14th International Conference on DEVELOPMENT AND APPLICATION SYSTEMS, Suceava, Romania, May 24-26, 2018

The Experimental Method for Tuning of PID Controller Based on the Maximum Stability Degree Criterion

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Abstract—**An experimental method for tuning of the PID controller and its variations, using the critical regime parameters of the closed loop control system is proposed in this paper. The method was developed based on the maximum stability degree criterion, that offers to the designed control system the high stability, robustness at the variation of the control objects' parameters, the transient processes with low overshoot and high performance. For efficacy analysis of the elaborated algorithms, there are presented some case studies and practical applications.**

Keywords—experimental method; typical control algorithms; control system; maximum stability degree criterion; performance of control system

I. INTRODUCTION

The practice of automation the technological processes demonstrates that a wide use have fixed structure and low-order controllers, which are operating based on the PID algorithm and its variations. This is explained based on the fact that within the limits of the automatic control system structure, the PID control algorithms are optimal if we refer to the specific functions of the controllers. The unsatisfactory operating of the automatic control system usually refers not to the unsatisfactory action of the PID algorithm (with condition that it was optimal tuning) but to the uncertainty over the information about the properties of the controlled process and the approximation of the mathematical model that describes the real process [1, 2, 3].

In this context, the highest interest present the experimental methods of tuning the typical controllers, which do not involve preliminary identification of the process. Nowadays, there exist many different experimental criterions for tuning of the PID controllers, which starts from the some simple assumptions and obtained values of the tuning parameters differs. Starting from these values, it is necessary to make some successive additional adjustments of the tuning parameters, in so way that the obtained performance of the automatic control system to be close to the imposed performance [2, 4].

The Ziegler-Nichols method is the most widely used method in practice, which was developed based on the minimum surface criterion with the 4:1 oscillation damping. The calculation of the optimal tuning values of the PID controller based on the Ziegler-Nichols criterion is simple and the procedure is based on the stability limit of the closed loop system. However, this method it is recommended only for the slow processes, with load disturbances which have a long period; it does not imply the synthesis of systems with imposed performance; it provides the oscillating transient processes with a low damping rate; it does not take into account the requirements related to the system stability reserve; the operation quality of the designed control systems based on this method usually is unsatisfactory and requires further adjustment. This method does not allow the optimization of the calculated tuning parameters [2, 5].

Based on these considerations, in this paper it is proposed to elaborate the experimental method for tuning of the typical controllers based on the maximum stability degree criterion. This criterion provides to the designed control system the maximum stability degree, robustness at the object's parameters variation, the transient processes with low overshoot and high performance in the transient and in the steady state regimes [6, 7].

II. EXPERIMENTAL ALGORITHMS FOR SYNTHESIS OF THE TYPICAL CONTROLLERS BASED ON THE MAXIMUM STABILITY DEGREE CRITERION

In the paper [7] it is proposed a method for synthesis of the typical PID controllers based on the maximum stability degree criterion. The method assumes that the mathematical model of the control object is described by the following transfer function

$$H_F(s) = \frac{k\exp(-\tau s)}{a_0 s^r + a_1 s^{r-1} + \ldots + a_{r-1}s + a_r}, \qquad (1)$$

where k is the transfer coefficient of the control object; τ -

time delay; a_i, $i=0,\dots,r$ - parameters which characterize the inertia of the control object; r - the degree of the object model.

The controller represents the generalized form of the PID controller and its variations, which it is characterized by the following transfer function

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_i + k_p s + k_d s^2}{s}, \qquad (2)$$

where k_p, k_i, k_d are the tuning parameters of the controller.

In the papers [8, 9], based on the maximum stability degree method, there were developed the analytical algorithms for tuning of the typical controllers P, PD, PI and PID. Thus, for synthesis of the PID controller, the following analytical expressions were obtained

$$\begin{aligned} J &= \sqrt[r-2]{\frac{6a_{r-2}}{(r^3-r)a_0}}, \\ k_p &= \frac{\exp(-\tau J)}{k}[(r+1)a_0 J^r - a_r], \qquad (3) \\ k_i &= \frac{\exp(-\tau J)}{k}[a_0 J^{r+1}], \\ k_d &= \frac{\exp(-\tau J)}{k}[0{,}5 a_0 (r^2+r) J^{r-1} - a_{r-1}], \end{aligned}$$

where J represents the maximum stability degree of the designed control system.

