

# Integration of Renewable Energy Sources using Generation Expansion Planning

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**Abstract** — Generation expansion planning (GEP) problem is required for maintaining the system reliability and ensuring demand supply. GEP is usually determined by the infrastructure aging and growth of demand. This paper describes a perspective of the independent system operator's (ISO) regarding the expansion of generation capacity. The model presented can be adapted by ISO for satisfying the needs of any other system that requires generation expansion. The planner is encouraging investors to build new generation units, by determining the optimal plan through solving the mathematical model that minimizes the investment costs and maximizing the social welfare.

**Keywords** — deterministic model; generation expansion planning; renewable energy sources.

## I. NOMENCLATURE

### Indices:

$b$  — buses;  
 $d$  — demands;  
 $h$  — hydro energy generation units;  
 $l$  — transmission lines;  
 $so$  — system operation conditions;  
 $t$  — thermal energy production units;

### Sets:

$d_b$  — demand supplied from bus  $b$ ;  
 $h_b$  — hydro energy production unit located at bus  $b$ ;  
 $r(l)$  — receiving node;  
 $s(l)$  — sending node;  
 $t_b$  — thermal energy production unit located at bus  $b$ ;  
 $wp_b$  — wind energy production unit possible to be built, located at bus  $b$ ;

### Parameters:

$B_l$  — susceptance of transmission line  $l$  [p.u.];  
 $c_h$  — cost of production of hydro unit  $h$  [€/MWh];  
 $c_{kt}$  — cost of production of thermal unit  $t$  [€/MWh];  
 $c_{wp}$  — cost of production of possible to be built wind unit  $wp$  [€/MWh];  
 $C_l$  — maximum capacity for transmission line  $l$  [MW];  
 $G_h$  — generation capacity of hydro unit  $h$  [MW];  
 $G_t$  — generation capacity of thermal unit  $t$  [MW];

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$IC_{wp}$  — investment cost for construction of new wind generating units [€/MW];

$IA_{wp}$  — annualized investment cost for new wind generating units [€/MW];

$L_d$  — consumption of demand  $d$  [MW];

$\sigma_{so}$  — weight of system operation condition  $so$  [h];

### Binary variables:

$u_{wp}^m$  — Binary variable that is 1 if capacity option  $m$  determines the capacity of the new generating unit  $wp$  and 0 otherwise;

### Continuous Variables:

$G_{wp}$  — generation capacity of new wind generation unit that is possible to be built  $wp$  [MW];

$P_h$  — power produced by hydro unit  $h$  [MW];

$P_t$  — power produced by thermal unit  $t$  [MW];

$P_{wp}$  — power produced by new wind generation unit that is possible to be built  $wp$  [MW];

$PF_l$  — power flow through transmission line  $l$  [MW];

$V_b$  — voltage angle at bus  $b$  [rad];

### Dual and Auxiliary Variables

$Z_{wp,m,so}^{AUX}$  — auxiliary variable;

$\lambda$  and  $\mu$  — dual variables.

## II. INTRODUCTION

The generation expansion planning (GEP) problem appears due to infrastructure aging and demand increase. After solving the mathematical model associated with the GEP problem, the planner should be able to take the best decision regarding the location and capacity of the new generating units that are possible to be built. For providing safety and reliability of the existing network, building new generating units must be considered. In order to encourage the investors to build new generation systems, the ISO is determining the optimal solution for building these units. The result will be the optimal location and capacity of these units and the aim of the problem described in this article is to minimize the investment costs and maximize the social welfare [1].

In [2-6], GEP mixed-integer linear programming (MILP) problems are carried on. Linear problems are proven to be

efficient and with a non-computational burden. Non-linear programming (NLP) problems are solvable in iterations and this is complicating the computational burden. In [7] is demonstrated that fast-response units can improve the reliability of the grid but the classic units are expensive and they produce emissions. In order to decrease the emissions and have fast-response units, GEP with wind generating units is carried on.

