An Analysis of Improved Nyquist Pulses Based on Pearson Distance

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***Abstract*— Modern telecommunications require high data rates and very good quality of service. The improved Nyquist pulses outperform the standard RC pulse in terms of error probability when sampled with a time offset. An analysis of these novel pulses in terms of performance and similarity in shape based on Pearson distance between the vectors that contain the samples of the impulse response and frequency characteristic has been carried out. The most performing pulses show a smaller value of Pearson distance with respect to the standard 'raised cosine' pulse. The Pearson distance diminishes for higher values of the excess bandwidth.**

***Keywords*— Intersymbol interference; Error probability; Nyquist pulse; Pulse analysis; Pearson distance**

I. Introduction

The quality performance of digital communications is appreciated by the symbol error rate that depends on several factors such as signal-to noise ratio (SNR), interymbol interference, co-channel interference, and jitter.

In order to increase the transmission performance when affected by jitter, Beaulieau et al. [1, 2] proposed the use of Nyquist filters that have an impulse response with an asymptotic decay rate (ADR) of t^{-2}, as opposed to the standard 'raised cosine' (RC) pulse that has an ADR of t^{-3}. The first novel pulse with an exponential frequency characteristic was termed 'better than raised cosine' (BTRC) [1] or 'flipped exponential', in short 'fexp' [4]. However, one must mention that this advantage is countered by the fact that a slowly decaying pulse, such as BTRC, when implemented practically needs a bigger truncation length that results in spectral regrowth, increased latency, and implementation costs.

These novel pulses outperform the standard RC pulse in terms of error probability when sampled with a time offset.

This line of research was followed by a plethora of scientists that produced on a heuristic basis many novel pulses that outperformed those reported earlier [4-38]. These pulses are known as improved Nyquist pulse (INPs) or in frequency domain as improved Nyquist filters (INFs).

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The error probability is calculated as in [3] assuming fixed values of normalized time offset t/T of 0.05, 0.1, 0.2, and sometimes 0.3, where T is the symbol duration.

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ M\ odd}}^{M} \left(\frac{exp\left(-m^2\omega^2/2\right) sin\left(m\omega g_0\right)}{m} \right) \prod_{k=N_1}^{N_2} cos\left(m\omega g_k\right) \tag{1}$$

Here M is the truncation length of the approximate Fourier series with the period T_f of noise complementary distribution function; $\omega = 2\pi/T_f$ -angular frequency; N_1 and N_2 give the number of interfering symbols before and after the transmitted symbol; respectively, and $g_k = p(t-kT)$ where *p(t)* is the pulse shape used.

The mechanism that determines this improved behavior was not completely understood so far. Most improved pulses have a frequency characteristic that is concave in the frequency range [*B*(1-*α*), *B*] and convex in the [*B*, *B*(1+*α*)] range, in view of the odd-symmetry around cut-off frequency B of the ideal low-pass brick-wall filter.

Here α represents the excess bandwidth factor. This leads to the conclusion that the improved behavior is obtained at the expense of transferring energy within the transition region [*B*(1-*α*), *B*(1+*α*)] from low-frequency area [*B*(1-*α*), *B*] to the high frequency range [*B*, *B*(1+*α*)] [10].

However, there are pulses with improved performance, denoted in short improved Nyquist pulses (INPs), that have a concave frequency characteristic in most of the frequency range [*B*(1-*α*), *B*] and convex in most of the frequency range [*B*, *B*(1+*α*)], such as CC2 and CC3 in [9]. This leads to the conclusion that a reverse energy transfer is involved from the [*B*, *B*(1+*α*)] range to the [*B*(1-*α*), *B*] range.

In order to get insight on the mechanism that determines the improved behavior of INPs when sampled with a time offset, an analysis of time responses and frequency characteristics samples for fixed values of time offset based on Pearson distance has been carried out.

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II. PEARSON CORRELATION

As with improved performance in terms of error probability the difference in the values of error probability of the new INPs due to intersymbol interference (ISI) produced by the sampling with the same time offset diminishes, it is expected that the values of the samples of the impulse response will not differ much. This can be verified by measuring the similarity in shape between the vectors that contain the samples of the impulse response [39]. For this purpose, Pearson correlation [x, y] was used [40].

Consider two vectors $\mathbf{X} = [x_1, x_2, \cdots x_N]$ and $\mathbf{Y} = [y_1, y_2, \cdots y_N]$ of equal length N, which contain samples of the impulse responses of two INFs that are sampled with a time offset.