It is noted, that tuning parameters of the PID controller (3) are functions of the parameters a_0, $a_1,\dots$, a_n, τ, k and the inertia degree r of the control object model (1). The variation of these parameters implicitly brings to the changes of the critical regime parameters of the control system, when it is brought to the limit of stability, namely: the period of the un-amortized oscillations T_{cr}, the critical transfer coefficient k_{cr} of the system and the amplitude of the un-amortized oscillations A_{cr}. It is proposed, based on the maximum stability degree method, to elaborate the synthesis algorithms of PID controller and its variations, using the parameters of the critical regime of the designed system.

Suppose that the control object is described by the transfer function with third order inertia

$$H_F(s) = \frac{k}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}. \qquad (4)$$

In the MATLAB software package it is simulated the control system which includes the control object (4) and PID controller (2). To elaborate the experimental algorithm for tuning of the PID controller, the following steps are passed:

1. The transfer coefficient k of the control object (4) it is fixed at the constant value and there are modified the time constants T_1, T_2, T_3. For each set of time constants values, according to the expressions (3), the controller's parameters are calculated and, at the same time, by the MATLAB simulation, the system is brought to the limit of stability and it is determined the period of the un-amortized oscillations T_{cr} of the system.

In the MATLAB software package are constructed the dependencies $k_{p1}=f(T_{cr})$, $k_{i1}=f(T_{cr})$, $k_{d1}=f(T_{cr})$ and, using the least squares method, there are determined the algebraic expressions which approximate the experimental curves

$$\begin{aligned} k_{p1} &= 2{,}38; \\ k_{i1} &= \frac{1}{0{,}38 T_{cr}}; \qquad (5) \\ k_{d1} &= 0{,}521 T_{cr}. \end{aligned}$$

The results of the approximations by the expressions (5) are presented in the Fig. 1, where the points represent the experimental data, and the continues line – the dependencies curves, which approximate the experimental data.

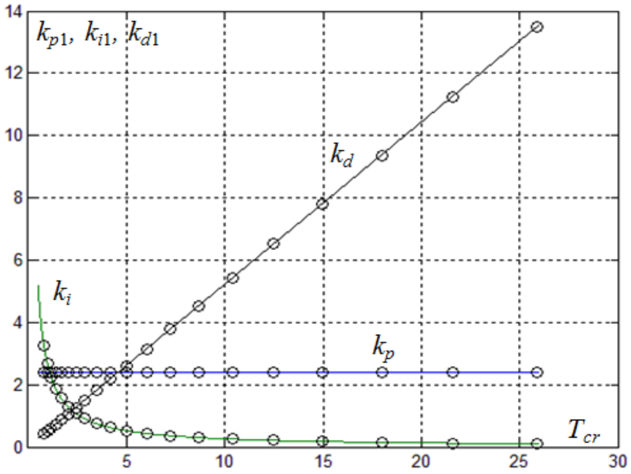


Fig. 1. Dependences $k_{p1}=f(T_{cr})$, $k_{i1}=f(T_{cr})$ and $k_{d1}=f(T_{cr})$.

2. For the one set of the temporal constants values of the control object (4) it is varied the transfer coefficient k. For each values of the transfer coefficient it is determined the critical transfer coefficient of the control system k_{cr}, it is calculated the tuning parameters of the controller according to the expressions (3) - k_0 and, at the same time by the expressions (5) - k_1, obtained at the first step. In order to exclude the influence of the object inertia to the dependence of the tuning parameters toward k_{cr}, for each value of k, there is determined the ratio between the optimal values of the tuning parameters, calculated based on the maximum stability degree method (3), and the tuning parameters determined by the expressions (5) - $k_2 = k_0 / k_1$.

In MATLAB there were obtained the dependencies $k_{p2}=f(k_{cr})$, $k_{i2}=f(k_{cr})$ şi $k_{d2}=f(k_{cr})$ and there were determined the algebraic expressions which approximated the experimental curves

$$k_{p2} = 0{,}0626 \cdot k_{cr};$$
$$k_{i2} = 0{,}0664 \cdot k_{cr}; \qquad (6)$$
$$k_{d2} = 0{,}0555 \cdot k_{cr}.$$

The approximation results by the expressions (6) are presented in the Fig. 2.