The European Union (EU) decided to decrease the greenhouse gas emissions by 20% by year 2020. In [8] a model with low-carbon economy with CO<sub>2</sub> emissions constraints is presented. Renewable energy sources (RES) are important to be considered for satisfying this change. In [9-12], RESs are considered for GEP problem, as are clean energy sources and are not polluting the environment.

In this paper, a MILP problem is proposed using a deterministic static approach. The initial model is a NLP problem that is linearized and reformulated as a MILP problem. In this paper, the new generating units that are possible to be built are wind power plants (WPP).

The article is structured as follows: section III presents the mathematical model of the considered GEP problem, section IV presents the considered case study. Section V reports the results obtained after solving the mathematical model, and finally section VI provides the conclusions regarding the results obtained.

### III. MATHEMATICAL MODEL

The approach that is chosen in this paper is a static one, thus the investment decision is taken once. The model is formulated as a single-stage model with the objective function of minimizing the investment and operational costs, and maximizing the social welfare. The planning horizon is equal to one year, the year chosen is at the end of the time period for which the problem is applied due to the fact that the demand is usually increasing over time, thus the last years are characterized by an increased level of consumption.

The uncertainties are ignored so the model is a deterministic one. In this case, the planner holds a good enough forecast of the operational condition of the system. Also, the losses are neglected in order to simplify the problem. It is important to mention that the deterministic static model described in this work is a DC power flow one.

The model contains products between continuous and binary variables, and these products are also reformulated and transformed into mixed-integer linear expressions that contain auxiliary variables, and a large enough positive constant M that in this paper is 300000. The resulted problem is a MILP problem that is solved with optimization software (branch-and-cut solver).

The objective function of the mathematical model applied to the analyzed grid is presented in (1):

$$\min \sum_{wp} IA_{wp} G_{wp} + \sum_{so} \sigma_{so} . \quad (1)$$

$$\cdot \left[ \sum_t c_t P_{t,so} + \sum_h c_h P_{h,so} + \sum_{wp} c_{wp} P_{wp,so} \right]$$

The constraints applied to the production capacity of the new WPPs are reformulated:

$$\begin{aligned} G_{wp} &= \sum_m u_{wp}^m P_{wp}^m, \forall wp \\ \sum_m u_{wp}^m &= 1, \forall wp \quad u_{wp}^m \in \{0,1\}, \forall wp, m \end{aligned} \quad (2)$$

The balance between production and consumption that must be maintained in every system operation condition is:

$$\begin{aligned} \sum_{t \in t_b} P_{t,so} + \sum_{h \in h_b} P_{h,so} + \sum_{wp \in wp_b} P_{wp,so} - \\ - \sum_{l|s(l)=b} PF_{l,so} + \sum_{l|r(l)=b} PF_{l,so} &= \sum_{d \in d_n} L_{d,so}, \forall b, so \end{aligned} \quad (3)$$

The power flow through the transmission lines is:

$$PF_{l,so} = B_l (V_{s(l),so} - V_{r(l),so}), \forall l, so \quad (4)$$

Expression (5) constraints the power flow to be limited by the capacity of the line:

$$-C_l^{max} \leq PF_{l,so} \leq C_l^{max}, \forall l, so \quad (5)$$

The power produced by each generator (existing and the possible ones) cannot be higher than their capacity:

$$\begin{aligned} 0 \leq P_{t,so} &\leq G_t, \forall t, so \\ 0 \leq P_{h,so} &\leq G_t, \forall t, so \\ 0 \leq P_{wp,so} &\leq G_{wp}, \forall wp, so \end{aligned} \quad (6)$$

The voltage angle must be between  $-\pi$  and  $\pi$ , while the voltage angle at the reference node is 0:

$$\begin{aligned} -\pi \leq V_{b,so} &\leq \pi, \forall b, so \\ V_h &= 0, b: ref \end{aligned} \quad (7)$$

Constraints (8) - (12) are the dual constraints of the dual problems:

$$\begin{aligned} c_t - \lambda_{b(t),so} + \mu_{t,so}^{G_t} &\geq 0, \forall t, so \\ c_h - \lambda_{b(h),so} + \mu_{h,so}^{G_h} &\geq 0, \forall h, so \\ c_{wp} - \lambda_{b(wp),so} + \mu_{wp,so}^{G_{wp}} &\geq 0, \forall wp, so \end{aligned} \quad (8)$$

