Combining the MIT and Lyapunov Stability Adaptive Methods for Second Order Systems

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Abstract— **In this paper, two adaptive control methods introduced in literature (e.g. the gradient and Lyapunov stability methods) are applied on several theoretical examples considering the second order plant for different time varying parameters. Further on, two new adaptive control laws are proposed. The first one is based on MIT rule and Lyapunov stability combination to allow usage of the same value for all adaptation gains. The second one, based on the first one, is developed for perturbation rejection by adding an artificial perturbation at the plant's input. Using Matlab / Simulink, the preferred approach for each use case is showcased.**

Keywords— ***adaptive control law, adaptation gain, Lyapunov stability, MIT rule, Model Reference Adaptive Control***

I. INTRODUCTION

In the 1950s, the aerospace engineering was looking for a solution to autopilots design. The proposed idea was to find a controller which could work well in flight conditions. Therefore, an intense research activity in adaptive domain had started, to help at designing such a controller. A sophisticated controller that could guarantee high performances was developed, such as the adaptive controller. Based on that, many adaptive flight control schemes were proposed and the most known is the one suggested by Whitaker et all, which can be found in many publications [1-4]. The method which supports the Whitaker's scheme, which is still used in our days, is named Model Reference Adaptive Control (MRAC) and can solve the autopilot problem [1, 3-4]. The results obtained are very well summarized in [1-2, 5]. At the beginning, this method was based on gradient method only and was named 'the MIT rule'. With this rule in place it was possible to automatically adjust the controller's parameters so that the system response is close to the one given by the reference model [2-3]. Furthermore, the MRAC method was improved in the 1960s by Park, by using the Lyapunov stability [1-3, 6].

The research continued in the 1970s, when Astrom and Egardt [1, 3, 6], developed the adaptive pole-placement method. However, the adaptive control scheme was unstable in case of small perturbations, high gains, high frequency and time varying parameters. All those issues forced in the 1980s to create a robust adaptive control [3, 6], helped by the evolution of microelectronics that eased the implementation in that times [1].

Until then, researches in the adaptive field were made for time varying plants. In the 1990s the focus was to extend the adaptive theory on discrete systems and nonlinear systems [6]. Another researching aspect was to find alternative methods for avoiding online parameter estimation [3].

To the present date, new adaptive methods have been developed. The L1-AC (L1 Adaptive Control) method is an adaptive method, which, as stated by their authors, can guarantee fast adaptation, transient response, decoupling fast adaptation and robustness [7].

Many tutorials [2-3, 8], and related papers [5, 9-13], show how the adaptive control methods can be applied in theory and also in many engineering fields (e.g. chemical, mechanical, robotics) [14-19].

In this paper, the fundamental adaptive techniques which are going to be applied on a theoretical process are presented in a structured and understandable way (Section II). In Section III, by using examples, a comprehensive analysis for the second order system (using MIT rule and Lyapunov stability) is done. The main issue of these standard techniques is that independently on how many plant parameters are time varying, different values for the adaptation gain (from controller adaptation mechanism) must be selected in such a way that key performance indicators are achieved. To solve this impediment, a new adaptive control law is proposed in Section III, which is a combination between the MIT rule and Lyapunov stability. This method is simulated with the same value for all adaptation gains.

The presence of perturbation is also treated in the present paper (Section III). To eliminate any perturbation which can appear at some moment and in any location in the system, in the adaptive control law an artificial perturbation is injected at the plant's input to reject the actual physical disturbance. All simulations validate the introduced theory. Finally, conclusions are presented in Section IV.

II. THEORETICAL ASPECTS

In the adaptive control theory, the Model Reference Adaptive System (MRAS) is the most known approach. The basic MRAS scheme, developed by Whitaker [1, 3-4, 19], is presented in Fig. 1.

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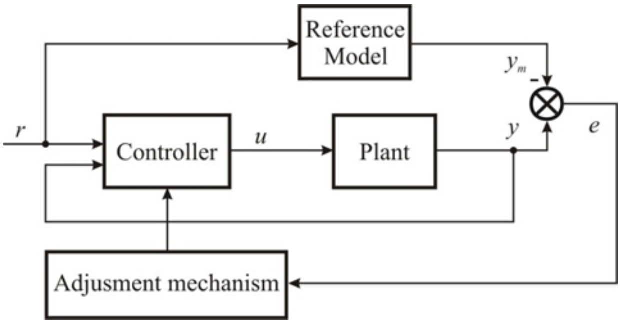

Fig. 1. Model reference adaptive scheme.

