead to low performance of the automatic control system and as result low quality of the output final product. Nowadays, there are proposed many methods for tuning the P, PI and PID controllers:

- experimental methods, which start from the some simple assumptions and obtained values of the tuning parameters. One of these methods is the Ziegler-Nichols method, where the calculation of the tuning parameters of the PID controller are simple. However, this method provides the oscillating transient processes with a low damping rate [1-4];

- the graph-analytical methods offer to the designed system the satisfactory performance, but require a big volume of calculations and graphical representation [1-4];

- the optimization methods which can offer so good performance of the control systems and can be applied to the complex model objects, but these methods are required big volume of calculations.

Besides this, nowadays the artificial intelligence approaches such as evolutionary algorithms, fuzzy logic, neuronal network are widely used in case of optimization problem of tuning the PID controller [14-17].

In [5, 10], it is proposed the maximum stability degree method for tuning the typical controllers. This method permits to obtain the high performance and good robustness of the automatic control system. But this method in the classical version it is not applied in case of tuning the PID controller to the model of object with first and second order inertia, and it is applied the iterative procedure for finding the tuning parameters [11-12].

According to this, in this paper was proposed to tune the PID controller to the model of object with first and second order inertia using the maximum stability degree method with iterations, but the iterative procedure of finding the tuning parameters of the PID controller, it was proposed to be optimized based on the evolutionary approach and namely the genetic algorithm.

II. Tuning of the PID Controller by the Maximum Stability Degree Method with Iterations

One of the criteria, which is used for designing of the automatic control systems, is the maximum stability degree criterion [13]. This criterion supposes the maximum displacement in the complex half-plane of the nearest characteristic equation's roots of the designed system to the imaginary axe $Re\ p_i \leq 0$ (Fig. 1). This permits to obtain the reducing of the duration of the transient response and to ensure the high stability degree of the system.

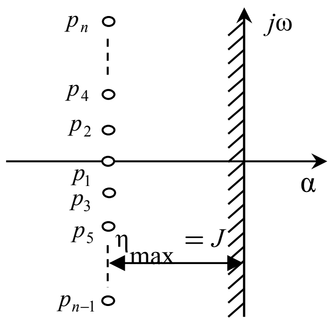

Fig.1. Placement of the characteristic equation's roots in the complex plane for the system with maximum stability degree.

Based on this criterion, it was proposed to tune PID controller to the two models of objects: with inertia first and with inertia second order. These models are widely used when it is need to describe the dynamic of some processes with model of object with low degree of inertia.

The PID control algorithm in the standard form is described by the following transfer function:

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \qquad (1)$$

where k_p, k_i, k_d – are the tuning parameters of the PID control algorithm.

It will be performed the tuning of the control algorithm (1) by the maximum stability degree method with iterations [10-12]. Bellow it is presented the analytical relations obtained in case of tuning the PID controller by the maximum stability degree method with iterations.

A. Tuning of the PID Controller to the Model of Object with First Order Inertia

The model of object with first order inertia is described by the following transfer function

$$H(s) = \frac{k}{(Ts+1)}, \qquad (2)$$

where k is the transfer coefficient of the model, T– time constant.

According to the maximum stability degree method the analytical expressions for calculations the tuning parameters of the PID controller are [11]:

$$k_p = \frac{1}{k}(2TJ-1) + 2k_d J; \qquad (3)$$

$$k_i = \frac{1}{k}(-TJ^2 + J) - k_d J^2 + k_p J; \qquad (4)$$

$$k_d = \frac{T}{k}. \qquad (5)$$

Further, based on the relations (3)-(5) can be calculated the tuning parameters of the PID algorithm.

B. Tuning of the PID Controller to the Model of Object with Second Order Inertia

The model of object with inertia second order is described by the following transfer function

$$H(s) = \frac{k}{(T_1 s+1)(T_2 s+1)} = \frac{k}{a_0 s^2 + a_1 s + a_2}, \qquad (6)$$

where k is the transfer coefficient of the model, T_1, T_2 – time constants, and $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$.

According to the maximum stability degree method the analytical expressions for calculations the tuning parameters of the PID controller are [12]:

$$k_p = \frac{1}{k}(-3a_0 J^2 + 2a_1 J - a_2) + 2k_d J = \frac{1}{k}(3a_0 J^2 - a_2); \qquad (7)$$

$$k_i = \frac{1}{k}(a_0 J^3 - a_1 J^2 + a_2 J) - k_d J^2 + k_p J; \qquad (8)$$

$$k_d = \frac{1}{k}(3a_0 J - a_1). \qquad (9)$$

Further, based on the relations (7)-(9) can be calculated the tuning parameters of the PID algorithm, and thus the synthesis problem can be solved.