The new dual constraints are represented by the following expressions:

$$\lambda_{s(l),so} - \lambda_{r(l),so} - \mu_{PF_l} + \mu_{c_l}^{max} - \mu_{c_l}^{min} = 0, \forall l, so \quad (9)$$

$$\sum_{l|s(l)=b} B_l \mu_{PF_l} - \sum_{l|r(l)=b} B_l \mu_{PF_l} + \mu_{b,so}^{Amax} - \mu_{b,so}^{Amin} = 0, \forall b | b:ref, \forall so \quad (10)$$

$$\sum_{l|s(l)=b} B_l \mu_{PF_l} - \sum_{l|r(l)=b} B_l \mu_{PF_l} + \mu_{b,so}^{A,ref} = 0, \forall b | b:ref, \forall so \quad (11)$$

The dual variables associated with the generators capacity constrains must be equal to 0 or positive:

$$\mu_{t,so}^{G_t} \geq 0, \forall t, so \quad \mu_{h,so}^{G_h} \geq 0, \forall h, so \quad (12)$$

Constraint (13) represents a strong duality equality which imposes that the market-clearing problem and its dual to have the same value of their objective functions, thus relation (15) result.

$$\mu_{wp,so}^{G_{wp}} \geq 0, \forall wp, so \quad (13)$$

The dual variables associated with the line capacity and the voltage angle constrains must be equal to 0 or positive, where  $A$  is the amortization rate:

$$\begin{aligned} \mu_{C_l}^{min}, \mu_{C_l}^{max} &\geq 0, \forall l \\ \mu_{b,so}^{Amin}, \mu_{b,so}^{Amax} &\geq 0, \forall b | b:ref, \forall so \end{aligned} \quad (14)$$

Constraint (15) represents the equality between the objective function of the market-clearing problem and its dual.

$$\begin{aligned} &\sum_t c_t P_{t,so} + \sum_h c_h P_{h,so} + \\ &+ \sum_{wp} c_{wp} P_{wp,so} = \sum_b \lambda_{b,so} \sum_{d \in d_n} L_{d,so} - \\ &- \sum_t \mu_{t,so}^{G_t} G_t - \sum_h \mu_{h,so}^{G_h} G_h - \\ &- \sum_{wp} \sum_m z_{wp,m,so}^{AUX} - \sum_l (\mu_{C_l}^{min} + \mu_{C_l}^{max}) C_l^{max} - \\ &- \sum_{b:ref} (\mu_{b,so}^{Amin} + \mu_{b,so}^{Amax}) \pi \end{aligned} \quad (15)$$

Expressions (16) resulted after reformulating the products between binary and continuous variables, thus  $z_{wp,m,so}^{AUX}$  and  $\hat{z}_{wp,m,so}^{AUX}$  are formulated as follows:

$$\begin{aligned} z_{wp,m,so}^{AUX} &= \mu_{wp,so}^{G_{wp}} P_{wp} - \hat{z}_{wp,m,so}^{AUX}, \forall wp, m, so \\ 0 \leq z_{wp,m,so}^{AUX} &\leq u_{wp}^m M, \forall wp, m, so \\ 0 \leq \hat{z}_{wp,m,so}^{AUX} &\leq (1 - u_{wp}^m) M, \forall wp, m, so \end{aligned} \quad (16)$$

A market is considered competitive when the producers offer their capacities at marginal costs. The dual variables  $\lambda$  describe the market marginal prices and they are associated to the balance between production and consumption at each bus.

#### IV. CASE STUDY

The mathematical model presented above is applied to a 6-bus network illustrated in Fig. 1.

The system contains 6 existing generating units (5 thermal generating units and 1 hydro generating unit) and two new wind generating units that are possible to be built, supplying three loads. The annualized investment cost  $IA_{wp}$  for new wind generating unit is 1000000 [€/MW]. Two system operation conditions are considered,  $so_1$  that is equal to 5700 [h] and  $so_2$  equal to 3060 [h].