It has two loops: the regular one (inner loop), which contains the plant and controller, and second one (outer loop), which adjusts the controller's parameters. Both loops have two common purposes: to ensure the system key performances specified through reference model and system stability, in such a way that the difference (the error) between the system's output (plant output) and the reference model output to be steered towards zero [1].

The components of MRAS are [1, 5, 8]: the reference model which states the key system performances (the ideal system behavior), the controller that is parameterized by a number of adjustable parameters, and the adaptation mechanism used to adjust the parameters used to compute the control law.

In the MRAC theory, there are two main mathematical approaches used to develop an adaptation mechanism [1, 6, 8]:

- The gradient approach, also called the MIT rule (because it was developed by the Instrumentation Laboratory at Massachusetts Institute of Technology).
- Lyapunov stability theory.

A. MIT Rule

The basic idea of the MIT rule is to find a minimization criterion which will update the controller's parameters to reduce the error to zero. If e is the error and θ the parameter vector, the chosen criterion is [1, 3, 6]:

$$J(\theta) = \frac{1}{2} e^2. \tag{1}$$

Parameters must be changed in the direction of a negative gradient. In such conditions, criterion J becomes small:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta}, \tag{2}$$

where, $\partial e / \partial \theta$ is the sensitivity, and γ is the adaptation gain.

B. Lyapunov stability theory

Lyapunov stability can be used as well in adaptive control, with an adaptation mechanism like the one used for MIT rule (the same general scheme presented in Fig. 1 can be used) [5-6, 8]:

It is supposed that the closed loop system is characterized by a differential equation:

$$\frac{dx(t)}{dt} = f(x(t)), f(0) = 0, \tag{3}$$

where, x represents the state vector of the system. If the system's dynamic is given by Equation (3), then a V function (called the Lyapunov function), which depends on the system's parameters, should be found so as to satisfy below conditions:

$$\begin{aligned} &1.\, V(x) > 0, x \neq 0, \\ &2.\, V(0) = 0, \\ &3.\, V \textit{ is differentiable}; \\ &4.\, \dot{V}(t) = \frac{\partial V}{\partial x} \dot{x}(t) \leq 0. \end{aligned} \tag{4}$$

If function V is positive definite (Eq. 4, conditions 1-2) and its derivative negative semi-definite (Eq. 4, conditions 3-4), then the closed loop system will be stable [1, 6, 8]. Still, the main issue on how to find such a function that meets requirements from Equation 4 remains open.

III. The Gradient and Lyapunov Stability Methods for Second Order Systems

The second order plant and model reference have been chosen with the following form [20]:

$$\begin{aligned} &\frac{d^2 y(t)}{dt^2} = -a_1 \frac{dy(t)}{dt} - a_2 y(t) + bu(t), && (a) \\ &\frac{d^2 y_m(t)}{dt^2} = -a_{m1} \frac{dy_m(t)}{dt} - a_{m2} y_m(t) + b_m r(t). && (b) \end{aligned} \tag{5}$$

A. MIT rule and Lyapunov Stability

No matter how many parameters are varying in time in the plant (e.g. one, two, or three), the proposed adaptive control law is [19]:

$$u(t) = k_1 r(t) - k_2 y(t) - k_3 \dot{y}(t), \tag{6}$$

where, the parameter vector θ is $\theta = [k_1\ k_2\ k_3]$. This control law was chosen to achieve a good reference model follow in steady state, which is done by the parameter k_1, and to correct the system dynamics (i.e. change closed-loop system pole locations), through parameters k_2 and k_3.