The obtained analytical expressions (3)-(5) and (7)-(9) for calculations the tuning parameters of the PID controller are the function of known parametrs of control object and of the unknown value J - named the stability degree of the control system: $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$. In concordance with the maximum stability degree method with iterations, the value of the stability degree is varied $J \geq 0$ and there are obtained the dependencies $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$ for the PID controller. From these dependencies can be chosen the different values of the tuning parameters of the PID controller and by the computer simulation it is verified the performance of the control system.

In this case it is proposed that the procedure of finding the optimal tuning parameters to be expanded by using genetic algorithm and this will permit to find the tuning parameters according to the maximum stability degree of the system.

III. Genetic Algorithm

Evolutionary algorithms are search and optimization methods that are inspired by the biological principles as: natural selection and the genetics, where is evolved a population of candidate solutions to a target problem, and it is doing a search in the space of candidate solutions according to the fitness function [7].

Genetic algorithms is a particular type of evolutionary algorithms where an individual (candidate solution) consists not only of data (variables or constants), but also of operators applied to the individual's data as selection, crossover, and mutation and it is manipulate not just with one potential solution but with a collection of potential solutions, that are called in genetic algorithm the population, and the potential solution in the population is called chromosomes.

Genetic algorithms are used to solve difficult search, optimization, and machine-learning problems and it has been demonstrated that it is capable of locating high performance areas in complex domains without experiencing the difficulties associated with high dimensionality.

A genetic algorithm is typically initialized with a random population consisting of between 20-100 individuals. The objective function assigns to each individual a corresponding number called its fitness and it is used to provide a measure of how individuals have performed in the problem domain [7-9].

Genetic algorithm consists from the following steps:

Step 1. Random generation of the populations of n chromosomes.

Step 2. Evaluating of the success of each x chromosome using the fitness function $f(x)$.

Step 3. Creating a new population by applying the following evolutionary operations until the new population is complete:

- Selection - Selection of two parents chromosomes from a population according to the function $f(x)$.

- Crossover - Two individual agents combine to produce an offspring. The main objective of crossover is to explore new areas within the search space.

- Mutation - During mutation individual agents endure the small random changes of genes that lead to the generation of new individuals.

Step 4. The above-mentioned steps are repeated until the swarm converges to an optimal or sub-optimal solution [8].

IV. Study Case and Computer Simulation

It was proposed to tune the PID controller to the model of objects with first order and second order inertia.

A. Synthesis of PID Controllers to the Model of Object with First Order Inertia

It is considered, that the technological process is described by the transfer function with inertia first order

$$H(s) = \frac{k}{Ts+1} = \frac{2}{10s+1}. \quad (10)$$

According to the maximum stability degree method with iterations using relations (3)-(5), to the model of object (10) was tuned the PID controller and there were constructed the dependences $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$ (Fig. 2).

From Fig. 2 it can be observed, that tuning parameters of the PID controller depend just only of the maximum stability degree (MSD) and according to this fact it was implemented the genetic algorithm for finding the optimal tuning parameters of the PID controller. The algorithm was implemented in MATLAB and fitness function verify the steady-state error.

The obtained results of tuning the PID controller by the maximum stability degree method with iterations and maximum stability degree method with genetic algorithm implementation were compared with parametric optimization (PA) from MATLAB.

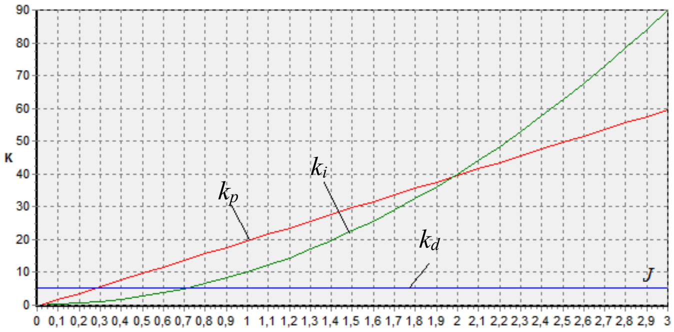

Fig. 2. Dependencies of the tuning parameters $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$ by degree of stability.