The considered data for the network are reported in Tables I-V. Table I reports the data for the transmission lines within the analyzed system illustrated in Fig. 1. Table II reports the demands for each system operation condition and the load shedding cost. Table III reports the data for thermal generating units, while Table IV provides the data for hydro generating units. The data for the new WPP are reported in Table V for each option capacity  $m$ .

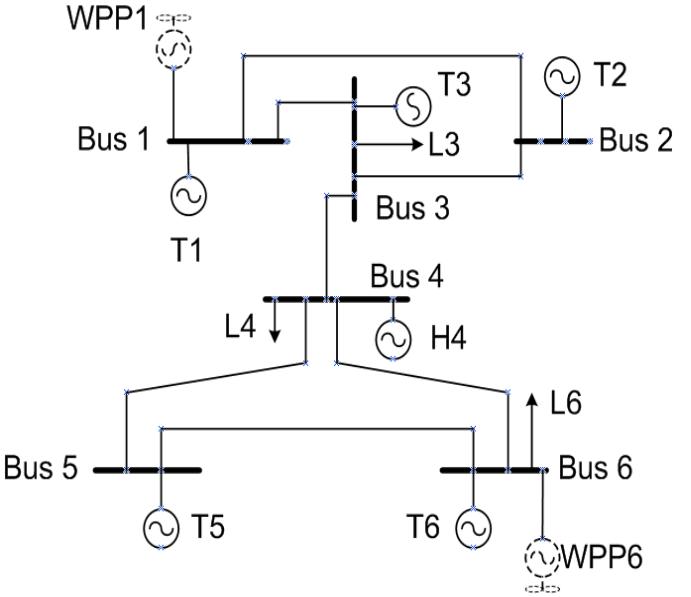


Fig. 1. System topology

TABLE I. DATA FOR TRANSMISSION LINES

Line	From bus	To bus	$B_l$ [p.u.]	$PF_l$ [MW]
I1	b1	b2	75,19	800
I2	b1	b3	106,38	900
I3	b2	b3	250	1060
I4	b3	b4	163,93	1060
I5	b4	b5	181,82	1060
I6	b4	b6	70,42	1060
I7	b5	b6	50,25	1060

TABLE II. DATA FOR DEMANDS

Demand	Bus	System operation condition		$c_{LS}^d$ [€/MWh]
		$so_1$	$so_2$	
d3	b3	133	130	1000
d4	b4	90	100	1000
d6	b6	130	100	1000

TABLE III. DATA FOR THERMAL GENERATING UNITS

Thermal unit	Bus	$G_t$ [MW]	$c_t$ [€/MWh]
t1	b1	50	40,88
t2	b2	50	42
t3	b3	50	55,55
t5	b5	62	78,51
t6	b6	62	81

TABLE IV. DATA FOR HYDRO GENERATING UNITS

Hydro unit	Bus	$G_h$ [MW]	$c_h$ [€/MWh]
h4	b4	50	32

TABLE V. DATA FOR NEW WPP

WPP	Capacity options [MW]		
	$m_1$	$m_2$	$m_3$
WPP1	25	18	21
WPP6	21	25	30

The production cost of new wind generation unit  $w_p$  is considered as 0 €/MWh.

## V. RESULTS

The location and the capacity of the new WPP that are possible to be built are obtained after solving the proposed mathematical model.

Fig. 2 illustrates the construction of new WPPs. WPP1 is required to be built with capacity option  $m_2$ , while WPP6 is required to be built with the capacity option  $m_1$ . Both WPPs are built for maintaining the balance between consumption and production, to minimize the investment and operational costs, and to maximize the social welfare. There is no load shedding, thus there are no load-shedding costs required to be payed considering this topology, thus the costs are minimized and the aim of the problem is fulfilled.

The capacities of the new WPPs are illustrated in Fig. 3, while the production of the existing generating units (thermal and hydro) is illustrated in Fig. 4.

