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TUNING OF LINEAR REGULATORS TO THE THIRD AND FOURTH ORDER ADVANCE DELAY MODELS OF OBJECTS WITH NONMINIMAL PHASE

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Abstract: *An algebraically procedure for tuning of typical regulators to the third and fourth order advance delay models of objects with nonminimal phase is proposed in this paper. As the typical regulator are used the P, PI, PID control algorithms. For tuning of typical control algorithms to the class of objects'models with known parameters the maximal stability degree method is used. For analysis of obtained results in conformity with this method the known tuning method are applied.*
Keywords: *the object with advance delay, regulator, tuning of regulators, the maximal and optimal stability degree of the system, nonminimal phase.*

Introduction

It is considered that the mathematical model of the regulated object is presented by the following transfer function [1]:

$$H_1(s) = \frac{(b_1 - b_0 s)}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \qquad (1)$$

$$H_2(s) = \frac{(b_1 - b_0 s)}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \qquad (2)$$

where b_0, b_1 the coefficients of numerator; a_0, a_1, a_2, a_3, a_4 the coefficients of denominator of the objects (1), (2) respectively. The models of objects of forms (1) and (2) are named the objects'models with third and fourth order advance delay objects. Assume that for the class of dynamic models (1), (2) with known parameters it's necessary to tune the following types of regulators P, PI, PID.
Because the models of objects (1), (2) are complex, some of known tuning methods of regulators for these objects can't be used (the Ziegler-Nichols method) or they are accompanied by difficult calculations (the parametrical optimization method).
In the work [2] for tuning of linear regulators to the inertial objects with time delay the maximal stability degree method of the designed monovariable system is used.
In the works [3-7] maximal stability degree (MSD) method is spreaded for tuning of P, PI, PID types of linear regulators for other categories of objects.
For getting over these difficulties the using of the maximal stability degree method to the designer stage of P, PI, PID typical algorithms for class of dynamic models, represented in forms (1), (2) is proposed in this paper.

The tuning algorithm of regulators

Assume that the control system is formed of object, which has forms (1), (2) and P, PI, PID regulators and after some transformation will have the forms:

- for the control system with the object (1):

$$A_1(p) = \frac{(a_0 p^3 + a_1 p^2 + a_2 p + a_3)}{b_1 - b_0 p} + k_p = 0; \qquad (3)$$

$$A_2(p) = \frac{(a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p)}{b_1 - b_0 p} + \qquad (4)$$
$$+ k_p p + k_i = 0;$$

$$A_3(p) = \frac{(a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p)}{b_1 - b_0 p} + \qquad (5)$$
$$+ k_d p^2 + k_p p + k_i = 0;$$

- for the control system with the object (2):

$$A_1(p) = \frac{(a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4)}{b_1 - b_0 p} + \qquad (6)$$
$$+ k_p = 0;$$

$$A_2(p) = \frac{(a_0 p^5 + a_1 p^4 + a_2 p^3 + a_3 p^2 + a_4 p)}{b_1 - b_0 p} +$$
$$+ k_p p + k_i = 0; \qquad (7)$$

$$A_3(p) = \frac{(a_0 p^5 + a_1 p^4 + a_2 p^3 + a_3 p^2 + a_4 p)}{b_1 - b_0 p} +$$
$$+ k_d p^2 + k_p p + k_i = 0, \qquad (8)$$

where k_p, k_i, k_d are the tuning parameters of regulators P, I, D respectively. The expressions (3)…(8) are rewritten using the substitution $p = -J$.

- For the control system with the object (1):

$$A_1(-J) = \frac{(-a_0 J^3 + a_1 J^2 - a_2 J + a_3)}{b_1 + b_0 J} + \qquad (9)$$
$$+ k_p = 0;$$

$$A_2(-J) = \frac{(a_0 J^4 - a_1 J^3 + a_2 J^2 - a_3 J)}{b_1 + b_0 J} - \qquad (10)$$
$$- k_p J + k_i = 0;$$

$$A_3(-J) = \frac{(a_0 J^4 - a_1 J^3 + a_2 J^2 - a_3 J)}{b_1 + b_0 J} + \qquad (11)$$
$$+ k_d J^2 - k_p J + k_i = 0;$$