The values of the tuning parameters are presented in the Table I.

TABLE I. Tuning Parameters of the PID Controller

No.	Synthesis method	PID Controller		
		k_p	k_i	k_d
1	**MSD**	57.5	84.1	5
2	**PO**	8.905	2.21	0.416
3	**Genetic Algorithm**	408.68	4185.5	5

The simulation results of control system with control object (10) are presented in the Fig. 3. The numbering of the curves corresponds with the numbering of the methods from the Table I.

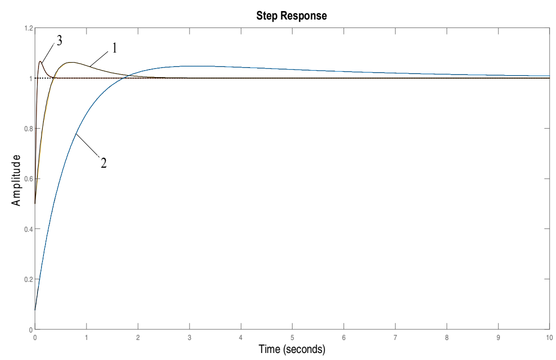

Fig. 3. The step responses of the control system with control object (10) and PID controller.

In the Table II are presented the performance of the automatic control system with PID controller tuned by the maximum stability degree method with iterations, maximum stability degree method with genetic algorithm implementation and parametrical optimization method. From this table it can be observed that the system with PID controller tuned by the maximum stability degree method with genetic algorithm implementation has the best performance.

In the Fig. 4 it is presented the distribution of poles and zeros of the closed loop system in the complex plan. According to this distribution in case of tuning the PID controller: by the maximum stability degree method with iterations was obtained stability degree η=2.9; by the parametrical optimization method was obtained the stability degree η=0.28; by the maximum stability degree method with genetic algorithm implementation was obtained the stability degree η=20.45. It can be concluded that the genetic algorithm permitted to find the tuning parameters that offer the highest stability degree of the system.

TABLE II. THE PERFORMANCE OF THE AUTOMATIC CONTROL SYSTEM

No.	$t_{settling}$, s	σ,%	t_{rise}, s
1	1.83	6.304	0.25
2	7.26	4.768	1.104
3	0.26	6.699	0.035

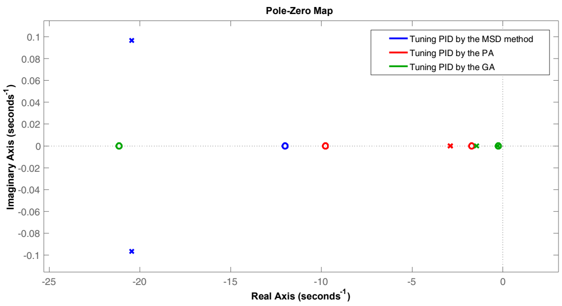

Fig. 4. Distribution of poles-zeros of the closed loop system.

B. Synthesis of PID Controllers to the Model of Object with Second Order Inertia

It is considered, that the technological process is described by the transfer function with second order inertia

$$H(s)=\frac{k}{a_0s^2+a_1s+a_2}=\frac{2}{s^2+7s+10}. \qquad (11)$$

According to the maximum stability degree method with iterations using relations (7)-(9) for the model of object (11) was tuned the PID controller and there were constructed the dependences $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ (Fig. 5).

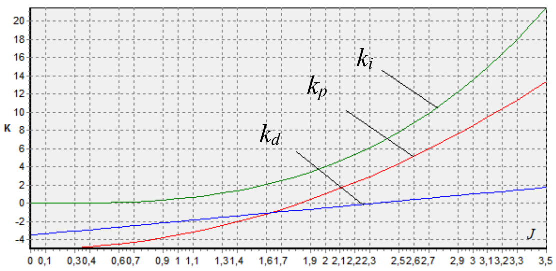

Fig. 5. Dependencies of the tuning parameters $k_p=f_p(J)$, $k_i=f(J)$, $k_d=f(J)$ by degree of stability.

Next, it was implemented in MATLAB the genetic algorithm for finding the tuning parameters of the PID controller, where the fitness function verified the steady-state error. The obtained results of tuning the PID controller by the maximum stability degree method with iterations and maximum stability degree method with genetic algorithm implementation were compared with parametrical optimization method from MATLAB. The values of the tuning parameters are presented in the Table III.