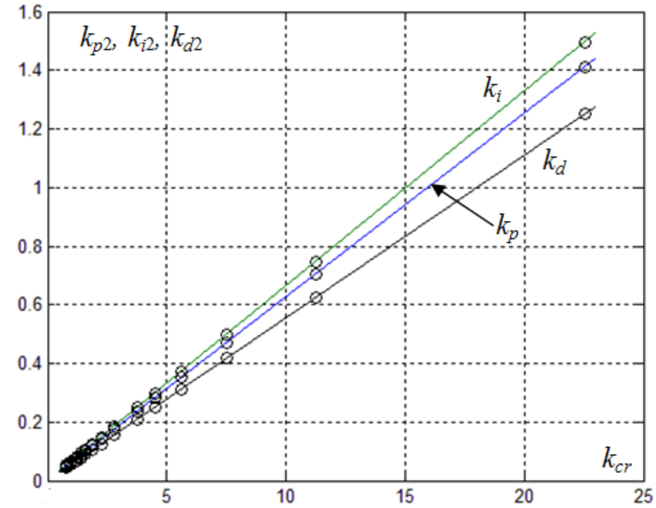


Fig. 2. Dependencies $k_{p2}=f(k_{cr})$, $k_{i2}=f(k_{cr})$ and $k_{d2}=f(k_{cr})$.

3. The expressions (5) and (6), obtained at the 1st and 2nd stapes, are multiplied and it is determined the dependence expressions of the controller's tuning parameters in the concordance with the period of the un-amortized oscillation T_{cr}, and the variation of the critical transfer coefficient k_{cr}: $k_{p3}=f(T_{cr},k_{cr})$, $k_{i3}=f(T_{cr},k_{cr})$ and $k_{d3}=f(T_{cr},k_{cr})$

$$k_{p3} = k_{p1}k_{p2} = 0{,}149\, k_{cr};$$
$$k_{i3} = k_{i1}k_{i2} = 0{,}1748\frac{k_{cr}}{T_{cr}}; \qquad (7)$$
$$k_{d3} = k_{d1}k_{d2} = 0{,}0289 T_{cr}k_{cr}$$

4. It is varied the inertia degree r of the control object (4). For each value r, it is determined the amplitude of the un-amortized oscillation of the control system A_{cr} and there are calculated the tuning values of the controller based on the expressions (3) - k_0 and, at the same time by the expressions (7) - k_3, obtained at the third step.

In order to exclude the influence of T_{cr} and k_{cr} over the dependencies of the tuning parameters in concordance with A_{cr}, for each value of the r, there were determined the ratio between the optimal values of the tuning parameters, calculated by the maximum stability degree method (3) and the tuning values obtained by the expressions (7) - $k_4= k_0/ k_3$.

As a result of the calculations, the following expressions that approximate the experimental results are obtained

$$k_{p4} = 38 A_{cr}^{-6};$$
$$k_{i4} = 127 \mathrm{A}_{cr}^{-7{,}5}; \qquad (8)$$
$$k_{d4} = 22 A_{cr}^{-5{,}5}.$$

The results of the approximations by the expressions (8) are presented in the Fig. 3.

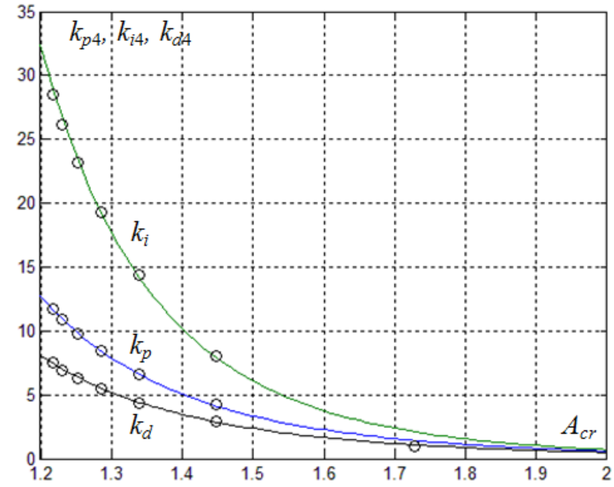


Fig. 3. Dependencies $k_p=f(A_{cr})$, $k_i=f(A_{cr})$, $k_d=f(A_{cr})$.