- for the control system with the object (2):

$$A_1(-J) = \frac{(a_0 J^4 - a_1 J^3 + a_2 J^2 - a_3 J + a_4)}{b_1 + b_0 J} + \qquad (12)$$
$$+ k_p = 0;$$

$$A_2(-J) =$$
$$= \frac{(-a_0 J^5 + a_1 J^4 - a_2 J^3 + a_3 J^2 - a_4 J)}{b_1 + b_0 J} - \qquad (13)$$
$$- k_p J + k_i = 0;$$

$$A_3(-J) =$$
$$= \frac{(-a_0 J^5 + a_1 J^4 - a_2 J^3 + a_3 J^2 - a_4 J)}{b_1 + b_0 J} + \qquad (14)$$
$$+ k_d J^2 - k_p J + k_i = 0,$$

where J is the maximal stability degree of the designed system and need to be determined. From the expressions (9)…(14) it is taken the first, second and third order derivatives, in conformity with the number of tuning parameters of respectively regulator, it is obtained the algebraic expressions which allow to determine the values of the maximal stability degrees of the designed systems. From algebraic equation, obtained through derivation on the variable J, algebraic expressions for the numeric values determination of the adjustment parameters are obtained for P, PI, PID regulators and these after some transformation will have the form:

- for the control system with the object (1):

- for the control system with P regulator:

$$-c_0 J^3 + c_1 J^2 + c_2 J - c_3 = 0, \qquad (15)$$

where $c_0 = 2a_0 b_0$; $c_1 = -3a_0 b_1 + b_0 a_1$; $c_2 = 2a_1 b_1$; $c_3 = a_2 b_1 + b_0 a_3$;

- for the control system with PI regulator:

$$c_0 J^5 + c_1 J^4 + c_2 J^3 + c_3 J^2 + c_4 J - c_5 = 0, \qquad (16)$$

where $c_0 = 6a_0 b_0^3$; $c_1 = 22a_0 b_0^2 b_1 - 2b_0^3 a_1$;

$c_2 = 28a_0 b_0 b_1^2 - 8a_1 b_0^2 b_1$;

$c_3 = 12a_0 b_1^3 - 12a_1 b_0 b_1^2$;

$c_4 = -6a_1 b_1^3 + 2a_2 b_0 b_1^2 + 2a_3 b_0^2 b_1$;

$c_5 = 2a_2 b_1^3 + 2a_3 b_0 b_1^2$;

- for the control system with PID regulator:

$$c_0 J^8 + c_1 J^7 + c_2 J^6 + c_3 J^5 + c_4 J^4 + + c_5 J^3 + c_6 J^2 + c_7 J - c_8 = 0, \quad (17)$$

where $c_0 = 1$; $c_1 = 8\beta$; $c_2 = 28\beta^2$; $c_3 = 56\beta^3$;

$c_4 = 69\beta^4 - \alpha_1\beta^3 - \alpha_2\beta^2 - \alpha_3\beta$;

$c_5 = 52\beta^5 - 4\alpha_1\beta^4 - 4\alpha_2\beta^3 - 4\alpha_3\beta^2$;

$c_6 = 22\beta^6 - 6\alpha_1\beta^5 - 6\alpha_2\beta^4 - 6\alpha_3\beta^3$;

$c_7 = 4\beta^7 - 4\alpha_1\beta^6 - 4\alpha_2\beta^5 - 4\alpha_3\beta^4$;

$c_8 = \alpha_1\beta^7 + \alpha_2\beta^6 + \alpha_3\beta^5$,

where

$\beta = b_1/b_0; \alpha_1 = a_1/a_0; \alpha_2 = a_2/a_0; \alpha_3 = a_3/a_0$;

- for the control system with the object (2):

- for the control system with P regulator:

$$c_0 J^4 + c_1 J^3 + c_2 J^2 + c_3 J - c_4 = 0, \quad (18)$$

where $c_0 = 3a_0 b_0$; $c_1 = 4a_0 b_1 - 2b_0 a_1$;

$c_2 = a_2 b_0 - 3a_1 b_1$; $c_3 = 2a_2 b_1$; $c_4 = a_3 b_1 + a_4 b_0$;

- for the control system with PI regulator:

$$-c_0 J^6 + c_1 J^5 + c_2 J^4 + + c_3 J^3 + c_4 J^2 + c_5 J + c_6 = 0, \quad (19)$$

where $c_0 = 1$; $c_1 = -3{,}5\beta + 0{,}5\alpha_1$;

$c_2 = -4{,}1667\beta^2 + 1{,}8333\alpha_1\beta - 0{,}1667\alpha_2$;

$c_3 = -1{,}1667\beta^3 + 2{,}3333\alpha_1\beta^2 - 0{,}6667\alpha_2\beta$;

$c_4 = \alpha_1\beta^3 - \alpha_2\beta^2$;

$c_5 = -0{,}5\alpha_2\beta^3 + 0{,}1667\alpha_3\beta^2 + 0{,}1667\alpha_4\beta$;

