TIME – OPTIMAL CONTROL SYNTHESIS USING NEURAL NETWORKS

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Abstract. *In the paper time-optimal control synthesis using neural networks is presented. The problem is reduced to the problem of determing the swithing hyperplane. The method of phase space is used for deriving the equition of the switching surface. Neural networks are used for its picewise linear approximation.*

Introduction

The problem of time-optimal control is well studied. But still the controllers have very complicated structure. In the present paper an attempt is made to simplify the time-optimal controller structure.

Time - optimal control synthesis

Let the continuous controllable plant in the system with feedback in the general case to be described by the equation of movement:

$$\frac{dx}{dt} = f(x,u,t)$$

where x is n-dimensional vector and u has r coordinates. The control action is bounded by the condition:

$$u \in \Omega(u)$$

where $\Omega(u)$ is admissible closed domain.

Let in the particular case the plant to be characterized by the equation:

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_n x = u(t)$$

and the control to be bounded by:

$$|u(t)| \geq M$$

Let the theorem about the n intervals to hold. The optimal process $x(t)$ consists of n intervals and the process in each of these intervals is described by:

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_n x = \sigma M$$

where the number is constant in each of the intervals and $\sigma = \pm 1$

The signs of σ alternate in neighbouring intervals. Thus the main problem is reduced to determing the control action in each point in the phase space in time t:

$$u = u(x,t)$$

In order to determine this dependence the phase space of the error is considered:

$$\varepsilon = x^* - x$$

with coordinates $\varepsilon_i = x_i^* - x_i$ (i =1, 2, ..., n) where x_i are the coordinates of the plant and x_i^* are the solution of the equation (1) with bounded control (2). For further convinience ε_i will be substituted by x_i. By virtue of the theorem for the n intervals, the value of u in each point of the phase space may be + M or - M i.e. if $u = \sigma M$ then σ = + 1 or σ = - 1. Thus value σ = + 1 or value σ = - 1 corresponds to each point of the phase space. Because of that in each time the whole phase space is devided into two domains which are chatacterized by the values σ = + 1 or σ = - 1. These domains are separated by a boundary which represents (n-1) - dimensional switching hyperplane. The synthesis problem is reduced to determing the hyperplane in each of these times.

Let $\tau_1, \tau_2, \ldots, \tau_{n-1}$ are the times of switching of the sign of σ and T is the minimal time for transition from the initial condition to the final condition. If in the equation:

$$a_0 \frac{d^n x}{dt^n} + a_1 \frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_n x = u(t)$$

we substitute the argument t by $\tau = T - t$ a new equation will be obtained:

$$a_0(-1)^n \frac{d^n x}{d\tau^n} + a_1(-1)^{n-1} \frac{d^{n-1}x}{d\tau^{n-1}} + \ldots$$
$$+ a_{n-1}(-1)\frac{dx}{d\tau} + a_n x = u(t)$$

This equation is solved for the "initial" conditions:

$$(x)_{\tau=0} = \left(\frac{dx}{d\tau}\right)_{\tau=0} = \ldots = \left(\frac{d^{n-1}x}{d\tau^{n-1}}\right)_{\tau=0} = 0$$

at $u = +M$ i.e. at $\sigma = +1$. Then the trajectory of the point movement at in the paramet form $x = x(\tau)$ will be obtained. At $\tau = \tau_1$ the coresponding of this trajectory point of the phase space has coordinates $x(\tau_1)$. If at this time the u = + M is substituted by u = - M then the solution of the equation (9) at time $\tau_2 > \tau_1$ will be function of τ_1 as well as of τ_2 i.e. $x = x(\tau_1, \tau_2)$. By switching the sign of σ in the times $\tau_1, \tau_2, \ldots, \tau_{n-1}$ the solution of the equation will be obtained in the form:

$$x = x(\tau_1, \tau_2, \ldots, \tau_{n-1})$$

The equation for the coordinates $x_1, x_2, \ldots, x_n$ of the vector x is as follows:

$$x_i = x_i(\tau_1, \tau_2, \ldots, \tau_{n-1}) \quad (i=1,2,\ldots,n)$$

These solutions represent the equation of the switching surface in the parametrical form and thus the synthesis problem is solved. By excluding of from equation (13) the parameters $\tau_1, \tau_2, \ldots, \tau_{n-1}$ it can be obtained the equation for switching surface in the form of the correlations relating the coordinates $x_1, x_2, \ldots, x_n$:

$$\psi(x_1, x_2, \ldots, x_n) = 0$$

Let the function Ψ is positive in one of the side of the hyperplane and negative in the other one. Let for example $\Psi > 0$ for points from the domain $\sigma = +1$ and $\Psi < 0$ for points from the domain $\sigma = +1$. Then it can be stated:

$$\sigma = sign\ \psi$$

and

$$u = M\sigma = Msign\ \psi = Msign\,\psi(x_1, x_2, \ldots, x_n)$$

Neural Networks

For signum function realization it can be used a perceptron. The response of the perceptron is as follows:

$$sign(v) = \begin{cases} +1 & v > 0 \\ -1 & v < 0 \end{cases}$$

where $v = w^T(n)x(n)$; w is the weight vector and $x(n)$ is the input vector.

The goal of the perceptron is to classify correctly the input set $x_1, x_2, \ldots, x_m$ into one of the two classes ϑ_1 or ϑ_2. The decision rule for the classification is to assign the points represented by the inputs $x_1, x_2, \ldots, x_m$ to class ϑ_1 if the perceptron output is +1 and to ϑ_2 if the perceptron output is –1.

As the switching hyperplane is, in general nonlinear the perseptron will realize a piecewise approximation of the nonlinear switching surface. For the purpose the phase space is quantized forming elementary hypercubes in which the control action is assumed constant. It is realized by using selforganizing maps.

Reference

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