5. The expressions (7) and (8), obtained at the 3rd and 4th stapes, are multiplied and there are obtained the dependencies of the tuning parameters in the concordance with amplitude variation of the un-amortized oscillation A_{cr}, period variation of the un-amortized oscillation T_{cr} and critic transfer coefficient of the system k_{cr}: $k_p=f(T_{cr},k_{cr},A_{cr})$, $k_i=f(T_{cr},k_{cr},A_{cr})$ and $k_d=f(T_{cr},k_{cr},A_{cr})$

$$k_p = 5{,}66 k_{cr} A_{cr}^{-6};$$
$$k_i = 22{,}2 k_{cr} A_{cr}^{-7{,}5} / T_{cr}; \qquad (9)$$
$$k_d = 0{,}636 k_{cr} T_{cr} A_{cr}^{-5{,}5}.$$

The algebraic expressions (9), thus obtained, represent the final expressions for calculation of the tuning parameters k_p, k_i and k_d of the PID controller, using the parameters of the critical regime of the closed loop control system, based on the maximum stability degree criterion.

Following the procedure presented above, there were obtained the algebraic expressions and for the calculation of the tuning parameters of the typical controllers P, PD and PI. The controllers synthesis algorithms to the model objects with inertia and time delay, based on the critical regime of the control system are presented in the Table I.

For the practical applications, the synthesis method of the typical controllers, based on the critical regime of the control system, consists from the following: for the PID controller it is fixed the $k_i = 0$, $k_d = 0$ and it is varied k_p until the output of the system y achieves the un-amortized oscillation regime, so the system is at the stability limit. The value of k_p, for which are obtained the un-amortized oscillations, represents the critical transfer coefficient of the system k_{cr}. The parameters of the critical regime: the transfer coefficient k_{cr}, the period T_{cr} and amplitude A_{cr} of the un-amortized oscillation are used to determinate the tuning parameters of controller according to the expressions presented in the Table I. A_{cr} represents the ratio between the maximum amplitude of the un-amortized oscillations and the reference value at the input of the system y_{max}/x_{sp}.

TABLE I. ALGORITHMS FOR SYNTHESIS OF THE TYPICAL CONTROLLERS

Type of controller	Expressions for calculation of the tuning parameters values
P	$k_p = (0.29A_{cr}^2 - 1.13\,A_{cr} + 1.11)k_{cr}$
PD	$k_p = (0.51A_{cr}^2 - 2.06\,A_{cr} + 2.13)k_{cr};$ $k_d = (0.145\,A_{cr}^2 - 0.532\,A_{cr} + 0.515)k_{cr}T_{cr}$
PI	$k_p = 257k_{cr}e^{-4.7A_{cr}};\ k_i = \dfrac{3751k_{cr}e^{-6A_{cr}}}{0.82T_{cr}+0.006}$
PID	$k_p = 5{,}66k_{cr}A_{cr}^{-6};$ $k_i = \dfrac{22{,}2k_{cr}A_{cr}^{-7{,}5}}{T_{cr}};$ $k_d = 0{,}636k_{cr}T_{cr}A_{cr}^{-5{,}5}$

III. CASE STUDIES AND PRACTICAL APPLICATIONS

In order to argue the applicability, effectiveness and quality of the proposed method for synthesis of the typical controllers based on the critical regime of the control system, there are presented some case studies and practical applications. For the comparative analysis of the efficacy of the developed algorithms, the maximum stability degree method [8], Ziegler-Nichols method [2,4,5], Coon [4] and Parametric Optimization from MATLAB methods were used.

Suppose that the technological process is characterized with model of object with forth order inertia and time delay

$$H_F(s) = \frac{k\exp(-\tau s)}{(T_1s+1)(T_2s+1)(T_3s+1)(T_4s+1)} = \frac{\exp(-0{,}5s)}{(0{,}5s+1)(s+1)(2s+1)(4s+1)} \qquad (10)$$

where k is the transfer coefficient; T_1, T_2, T_3, T_4 - time constants of the object model; τ - time delay.

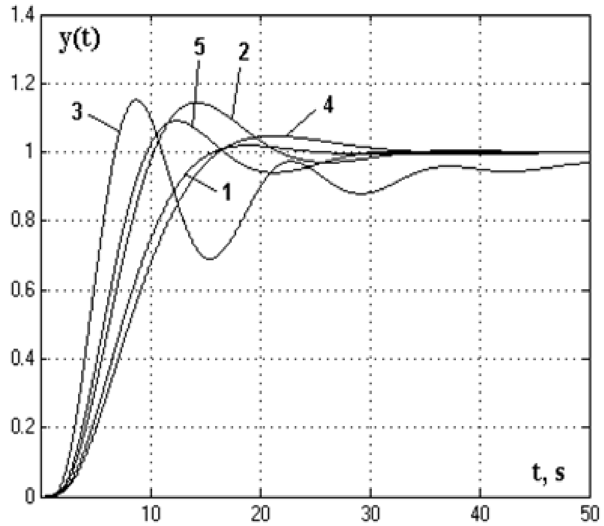
It is necessary to synthesize the PI and PID controllers to the model of object (10). The synthesis results of the controllers are presented in the Table II.