The adaptation mechanisms are:

- for the standard MIT rule, the adaptation mechanism is [19]:

$$\frac{dk_1(t)}{dt} = -\gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} r(t)\right)e(t),$$

$$\frac{dk_2(t)}{dt} = \gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} y(t)\right)e(t), \quad (7)$$

$$\frac{dk_3(t)}{dt} = \gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} \dot{y}(t)\right)\dot{e}(t).$$

- for the standard Lyapunov rule, the proposed adaptation function V is:

$$V(e, \dot{e}, k_1, k_2, k_3) = \frac{1}{2}(\dot{e}^2(t) + a_{m2}e^2(t) + + \frac{1}{b\gamma}(a_{m1} - a_1 - bk_3)^2 + \frac{1}{b\gamma}(a_{m2} - a_2 - bk_2)^2 + + \frac{1}{b\gamma}(bk_1 - b_m)^2), \quad (8)$$

with parameter adaptation:

$$\frac{dk_1(t)}{dt} = -\gamma r(t)\dot{e}(t),$$

$$\frac{dk_2(t)}{dt} = \gamma y(t)\dot{e}(t), \quad (9)$$

$$\frac{dk_3(t)}{dt} = \gamma \dot{y}(t)\dot{e}(t).$$

Simulations (Fig. 2 – using a square wave as input signal) with three different values for adaptation gain, $\gamma 1 = 10$ (k_1), $\gamma 2 = 10$ (k_2), $\gamma 3 = 30$ (k_3), are done for a relatively slow, badly damped plant with the nominal transfer function:

$$G(s) = \frac{2}{s^2 + 1.6s + 4}, \quad (10)$$

with an optimal transfer function chosen for the reference model (in accordance to module criterion, i.e. damping ratio equal to 0.7):

$$G_m(s) = \frac{9}{s^2 + 4.2s + 9}. \quad (11)$$

Choosing different values for the adaptation gain represents the main issue of the standard techniques. In practice, such values are difficult to be set up. If the same value is chosen for all the adaptation gain, then, the adaptation mechanisms from (7) and (9) do not perform with expected results (see Fig. 3, with $\gamma = 10$).

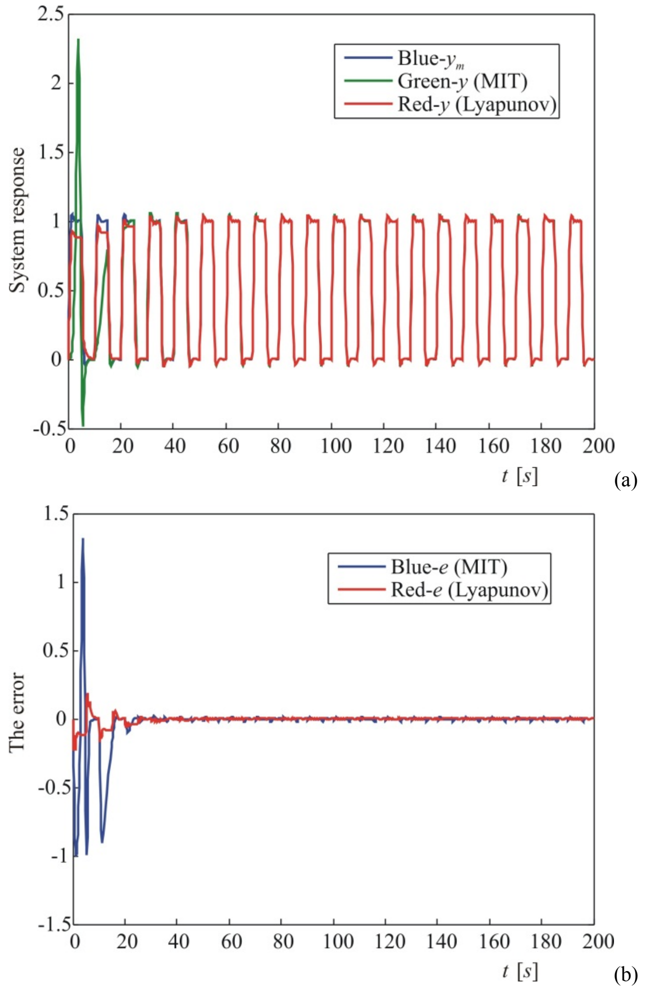

Fig. 2. System response (a) and error (b) for the second order system with different values for the adaptation gain.

In the MIT case, the controlled plant does not achieve the desired system dynamic (e.g. it has a larger overshoot, see Fig. 3), while in the Lyapunov case there is steady state error (more visible when applying step reference input).