$c_6 = 0{,}1667\alpha_3\beta^3 + 0{,}1667\alpha_4\beta^2$;

- for the control system with PID regulator:

$$-c_0 J^9 + c_1 J^8 + c_2 J^7 + c_3 J^6 + c_4 J^5 + + c_5 J^4 + c_6 J^3 + c_7 J^2 + c_8 J - c_9 = 0, \quad (20)$$

where $c_0 = 1$; $c_1 = -7{,}75\beta + 0{,}25\alpha_1$;

$c_2 = -26\beta^2 + 2\alpha_1\beta$;

$c_3 = -49\beta^3 + 7\alpha_1\beta^2$;

$c_4 = -56\beta^4 + 14\alpha_1\beta^3$;

$c_5 = -38{,}75\beta^5 + 17{,}25\alpha_1\beta^4 - - 0{,}25\alpha_2\beta^3 - 0{,}25\alpha_3\beta^2 - 0{,}25\alpha_4\beta$;

$c_6 = -15\beta^6 + 13\alpha_1\beta^5 - - \alpha_2\beta^4 - \alpha_3\beta^3 - \alpha_4\beta^2$;

$c_7 = -2{,}5\beta^7 + 5{,}5\alpha_1\beta^6 - - 1{,}5\alpha_2\beta^5 - 1{,}5\alpha_3\beta^4 - 1{,}5\alpha_4\beta^3$;

$c_8 = \alpha_1\beta^7 - \alpha_2\beta^6 - \alpha_3\beta^5 - \alpha_4\beta^4$;

$c_9 = 0{,}25\alpha_2\beta^7 + 0{,}25\alpha_3\beta^6 + 0{,}25\alpha_4\beta^5$,

where in the expressions (19) and (20):

$\beta = b_1/b_0$; $\alpha_1 = a_1/a_0$; $\alpha_2 = a_2/a_0$;

$\alpha_3 = a_3/a_0$; $\alpha_4 = a_4/a_0$.

The solution of equations (15)…(20) allows to obtain the numerical values of the maximal stability degrees of control system with respectively regulators *P, PI* and *PID.*

The value of optimum stability degree of control system is choosed from condition [2]:

$$J = J_{opt} = -\min \max \operatorname{Re} p_i , \quad (21)$$

where Rep_i are the real roots or the real parts of the complex roots of the algebraic equations (15)-(20) obtained from the characteristical equations.

The value of the optimum degree is the smallest positive root (or real part) of algebraic equations (15)-(20). In some cases the values of the maximal stability degree can be determined by approximations in opposite of the optimal degree parameter.

The expressions for determination of tuning parameters of respectively regulators *P, PI, PID* after some transformations have the form:

- for the control system with the object (1):

- for P regulator:

$$k_p = \frac{a_0 J^3 - a_1 J^2 + a_2 J - a_3}{b_1 + b_0 J}; \quad (22)$$

- for PI regulator:

$$k_p = \frac{d_0 J^4 + d_1 J^3 + d_2 J^2 + d_3 J - d_4}{(b_1 + b_0 J)^2}, \quad (23)$$

where

$d_0 = 3a_0b_0;\ d_1 = 4a_0b_1 - 2a_1b_0;$
$d_2 = a_2b_0 - 3a_1b_1;\ d_3 = 2a_2b_1; d_4 = a_3b_1;$

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J^2 + a_3J}{b_1 + b_0J} + k_pJ; \quad (24)$$

- for PID regulator:

$$k_d = \\ = \frac{-d_0J^5 - d_1J^4 - d_2J^3 - d_3J^2 - d_4J - d_5}{2(b_1 + b_0J)^4}, \quad (25)$$

where $d_0 = 6a_0b_0^3;\ d_1 = 22a_0b_0^2b_1 - 2b_0^3a_1;$
$d_2 = 28a_0b_0b_1^2 - 8a_1b_0^2b_1;$
$d_3 = 12a_0b_1^3 - 12a_1b_0b_1^2;$
$d_4 = -6a_1b_1^3 + 2a_2b_0b_1^2 + 2a_3b_0^2b_1;$
$d_5 = 2a_2b_1^3 + 2a_3b_0b_1^2;$

$$k_p = \frac{d_0J^4 + d_1J^3 + d_2J^2 + d_3J - d_4}{(b_1 + b_0J)^2} + \\ + 2k_dJ, \quad (26)$$

where
$d_0 = 3a_0b_0;\ d_1 = 4a_0b_1 - 2a_1b_0;$
$d_2 = a_2b_0 - 3a_1b_1;\ d_3 = 2a_2b_1; d_4 = a_3b_1;$