TABLE III. TUNING PARAMETERS OF THE PID CONTROLLER

No.	Synthesis method	PID Controller		
		k_p	k_i	k_d
1	**MSD**	14,4	23,3	1,9
2	**PO**	42,81	105,31	4,35
3	**Genetic Algorithm**	41,01	84,94	4,807

The simulation results of control system with control object (11) are presented in the Fig. 6. In the Fig. 7 it is presented the distribution of poles and zeros of the closed loop system in complex plan. The numbering of the curves corresponds with the numbering of the methods from the Table III.

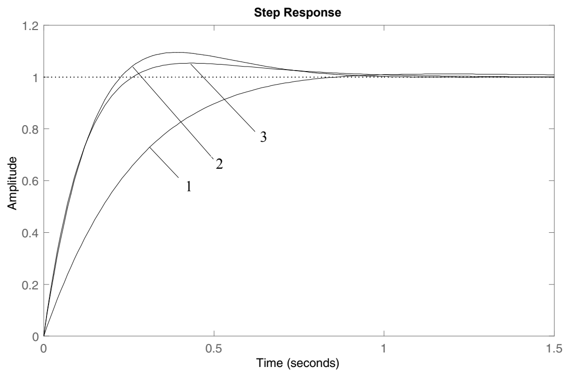

Fig. 6. The step responses of the control system with control object (11) and PID controller.

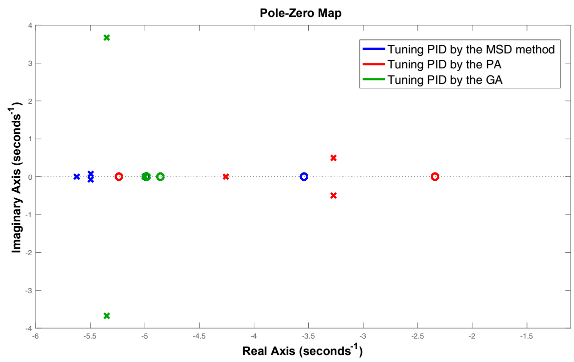

Fig. 7. Distribution of poles-zeros of the closed loop system.

According to this distribution in case of tuning the PID controller: by the maximum stability degree method with iterations was obtained stability degree η=3.2711; by the parametrical optimization method was obtained the stability degree η=5.005; by the maximum stability degree method with genetic algorithm implementation was obtained the stability degree η=5.495. In this case the maximum stability degree method with genetic algorithm implementation permitted to find the tuning parameters of the PID controller that offered the highest stability degree to the system.

In the Table IV are presented the performance of the automatic control system with PID controller tuned by the maximum stability degree method, genetic algorithm and parametric optimization.

TABLE IV. THE PERFORMANCE OF THE AUTOMATIC CONTROL SYSTEM

No.	$t_{settling}$, s	σ,%	t_{rise}, s
1	0.72	0	0.72
2	0.77	9.503	0.1629
3	0.79	5.331	0.1741

From the Table IV can be observed that the highest performances of the designed control system were obtained for the case of tuning the controller by the maximum stability degree method with iterations.

V. CONCLUSIONS

The approximation of the process with model of objects with first or second order inertia is widely used when it is necessary to describe some technological process. There are a lot of tuning methods of the PID controller to this model of objects. In this paper was proposed to tune the PID controller by the maximum stability degree method with iterations and it was proposed to improve the procedure of finding the tuning parameters by introducing genetic algorithm, where the fitness function verified the steady-state error.

Based on the theoretical calculations and computer simulations, it can be concluded:

- It was done the tuning of the PID controller to the model of object with first and second order inertia, using maximum stability degree method with iterations, maximum stability degree method with genetic algorithm implementation and parametric optimization method. The maximum stability degree method with genetic algorithm implementation showed the good results. In this case, the obtained performance of the designed automatic control system are higher in comparison with parametrical optimization method and the system has the highest stability degree.
- In case of tuning the PID controller by the maximum stability degree method with iterations were obtained good performance and high stability degree, but this method involved using the manual iterative procedure for finding these parameters.
- From these considerations, the genetic algorithm can be easily integrated in the maximum stability degree method for improving the procedure of finding the optimal tuning parameters of the typical controllers. The obtained results demonstrated that genetic algorithm permits to obtain high performance of the automatic control system with high stability degree.

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