TABLE II. SYNTHESIS RESULTS OF THE CONTROLLERS

No.	Synthesis method	Type of the controller				
		PI		PID		
		k_p	k_i	k_p	k_i	k_d
1	**Critical Regime method**	0,633	0,13	1,55	0,279	2,352
2	**Maximum stability degree**	0,861	0,1845	2,11	0,376	3,11
3	**Ziegler-Nichols**	2,143	0,117	3,57	0,156	1,066
4	**Coon**	0,5	0,125	1	0,189	1,336
5	**Parametrical optimization**	1,094	0,169	2,3	0,323	3,848

The obtained transient processes of the designed control system are presented in the Fig.4. The numbering of the curves corresponds with the numbering of the methods from the Table II.

PI controller



PID controller

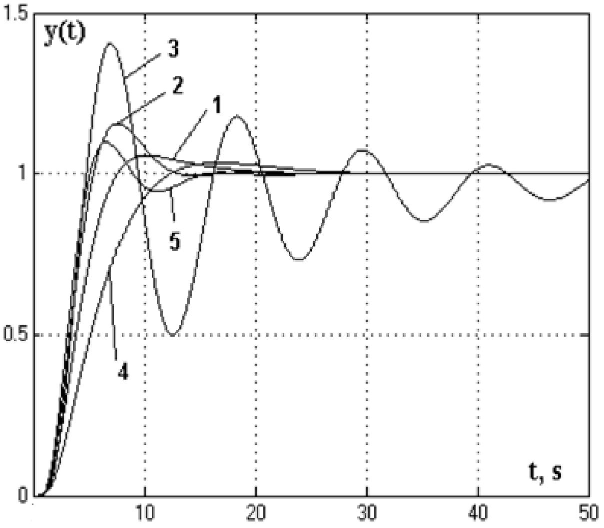


Fig. 4. Transient processes of the control system.

According to the simulation results there were obtained the performance of the control system ($\varepsilon_{st} = \pm 5\%$), presented in the Table III.

TABLE III. PERFORMANCE OF THE CONTROL SYSTEM, OBTAINED AS A RESULT OF SIMULATION

No.	**Synthesis method**	**Type of controller**	**Performance of the control system**				
			t_r, s	t_s, s	σ, %	λ	ψ
1	**Critical Regime method**	**PI**	10	13,6	-	-	-
		PID	5,9	11,8	5,7	1	1
2	**Maximum stability degree**	**PI**	6,3	19,4	14	1	1
		PID	3,3	10,9	15	1	1
3	**Ziegler-Nichols**	**PI**	4	45	15	-	-
		PID	2,6	60	40	5	0,5
4	**Coon**	**PI**	10,7	24	6	1	1
		PID	8,26	10,6	4,6	-	-
5	**Parametrical optimization**	**PI**	6	23,5	9	1	1
		PID	3,1	12	10	1	1

As a practical application, in order to verify the elaborated method, it is considered the control of the thermal process in the extruder of the 3D printer.

The extruder is that part of the printer, where the plastic is melted at a certain temperature. It is very important to keep the temperature in the extruder with high precision and in the settled range, because the quality of the printed objects depends on the temperature variation in the extruder and, from this reason, temperature control is done based on the PID algorithm.

It is proposed to tune the PID controller based on the designed method and to compare the obtained results with the Ziegler-Nichols method and the auto-tuning regime that is implemented in the 3D printer software. The synthesis results of the PID controller are presented in the Table IV.

TABLE IV. RESULTS OF SYNTHESIS THE PID CONTROLLER

No.	**Synthesis method**	**Parameters of the critical regime**			**Tuning parameters of the controller**		
		k_{cr}	T_{cr}	A_{cr}	k_p	k_i	k_d
1	**Critical Regime method**	100	22 s	0,99	584,2	108,8	1478,7
2	**Ziegler-Nichols**				75	13,2	2,2
3	**Auto-tuning**	-	-	-	30,43	3,27	70,83

The transient processes of the temperature control system in the 3D printer's extruder are presented in the Fig. 5.