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J^2 + a_3J}{b_1 + b_0J} - \\ - k_dJ^2 + k_pJ. \quad (27)$$

- for the control system with the object (2):

- for P regulator:

$$k_p = \frac{-a_0J^4 + a_1J^3 - a_2J^2 + a_3J - a_4}{b_1 + b_0J}; \quad (28)$$

- for PI regulator:

$$k_p = \frac{-d_0J^5 + d_1J^4 + d_2J^3 + d_3J^2 + d_4J - d_5}{(b_1 + b_0J)^2}, \quad (29)$$

where

$d_0 = 4a_0b_0;\ d_1 = 3a_1b_0 - 5a_0b_1;$
$d_2 = 4a_1b_1 - 2a_2b_0;\ d_3 = a_3b_0 - 3a_2b_1;$
$d_4 = 2a_3b_1; d_5 = a_4b_1;$

$$k_i = \frac{a_0J^5 - a_1J^4 + a_2J^3 - a_3J^2 + a_4J}{b_1 + b_0J} + k_pJ; \quad (30)$$

- for PID regulator:

$$k_d = \frac{d_0J^6 - d_1J^5 - d_2J^4 - d_3J^3 - d_4J^2 - d_5J - d_6}{2(b_1 + b_0J)^4}, \quad (31)$$

where $d_0 = 12a_0b_0^3;\ d_1 = -42a_0b_0^2b_1 + 6b_0^3a_1;$
$d_2 = -50a_0b_0b_1^2 + 22a_1b_0^2b_1 - 2a_2b_0^3;$
$d_3 = -20a_0b_1^3 + 28a_1b_1^2b_0 - 8a_2b_0^2b_1;$
$d_4 = 12a_1b_1^3 - 12a_2b_0b_1^2;$
$d_5 = -6a_2b_1^3 + 2a_3b_0b_1^2 + 2a_4b_1b_0^2;$
$d_6 = 2a_3b_1^3 + 2a_4b_0b_1^2;$

$$k_p = \frac{-d_0J^5 + d_1J^4 + d_2J^3 + d_3J^2 + d_4J - d_5}{(b_1 + b_0J)^2} + \\ + 2k_dJ, \quad (32)$$

where
$d_0 = 4a_0b_0;\ d_1 = 3a_1b_0 - 5a_0b_1;$
$d_2 = 4a_1b_1 - 2a_2b_0;\ d_3 = a_3b_0 - 3a_2b_1;$
$d_4 = 2a_3b_1; d_5 = a_4b_1;$

$$k_i = \frac{a_0J^5 - a_1J^4 + a_2J^3 - a_3J^2 + a_4J}{b_1 + b_0J} - \\ - k_dJ^2 + k_pJ. \quad (33)$$

With thise calculations the tuning algorithm of regulators for given models of objects forms (1), (2) is over.

Application and computer simulation

To show the efficiently to the proposed algorithms for tuning the P, PI, PID typical regulators we'll study an example with the model of object which has the following parameters:

- for the object (1):
$b_0 = 0.25;\ b_1 = 0,5;\ a_0 = 5;\ a_1 = 13,5;\ a_2 = 7,5;$
$a_3 = 1;$ and $\beta = 2;\ \alpha_1 = 2,7;\ \alpha_2 = 1,5; \alpha_3 = 0.2;$

- for the object (2):

$b_0 = 0,1667;\ b_1 = 0,5; a_0 = 1,6667;\ a_1 = 4,5;$
$a_2 = 12,5;\ a_3 = 7,3333;\ a_4 = 1;$ and $\beta = 3;$
$\alpha_1 = 2,7;\ \alpha_2 = 7,5;\ \alpha_3 = 4,4;\ \alpha_4 = 0,6.$

Is required to tune the following regulators P, PI, PID:
Doing the respectively calculations in conformity with the elaborated algorithm for the given object we obtained the following results:

- for the control system with the object (1):
- for the control system with P regulator:
  $J = 0,34;\ k_p = 0,318;$
- for the control system with PI regulator:
  $J = 0,21;\ k_p = 0,99;$
  $k_i = 0,21;\ (T_i = 5,097s).$
- for the control system with PID regulator:
  $J = 0,5755;\ k_p = 6,17;$
  $k_i = 1,0961;\ (T_i = 0,91s);$
  $k_d = 7,957s.$

- for the control system with the object (2):