B. Combining the MIT Rule and Lyapunov Stability

The goal is to have the same value for all adaptation gain. To reach it, a new control law is proposed: a combination between the MIT rule and Lyapunov rule. In this context, the MIT rule is used to eliminate steady state error and Lyapunov stability theory to ensure the prescribed system dynamics. It is proposed in (6) to consider the input signal equal to zero ($r(t) = 0$). Starting from the control law $u(t) = -k_2 y(t) - k_3 \dot{y}(t)$, the V function is defined:

$$V(e, \dot{e}, k_2, k_3) = \frac{1}{2}(\dot{e}^2(t) + a_{m2}e^2(t) + + \frac{1}{b\gamma}(a_{m1} - a_1 - bk_3)^2 + \frac{1}{b\gamma}(a_{m2} - a_2 - bk_2)^2, \quad (12)$$

with parameter adaptation:

$$\frac{\mathrm{d}k_2(t)}{\mathrm{d}t} = \gamma y(t)\dot{e}(t),$$
$$\frac{\mathrm{d}k_3(t)}{\mathrm{d}t} = \gamma \dot{y}(t)\dot{e}(t). \tag{13}$$

The parameters computed using the Lyapunov mechanism result into an ideal adaptive PD controller on the feedback path for inner loop. This controller ensures the desired system dynamics by modifying the closed loop poles to match the locations of the reference model poles. Parameter k_1 is used to adapt the overall DC gain of the control system to match the reference model in steady state. For the adaptation of k_1 parameter, the MIT rule is used with the following adaptation mechanism:

$$\frac{\mathrm{d}k_1(t)}{\mathrm{d}t} = -\gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} r(t)\right)e(t). \tag{14}$$

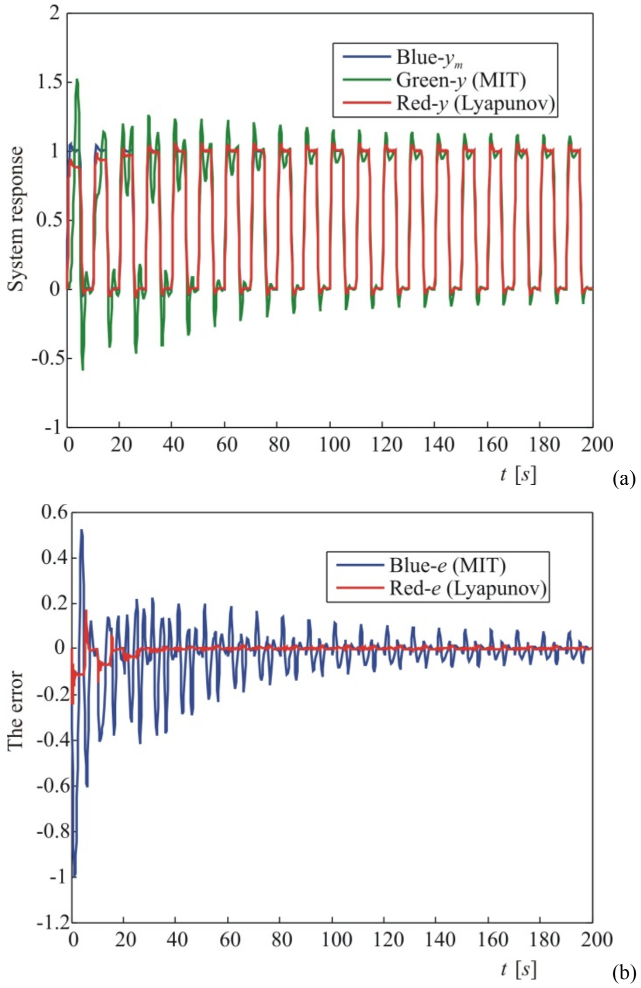

Fig. 3. System response (a) and error (b) for the second order system with the same value for the adaptation gain.

In the presence of the same input signal with $\gamma = 10$, the simulations presented in Fig. 4 demonstrate this theory.

Furthermore, the adaptation mechanism proposed in (12)-(14) was applied for unstable plant, plant with real poles, and plant with one or two parameters which were time varying.