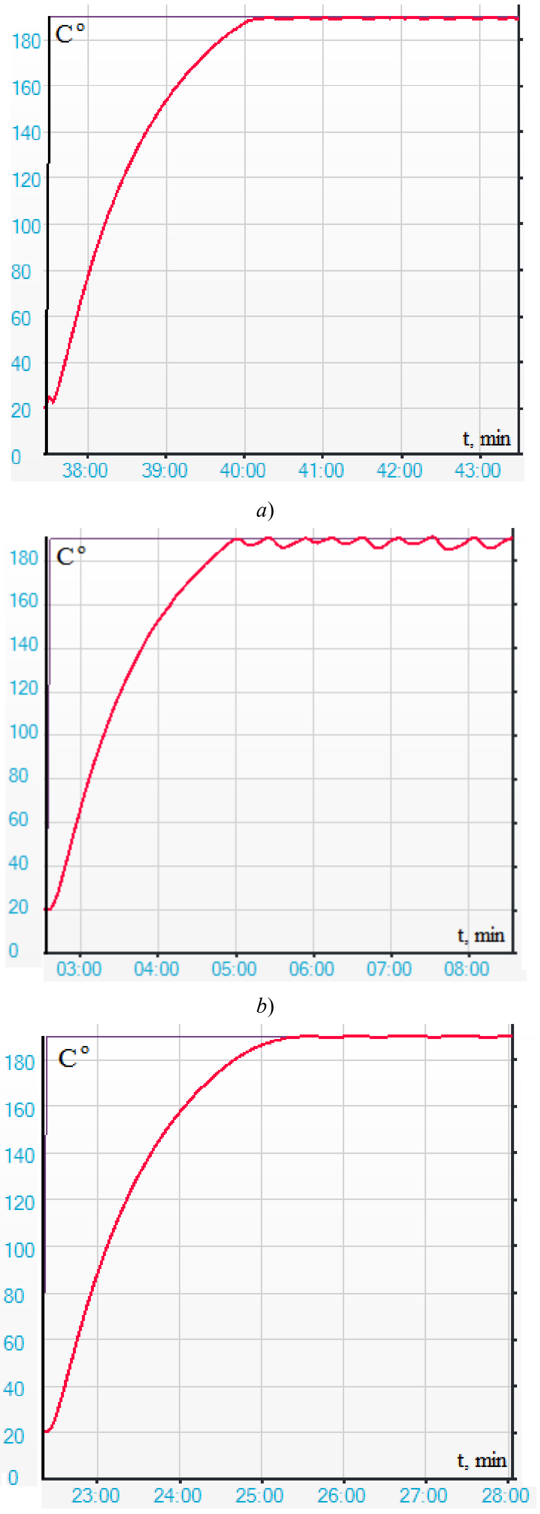


Fig. 5. The transient processes of the temperature control system in the 3D printer's extruder: *a*) Critical regime method; *b*) Ziegler-Nichols method; *c*) auto-tuning method.

IV. CONCLUSIONS

In this paper it is proposed a new experimental method of tuning the PID controller and its variations, using the critical regime parameters of the closed loop system: the period of the un-amortized oscillation T_{cr}, the critical transfer coefficient k_{cr} of the system and the amplitude of the un-amortized oscillations A_{cr}. The method was developed based on the analytical algorithms [8], determined according to the maximum stability degree criterion of designed system.

In comparison with Ziegler-Nichols method, where it is taken into account only the period of the un-amortized oscillation T_{cr} and critical transfer coefficient k_{cr} of the control system, in the proposed method it is used the third parameter - amplitude of the un-amortized oscillation A_{cr}, which is a function of the controlled object degree. Based on these three critical parameters of the system are calculated the optimal tuning values of the controller, according to the proposed method.

It is a simple experimental method, that does not require to be known the mathematical model of the control object, thus eliminating the difficulties connected with identification procedure of the processes, as well as the random nature of the perturbations that can effect the system.

After analysing the synthesis results of the typical controllers based on the elaborated method and, for comparison, on the other known tuning methods, both for computer simulation and for real systems, it was observed that the proposed method offers to the designed control system the transient processes with low overshoot and high performance.

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