- for the control system with P regulator:
  $J = 0,33;\ k_p = 0,362;$
- for the control system with PI regulator:
  $J = 0,221;\ k_p = 1,1;$
  $k_i = 0,221;\ (T_i = 4,523s).$
- for the control system with PID regulator:
  $J = 0,6271;\ k_p = 13,1431;$
  $k_i = 2,8822;\ (T_i = 0,347s);$
  $k_d = 14,8555s.$

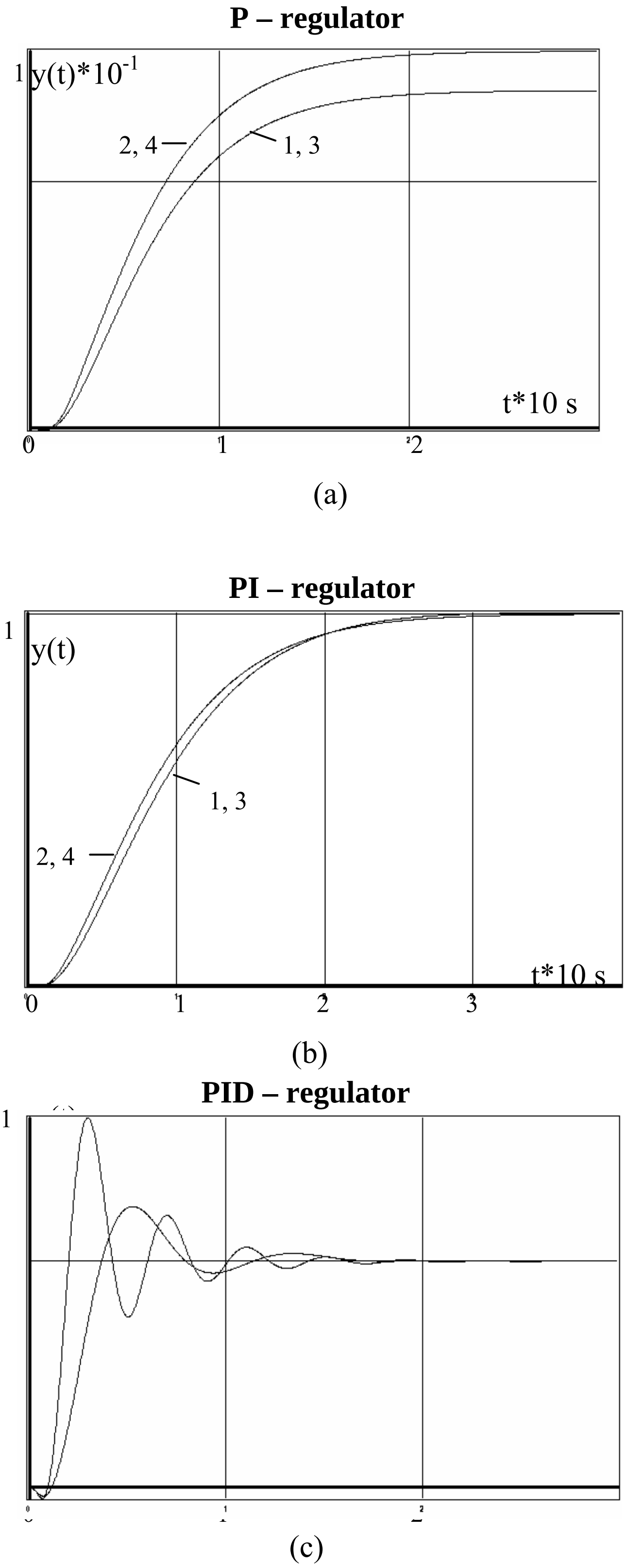

Fig. 1. The transient responses of control systems

On the computer was simulated the control system with the given objects'models forms (1), (2) and respectively regulator P, PI, PID. The results of computer simulations are represented in the figure 1 (form (1), curves 1 and form (2), curves 2).
For comparison the obtained results for tuning of regulators in conformity with proposed method were applied the parametrical optimization method (Poisk-2) and have been obtained approximately the same results of values of tuning parameters. The results of computer simulations are represented in figure 1 (form (1), curves 3 and form (2), curves 4).

Conclusions

As a result of the study, which was made for given class of objects models, the following conclusion can be made:

1. The proposed tuning algorithm for linear regulators P, PI, PID to given third and fourth order advance delay objects'models with nonminimal phase (1), (2) represents an algebraic method.
2. The proposed tuning algorithm represents a simple procedure which consists of following stages:

- the value of optimum stability degree of the designed system with respectively type of regulator is determined;
- the tuning parameters of respectively regulators are determined from algebraic expressions.

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