C. MIT Rule and Lyapunov Stability in the presence of disturbances

When a perturbation appears in the system at a given moment, its effect must be eliminated and the plant must be forced to follow the reference model. It is supposed that a disturbance, with the Laplace transform of its effect:

$$G_{pert}(s) = \frac{5e^{-90s}}{s(s+4)}, \tag{15}$$

appears at the system's output after 90s. By using the adaptation mechanism and control law ((12)-(14)), it can be seen (Fig. 5) that the system's output does not track the reference model anymore and the error does not converge to zero.

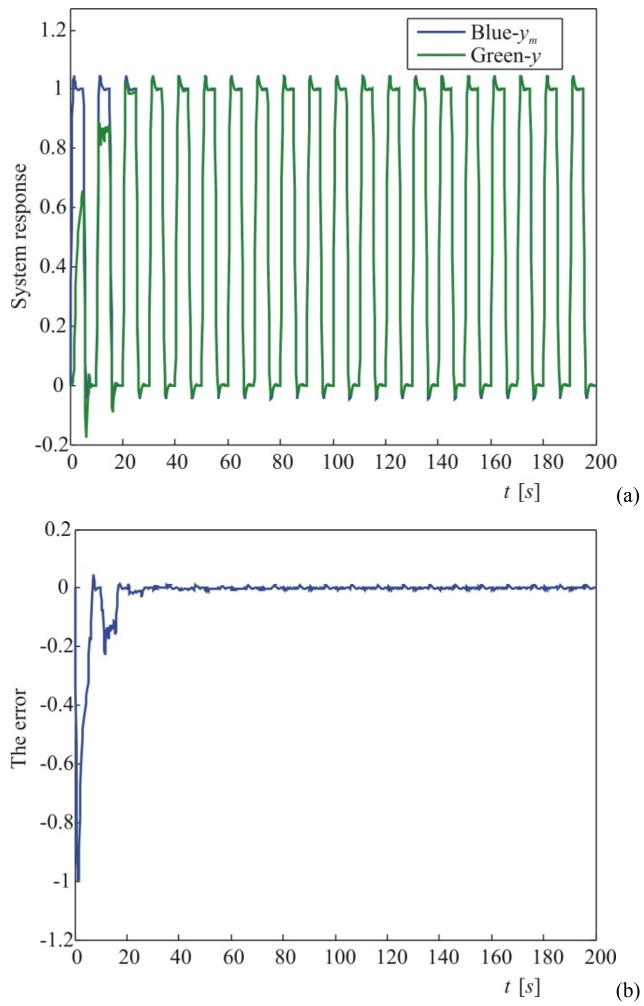

Fig. 4. System response (a) and error (b) for the second order system (adaptation mechanism from (12)-(13)).