Let

$$x_i^{'} = x_i - \overline{x} \tag{2}$$

where $\overline{x}$ is the mean of all the entries of **X** [39].

$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{3}$$

Also

$$y_i^{'} = y_i - \overline{y} \tag{4}$$

and

$$\overline{y} = \frac{\sum_{i=1}^{N} y_i}{N} \tag{5}$$

The standard deviation of the entries of a vector **Z** [39] is given by:

$$\sigma_Z = \sum_{i=1}^{N} \frac{(z_i - \overline{z})^2}{N} \tag{6}$$

The Pearson correlation distance is defined [39] as:

$$d = 1 - r \tag{7}$$

where r is the dot product of the scores of the vectors **X** and **Y** that are obtained by subtracting form each vector its mean and dividing the result by the standard deviation of each vector. **X'** and **Y'** are vectors that have the entries $x_i^{'}$ and $y_i^{'}$, respectively, [39].

$$r = \frac{X' \cdot Y'}{\sigma_X \sigma_Y} \tag{8}$$

The Pearson correlation distance is sensitive to a linear relationship between two vectors or variables [40]. In order to evidence also inverse relationships between two vectors the Pearson Squared distance is used [39]. This is defined as:

$$d_2 = 1 - 2r \tag{9}$$

III. ANALYSIS OF TIME RESPONSE

Tables I, II, and III report the values of error probability of several INPs reported in the literature, for several cases of practical interest, namely α=0.25, α=0.35, and α=0.5, respectively and three values of the normalized time offset t/T, 0.05, 0.1, and 0.2. The INPs include RC, BTRC [1] also known as fexp[3], fsech [3], farcsech [3], Poly [5], acos [7], acos[acos] [7], acoxs[asech] [7], acos[log] [7], asech[acos] [7], asech[asech] [7], asech[log] [7], asech [exp] [7], acos[exp] [7], Power [8], acos[asinh] [6], acos[atan] [6], and sin[acosh] [6]. The calculations involved 50 samples of the impulse response

For the sake of compactness, we have denoted the normalized time offset t/T as ε. Here d_{1_n} represents the Pearson distance between RC pulse (number 1) and pulse number n in terms of impulse response. The pulses were listed based on error probability performance. The most performing pulses for a particular value of the normalized time offset t/T were assigned to a higher value of n.

Most pulses have a negative value of the Pearson correlation coefficient in time domain, which indicates a negative association. Notable exceptions are the poly [6], and fexp [1, 4] pulses that exhibit positive values of the Pearson correlation coefficient in time domain.

In addition to the afore mentioned, it is worth noting that for $\alpha = 0.25, 0.35, 0.5$ and different values of normalized time offset t/T, the polynomial pulse [6] has the smallest value of the absolute value of Pearson distance with respect to the standard RC pulse.

We observe that after the poly pulse, the next positive value of Pearson distance with respect to the standard RC pulse is found to be for the fexp pulse. The fexp pulse has received recently much attention in a variety of applications including cochannel interference systems, intersymbol interference systems, direct sequence code division multiple access (DS-CDMA) systems, orthogonal frequency division multiplexing (OFDM) systems, and multiple input-multiple output (MIMO) systems. The fexp pulse has the second smallest value of the absolute value of Pearson distance with respect to the standard RC pulse only for $\alpha = 0.5$.

The pulses with better performance in terms of error probability when sampled with a timing offset show a smaller value of the absolute value of Pearson distance with respect to the standard RC pulse. This is also valid for the frequency characteristic. There is no direct relationship between the improvement in error probability and the decrease of Pearson distance. However, we may observe that the pulses with improved performance show smaller values of the Pearson distance.

IV. ANALYSIS OF FREQUENCY RESPONSE

To infer the influence of the transition region frequency response on the probability of error, 100 samples of the frequency characteristic were taken in the frequency interval [B(1-α), B(1+α)] and the Pearson distance between the RC pulse and pulse number n was calculated and denoted as D_{1_n}.

The results are tabulated in the last column of the Tables I to III for compactness' sake. Table IV provides the values of Pearson distance and Pearson Squared distance for the excess bandwidth α=0.5.

On the same note, as in the precedent section we observe now that the pulse with the smallest value of error probability (POWER pulse) has the smallest absolute value of the Pearson distance with respect to the standard RC in frequency domain, D_{1_n} for the roll off factor $\alpha = 0.35, 0.5$ in frequency domain. The polynomial pulse presents the smallest absolute value for D_{1_n} only for $\alpha = 0.25$. The POWER pulse presents also and the smallest value of the Pearson Squared distance for the excess bandwidth α=0.5.