The construction of new WPP

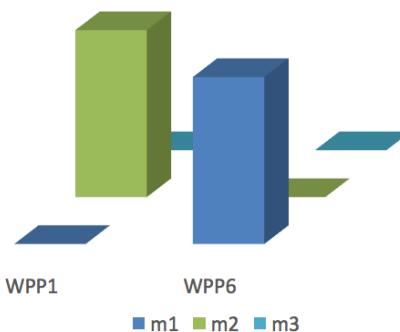


Fig. 2. The construction of the new WPPs.

Power produced by the new WPP [MW]

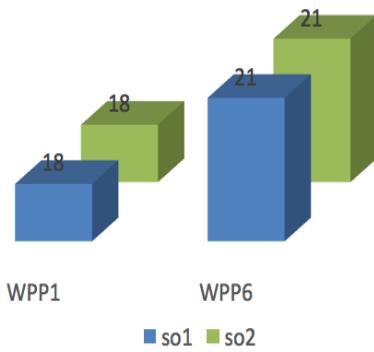


Fig. 3. The power generated by each WPP for system operation conditions  $so_1$  and  $so_2$

Power produced by the existing generating units [MW]

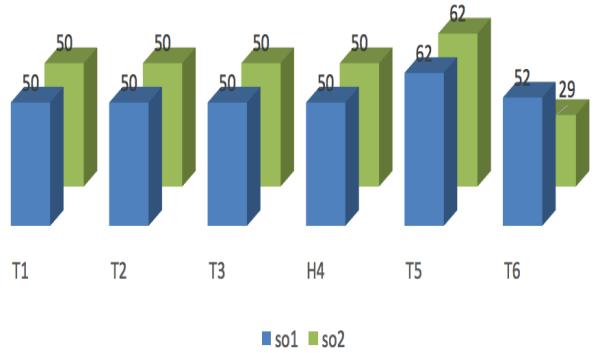


Fig. 4. The power generated by thermal and hydro generating units in each system operation condition

The final topology of the network is showed in Fig. 5. Thermal unit T6 is producing less than maximum power due to the construction of the new WPP. As the production costs of WPPs are 0, the WPPs are supplying the demands with a smaller cost as the one of the thermal units, thus operational costs are minimized. The value of the objective function is 187485031 €.

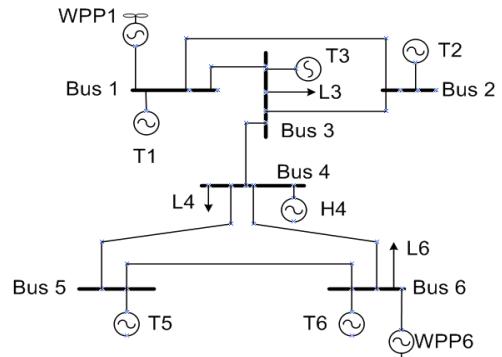


Fig. 5. The final topology of the considered grid

## VI. CONCLUSIONS

The generation expansion planning is required as the existing infrastructure is aging and the demand is growing in order to maintain the system reliability. The expansion of the grid is necessary to be considered.

The mathematical model that is applied to the 6-bus system is a deterministic static MILP problem. The network topology can be replaced with any other grid (smaller or bigger than the one considered in this paper).

The GEP problem is solved with the help of GAMS (General Algebraic Modeling System) optimization software in which the network data reported in tables and the mathematical model is introduced.

The results presented in section V were provided by the optimization software used.

The mathematical model can be modified by the ISO in order to determine the optimal solution for other systems that need generation expansion.

The ISO is encouraging investors to build new generating units after the problem is solved. The aim of the problem is to minimize the investment costs and operational costs.

The objective function (1) of the optimization problem is minimizing the operational and investment costs. The three problems that were reformulated in a single one represented by equations (1) - (16) have the aim to minimize the costs and maximize the social welfare.

In this approach, the uncertainties are ignored and the investment decision is taken once. Also, a DC power flow model is applied and the loses are neglected.

In this paper, only WPPs were considered as new generating units that are possible to be built. The model can be adapted for any other new type of generating units, e.g. photovoltaics and also for generating units that are considered to be non-renewable energy sources.

The investment budgets and operational costs that were considered can also be modified.

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