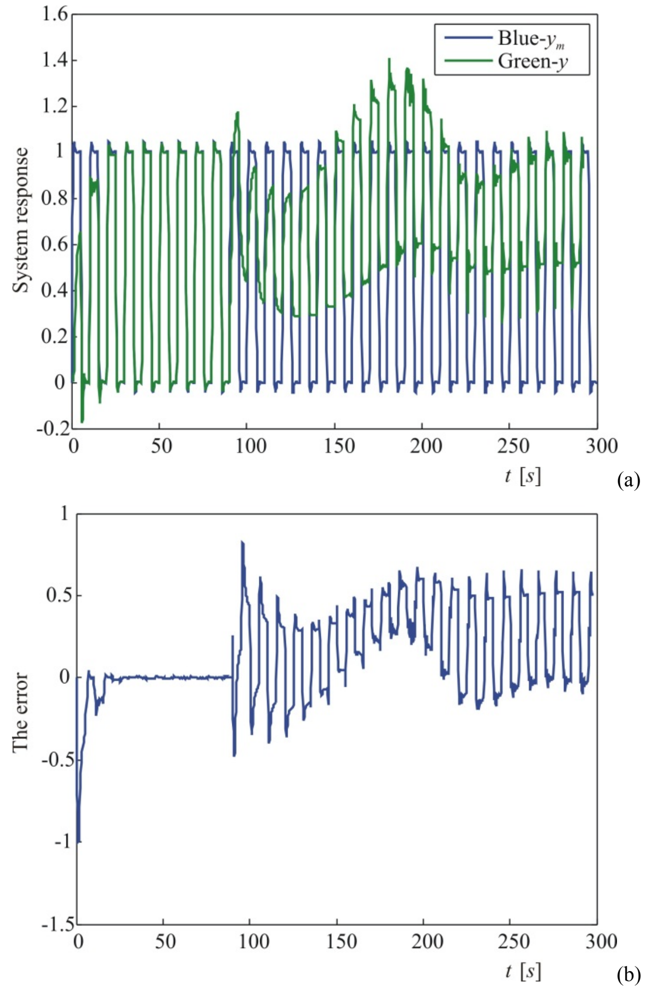

Fig. 5. Second order system response (a) and error (b) in the presence of perturbation.

To overcome this impediment and to push the output of the system to follow the reference model, another adaptive control law is proposed:

$$u(t) = k_1 r(t) - k_2 y(t) - k_3 \dot{y}(t) + k_4 p(t), \tag{16}$$

where, $p(t)$ is an artificial zero-delay unit step (i.e. $p(t) = 1, \forall\, t > 0$) perturbation injected at the plant's input to reject the actual physical perturbation. Thus, the parameter vector θ is $\theta = [k_1\ k_2\ k_3\ k_4]$. Basically, by adding this term in the adaptive control law, any perturbation which may appear in the system is eliminated (see Fig. 6).

For parameters k_2 and k_3, the same adaptive mechanism presented in (12)-(13) is adopted, considering $r(t) = 0$ and no external disturbances. The adaptation mechanism for parameters k_1 and k_4 is developed using the MIT rule and considering the inner loop to have the same dynamics (i.e. pole locations) as the reference model:

$$\begin{aligned} \frac{\mathrm{d}k_4(t)}{\mathrm{d}t} &= -\gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} p(t)\right)e(t), \\ \frac{\mathrm{d}k_1(t)}{\mathrm{d}t} &= -\gamma\left(\frac{1}{p^2 + a_{m1}p + a_{m2}} r(t)\right)e(t). \end{aligned} \tag{17}$$

The simulations from Fig. 7 with $\gamma = 10$, validate the theory mentioned above (also tested on first order systems).

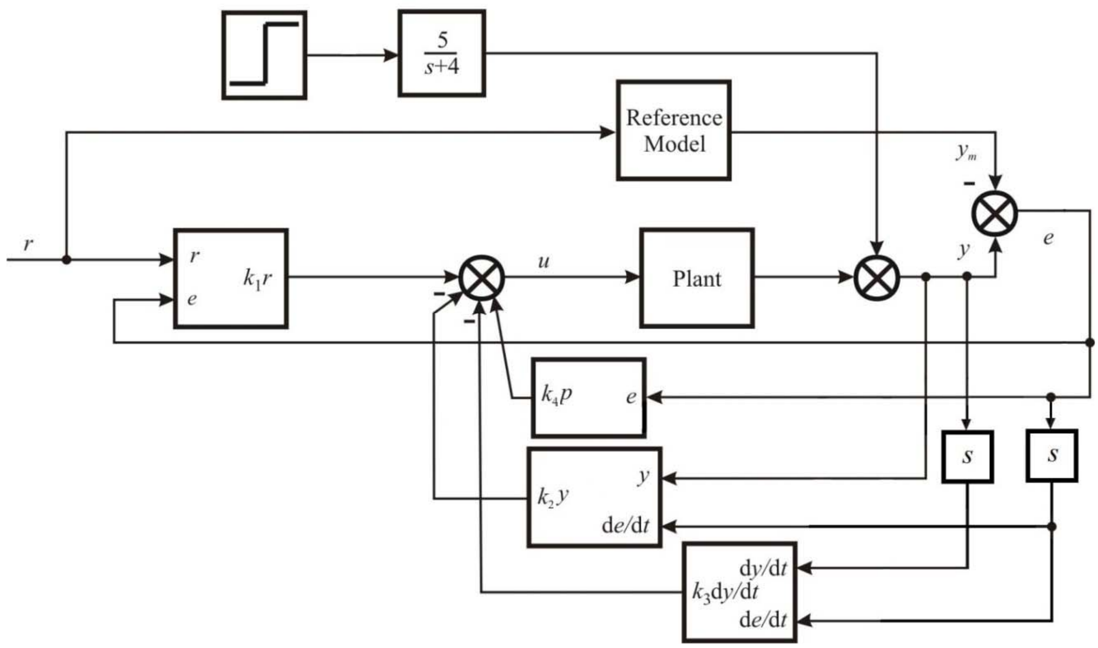

Fig. 6. Simulink scheme for the second order system with external perturbation.