TABLE I. PEARSON DISTANCES AND ISI ERROR PROBABILITIES OF SEVERAL NYQUIST PULSES FOR $N = 2^9$ INTERFERING SYMBOLS, $T_f = 40$, $M = 61$, $SNR = 15dB$, AND $\alpha = 0.25$

n	Pulse $\alpha = 0.25$	P_e $\varepsilon = \pm 0.05$	d_{1_n}	P_e $\varepsilon = \pm 0.1$	d_{1_n}	P_e $\varepsilon = \pm 0.2$	d_{1_n}	D_{1_n}
1	rcos	8.2189e-8	1	2.8184e-6	1	9.7462e-4	1	1
2	fsech	7.5579e-8	-0.9214	2.3337e-6	-0.9352	7.7201e-4	-0.9557	-0.8025
3	asech[log]	6.1687e-8	-0.9178	1.4808e-6	-0.9325	4.2640e-4	-0.9509	**-0.8049**
4	fexp	5.8117e-8	**0.9744**	1.2980e-6	**0.9718**	3.5678e-4	**0.9662**	**-0.7222**
5	farcsech	5.3996e-8	-0.9186	1.1011e-6	-0.9300	2.8405e-4	-0.9464	-0.7395
6	acos[log]	5.3333e-8	-0.9185	1.0726e-6	-0.9297	2.7420e-4	-0.9458	-0.7323
7	acos[atan]	5.3114e-8	-0.9184	1.0636e-6	-0.9296	2.7117e-4	-0.9456	-0.7263
8	sin[acosh]	5.2941e-8	-0.9184	1.0565e-6	-0.9295	2.6878e-4	-0.9454	-0.7230
9	acos[asinh]	5.1480e-8	-0.9180	9.9816e-7	-0.9288	2.4946e-4	-0.9440	-0.7013
10	acos[exp]	5.1036e-8	-0.9179	9.8109e-7	-0.9285	2.4406e-4	-0.9434	-0.6923
11	acos[asech]	5.0695e-8	-0.9178	9.6775e-7	-0.9283	2.3971e-4	-0.9431	-0.6909
12	acos	5.0636e-8	**-0.9479**	9.6618e-7	-0.9461	2.3940e-4	-0.9430	-0.6850
13	acos[acos]	4.9480e-8	-0.9173	9.2585e-7	-0.9274	2.2747e-4	-0.9413	-0.6579
14	asech[exp]	4.9219e-8	-0.9172	9.1771e-7	-0.9272	2.2534e-4	-0.9409	-0.6483
15	asech[asech]	4.8700e-8	-0.9170	9.0060e-7	-0.9268	2.2053e-4	-0.9400	-0.6387
16	asech[acos]	4.8697e-8	-0.9170	9.0126e-7	-0.9267	2.2096e-4	-0.9399	-0.6338
17	Poly (40, −100, 85)	4.7582e-8	**0.8637**	8.8156e-7	**0.8479**	2.2060e-4	**0.8119**	**-0.5713**
18	Power ($\beta = 0.29$)	**4.6192e-8**	-0.9157	**8.2832e-7**	-0.9243	**2.0300e-4**	-0.9350	-0.6029

TABLE II. PEARSON DISTANCES AND ISI ERROR PROBABILITIES OF SEVERAL NYQUIST PULSES FOR $N = 2^9$ INTERFERING SYMBOLS, $T_f = 40$, $M = 61$, $SNR = 15dB$, AND $\alpha = 0.35$