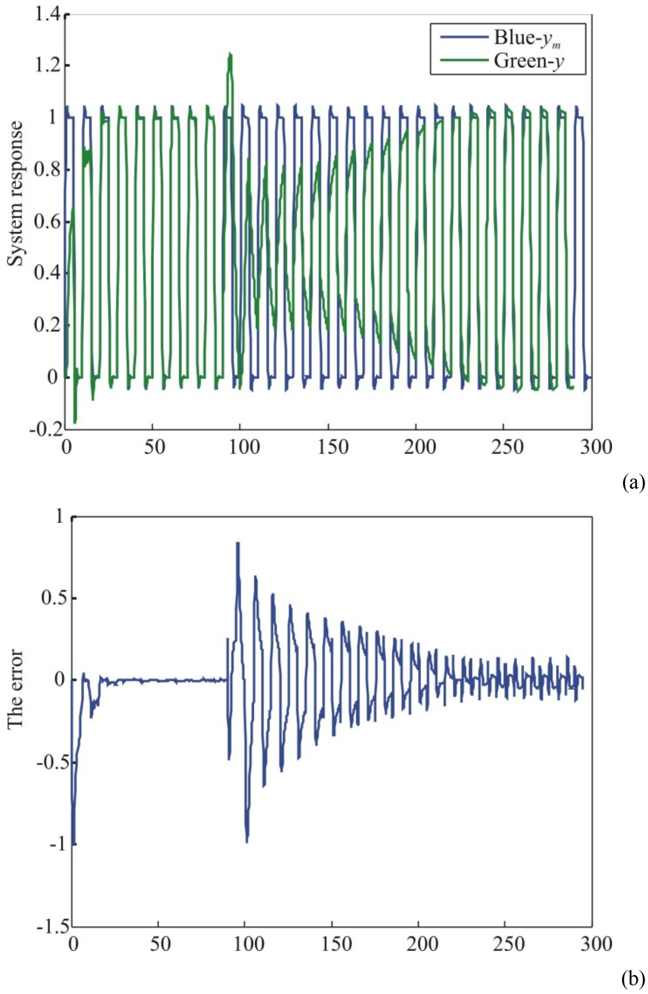

Fig. 7. Second order system response and error with perturbation rejection.

IV. CONCLUSIONS

A list of conclusions can be sketched out:

- The two presented adaptive methods (MIT and Lyapunov stability) are quite similar regarding the adaptation mechanisms, with one difference in the Lyapunov case, where the signals are not filtered anymore by the reference model;
- The Lyapunov rule is not as sensible as MIT rule to the adaptation gain value;
- The proposed control laws and adaptation schemes are robust and applicable to a wide range of plant parameters and have a good potential to stabilize unstable plants;
- Our final proposed control law can reject external disturbances, independently to their impact in the closed loops system.

Even though the models used for all simulations were time invariant, the results are relevant due to initial values of θ (the controller parameter vector) which were equal to zero. Therefore, by using the presented adaptation mechanisms the θ values were updated to fulfill the prescribed performance and stability requirements.

Moreover, an analysis regarding the second order system was made. The main goal was to use the same value for all adaptation gains. To accomplish this condition, a new adaptive control law was proposed, which met the expectations. Another approached subject for the second order system is how the closed loop system behaves in presence of perturbations. The dynamic of the system and associated performances are achieved by another new adaptive control law proposed and tested in this paper.

Different adaptation laws were presented and experimentally evaluated in this paper. The current study can provide guidelines to other researchers to apply newly proposed adaptive control laws on different engineering processes.

As a future work, the new adaptive control theory related to linear continuous time systems can be extended to cover discrete time systems, multi-input-multi-output systems and nonlinear systems.

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