n	Pulse $\alpha = 0.35$	P_e $\varepsilon = \pm 0.05$	d_{1_n}	P_e $\varepsilon = \pm 0.1$	d_{1_n}	P_e $\varepsilon = \pm 0.2$	d_{1_n}	D_{1_n}
1	cos	5.9997e-8	1	1.3896e-6	1	3.9084e-4	1	1
2	fsech	5.4002e-8	-0.9534	1.0944e-6	-0.9660	2.8000e-4	-0.9779	-0.8025
3	asech[log]	4.2145e-8	-0.9520	6.2866e-7	-0.9647	1.2567e-4	-0.9736	-0.8049
4	fexp	3.9253e-8	**0.9575**	5.4021e-7	**0.9524**	1.0129e-4	**0.9403**	-0.7518
5	farcsech	3.5970e-8	-0.9508	4.4580e-7	-0.9636	7.6203e-5	-0.9694	-0.7395
6	acos[log]	3.5470e-8	-0.9506	4.3365e-7	-0.9634	7.3486e-5	-0.9688	-0.7323
7	acos[atan]	3.5310e-8	-0.9506	4.3008e-7	-0.9634	7.2778e-5	-0.9686	-0.7263
8	sin[acosh]	3.5182e-8	-0.9505	4.2722e-7	-0.9633	7.2196e-5	-0.9685	-0.7230
9	acos[asinh]	3.4124e-8	-0.9501	4.041e-7	-0.9630	6.7653e-5	-0.9670	-0.7013
10	acos[exp]	3.3806e-8	-0.9500	3.9786e-7	-0.9628	6.6617e-5	-0.9665	-0.6923
11	acos[asech]	3.3558e-8	-0.9499	3.9255e-7	-0.9627	6.5582e-5	-0.9661	-0.6909
12	acos	3.3527e-8	**-0.9665**	3.9249e-7	**-0.9662**	**6.5764e-5**	-0.9660	-0.6850
13	Poly (31,−80, 69)	3.2897e-8	**0.8748**	3.8388e-7	**0.8573**	6.5629e-5	**0.8151**	**-0.7854**
14	acos[acos]	3.2753e-8	-0.9495	3.7964e-7	-0.9623	6.4348e-5	-0.9644	-0.6579
15	asech[exp]	3.2591e-8	-0.9493	3.7775e-7	-0.9622	**6.4445e-5**	-0.9639	-0.6483
16	asech[acos]	3.2264e-8	-0.9491	3.7363e-7	-0.9619	**6.4494e-5**	-0.9630	-0.6338
17	asech[asech]	3.2255e-8	-0.9491	3.7275e-7	-0.9620	6.4110e-5	**-0.9631**	**-0.6387**
18	Power ($\beta = 0.32$)	3.0955e-8	-0.9481	3.5466e-7	-0.9606	6.4326e-5	-0.9591	-0.6155

TABLE III. PEARSON DISTANCES AND ISI ERROR PROBABILITIES OF SEVERAL NYQUIST PULSES FOR $N = 2^9$ INTERFERING SYMBOLS, $T_f = 40$, $M = 61$, $SNR = 15dB$, AND $\alpha = 0.5$

n	Pulse $\alpha = 0.5$	P_e $\varepsilon = \pm 0.05$	d_{1_n}	P_e $\varepsilon = \pm 0.1$	d_{1_n}	P_e $\varepsilon = \pm 0.2$	d_{1_n}	D_{1_n}
	rcos	3.9723e-8	1	5.4890e-7	1	1.0217e-4	1	1
2	fsech	3.4949e-8	-0.9853	4.1186e-7	-0.9903	6.6009e-5	-0.9962	**0.9443**
3	asech[log]	2.6157e-8	-0.9842	2.1763e-7	-0.9886	2.5364e-5	-0.9940	0.9966
4	fexp	2.4134e-8	**0.9157**	1.8580e-7	**0.9009**	2.0878e-5	**0.8611**	0.9813
5	farcsech	2.1875e-8	-0.9833	1.4916e-7	-0.9871	1.5344e-5	-0.9915	0.9847
6	acos[log]	2.1559e-8	-0.9833	1.4514e-7	-0.9869	1.4987e-5	-0.9912	0.9826
7	acos[atan]	2.1462e-8	-0.9832	1.4410e-7	-0.9868	1.4953e-5	-0.9910	0.9810
8	sin[acosh]	2.1386e-8	-0.9832	1.4323e-7	-0.9867	1.4911e-5	-0.9909	0.9801
9	acos[asinh]	2.0758e-8	-0.9829	1.3617e-7	-0.9862	1.4609e-5	-0.9899	0.9727
10	acos[exp]	2.0583e-8	-0.9828	1.3446e-7	-0.9860	**1.4657e-5**	-0.9896	0.9694
11	Poly (25, −64, 55)	2.0574e-8	**0.7749**	1.3539e-7	**0.7273**	**1.5197e-5**	**0.5964**	**0.9704**
12	acos[asech]	2.0438e-8	-0.9827	1.3273e-7	-0.9859	1.4563e-5	-0.9893	0.9685
13	acos	2.0431e-8	**-0.9865**	**1.3300e-7**	**-0.9873**	**1.4717e-5**	-0.9892	0.9666
14	acos[acos]	2.0054e-8	-0.9824	1.3014e-7	-0.9852	**1.5328e-5**	-0.9880	0.9553
15	asech[exp]	1.9992e-8	-0.9823	1.3005e-7	-0.9850	**1.5658e-5**	-0.9876	0.9512
16	asech[acos]	1.9865e-8	-0.9821	1.2958e-7	-0.9847	**1.6248e-5**	-0.9870	0.9443
17	asech[asech]	1.9845e-8	-0.9821	1.2902e-7	-0.9847	**1.6057e-5**	-0.9870	**0.9462**
18	Power ($\beta = 0.37$)	1.9451e-8	-0.9817	1.2504e-7	-0.9839	**1.6619e-5**	-0.9854	0.9405

TABLE IV. ISI ERROR PROBABILITIES OF SEVERAL NYQUIST PULSES FOR $N = 2^9$ INTERFERING SYMBOLS, $T_f = 40$, $M = 61$, $SNR = 15dB$, AND $\alpha = 0.5$

n	Pulse $\alpha = 0.5$	P_e $\varepsilon = 0.05$	P_e $\varepsilon = 0.1$	P_e $\varepsilon = 0.2$	D_{1_n}	D_{2_n}
	rcos	3.9723e-8	5.4890e-7	1.0217e-4	1	-1
2	fsech	3.4949e-8	4.1186e-7	6.6009e-5	**0.9443**	-0.8886
3	asech[log]	2.6157e-8	2.1763e-7	2.5364e-5	0.9966	-0.9932
4	fexp	2.4134e-8	1.8580e-7	2.0878e-5	0.9813	-0.9626
5	farcsech	2.1875e-8	1.4916e-7	1.5344e-5	0.9847	-0.9695
6	acos[log]	2.1559e-8	1.4514e-7	1.4987e-5	0.9826	-0.9652
7	acos[atan]	2.1462e-8	1.4410e-7	1.4953e-5	0.9810	-0.9621
8	sin[acosh]	2.1386e-8	1.4323e-7	1.4911e-5	0.9801	-0.9601
9	acos[asinh]	2.0758e-8	1.3617e-7	1.4609e-5	0.9727	-0.9454
10	acos[exp]	2.0583e-8	1.3446e-7	**1.4657e-5**	0.9694	-0.9389
11	Poly (25, −64, 55)	2.0574e-8	1.3539e-7	**1.5197e-5**	**0.9704**	-0.9408
12	acos[asech]	2.0438e-8	1.3273e-7	1.4563e-5	0.9685	-0.9370
13	acos	2.0431e-8	**1.3300e-7**	**1.4717e-5**	0.9666	-0.9332
14	acos[acos]	2.0054e-8	1.3014e-7	**1.5328e-5**	0.9553	-0.9107
15	asech[exp]	1.9992e-8	1.3005e-7	**1.5658e-5**	0.9512	-0.9025
16	asech[acos]	1.9865e-8	1.2958e-7	**1.6248e-5**	0.9443	-0.8886
17	asech[asech]	1.9845e-8	1.2902e-7	**1.6057e-5**	**0.9462**	-0.8924
18	Power ($\beta = 0.37$)	1.9451e-8	1.2469e-7	**1.6619e-5**	0.9405	-0.8809

I. CONCLUSION

The improved Nyquist pulses display a smaller value of the absolute value of Pearson distance with respect to the standard RC pulse both for the impulse response and the frequency characteristic. Unfortunately, no direct relationship between the improvement in error probability and the decrease of Pearson distance has been found. Most pulses have a negative value of the Pearson correlation coefficient in time domain, which indicates a negative association with respect to the standard RC pulse. This shows that as the value of the RC impulse response samples increase, the value of most INPs impulse responses decrease. Notable exceptions are the poly [6], and fexp [1, 4] pulses that exhibits positive values of the Pearson correlation coefficient in time domain, which decrease with increased normalized time offset. However, the improved pulses display smaller values of the Pearson distance with respect to the standard RC pulse for higher values of the excess bandwidth.

One outstanding question which arises from the description above is: how do we correlate the behavior of the improved pulses with the relationship between the improvement in error probability and the decrease of Pearson distance? This discussion will facilitate our further research and provide a basis for obtaining new improved Nyquist pulses, which outperform the standard RC pulse and previously reported INPs and are suitable for a variety of applications such as communication systems perturbed by co-channel and intersymbol interference, direct sequence code division multiple access (DS-CDMA) systems, orthogonal frequency division multiplexing (OFDM) systems, and multiple input-multiple output (MIMO) systems.

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