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THE APPROXIMATE MODELS OF OBJECTS WITH SECOND ORDER INERTIA AND TIME DELAY AND TUNING OF CONTROLLERS

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Abstract: *The analysis of the approximating of the term with time delay of transfer function of the control object's models with second order inertia and time delay using minimal and nonminimal phase approximants Pade with purpose to represented the transfer function of object in rational form is proposed in this paper. For these type of approximate objects' models the typical controllers are tuning in conformity of the maximal stability degree method. The tuning procedure represents an algebraic method and needs a limited volume of calculations. The results obtained after tuning of controllers to the approximate objects' models in conformity of the proposed method are compare with the results obtained in conformity of the other known method.*
Keywords: *models of objects, time delay, approximants Pade, nonminimal phase, tuning of regulators, maximal stability degree method.*

Introduction

At the automation of many slow technological processes the mathematical models of these control processes usually are represented as models with inertia with respectively order and have time delay. In consequence the object's model is an irrational function and practical realization of this model is difficult. To bypass these above-cited inconveniences the term with time delay from transfer function of object is approximated with rational forms which are known as approximate Pade [1, 2].
It is considered that the control object is represented by the object's models with second order inertia and time delay:

$$H(s)=\frac{k\,e^{-\tau s}}{(T_1 s+1)(T_2 s+1)}=\frac{k\,e^{-\tau s}}{a_0 s^2+a_1 s+a_2}, \qquad (1)$$

where k, T_1, T_2, τ are the parameters of object, and $a_0=T_1T_2$, $a_1=T_1+T_2$, $a_2=1$.

The term $\exp(-\tau s)$ from (1) which represented the time delay of process presented a transcendent expression.

The approximate models of object's transfer function

In most cases the term $\exp(-\tau s)$ is approximated by approximants Pade [1,2]. Mostly the following approximants Pade are using:

$$\exp(-\tau s)\approx\frac{1}{1+\tau s}; \qquad (2)$$

$$\exp(-\tau s)\approx\frac{1-0{,}5\tau s}{1+0{,}5\tau s}; \qquad (3)$$

$$\exp(-\tau s)\approx\frac{1}{1+\tau s+0{,}5\tau^2 s^2}; \qquad (4)$$

$$\exp(-\tau s)\approx\frac{1-\frac{\tau s}{3}}{1+\frac{\tau s}{3}+\frac{\tau^2 s^2}{6}}, \qquad (5)$$

where (2), (4) are minimal phase and (3), (5), are nonminimal phase approximants Pade.
In [2] is used the following minimal phase approximate:

$$\exp(-\tau s) \approx \frac{1}{(\frac{\tau s}{n}+1)^n}. \quad (6)$$

The transfer function (1) by approximants Pade (2)…(6) is represented in following forms respectively:

$$H(s) = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \quad (7)$$

where $a_0 = T_1 T_2 \tau$;

$a_1 = T_1 T_2 + \tau(T_1 + T_2)$;

$a_2 = T_1 + T_2 + \tau$; $a_3 = 1$;

$$H(s) = \frac{b_1 - b_0 s}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \quad (8)$$

where $a_0 = 0{,}5 T_1 T_2 \tau$;

$a_1 = T_1 T_2 + 0{,}5\tau(T_1 + T_2)$;

$a_2 = T_1 + T_2 + 0{,}5\tau$; $a_3 = 1$;

$b_0 = 0{,}5\tau k$; $b_1 = k$;

$$H(s) = \frac{k}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \quad (9)$$

where $a_0 = 0{,}5 T_1 T_2 \tau^2$;

$a_1 = \tau T_1 T_2 + 0{,}5\tau^2 (T_1 + T_2)$;

$a_2 = T_1 T_2 + \tau(T_1 + T_2) + 0{,}5\tau^2$;

$a_3 = \tau + T_1 + T_2$; $a_4 = 1$;

$$H(s) = \frac{b_1 - b_0 s}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \quad (10)$$

where $a_0 = \frac{T_1 T_2 \tau^2}{6}$; $a_1 = \frac{\tau T_1 T_2}{3} + \frac{\tau^2 (T_1 + T_2)}{6}$;

$a_2 = T_1 T_2 + \frac{\tau(T_1 + T_2)}{3} + \frac{\tau^2}{6}$;

$a_3 = T_1 + T_2 + \frac{\tau}{3}$; $a_4 = 1$; $b_0 = \frac{\tau k}{3}$; $b_1 = k$;

$$H(s) = \frac{k}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \quad (11)$$

where $n = 2$; $a_0 = 0{,}25 T_1 T_2 \tau^2$;

$a_1 = \tau T_1 T_2 + 0{,}25\tau^2 (T_1 + T_2)$;

$a_2 = T_1 T_2 + \tau(T_1 + T_2) + 0{,}25\tau^2$;

$a_3 = T_1 + T_2 + \tau$; $a_4 = 1$;

After approximates have been obtained two classes of objects' models:

- the models (7), (9), (11) represent models with third and fourth order inertia respectively;
- the models (8), (10), represent models with third and fourth order advance delay respectively and with nonminimal phase.

Further analysing the influence of obtained approximates (7)…(11) on performances of the automatic control system with approximated models in comparison with the performances of the control system with initial object's model (1).

Tuning of regulators to the approximate models

To the object's model (1) have been tuned the typical regulators P, PI, PID using the maximal stability degree method and the results are presented in the article [7].

For tuning of typical P, PI, PID regulators to the approximate models (7)…(11) it is using maximal stability degree method and the results are presented in the authors' paper in the same volume.

For automatic control system with object's model (7) and with:

- P regulator:

$$3a_0 J^2 - 2a_1 J + a_2 = 0; \quad (12)$$

$$k_p = (1/k)(a_0 J^3 - a_1 J^2 + a_2 J - 1); \quad (13)$$

- PI regulator:

$$6a_0 J^2 - 3a_1 J + a_2 = 0; \quad (14)$$

$$k_p = (1/k)(4a_0 J^3 - 3a_1 J^2 + 2a_2 J - a_3); \quad (15)$$

$$k_i = (1/k)(-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J); \quad (16)$$

- PID regulator:

$$J = a_1/(4a_0); \quad (17)$$

$$k_d = \frac{1}{k}(-6a_0 J^2 + 3a_1 J - a_2) = \frac{1}{k}\left(\frac{3a_1^2}{8a_0} - a_2\right); \quad (18)$$

$$k_p = \frac{1}{k}(4a_0 J^3 - 3a_1 J^2 + 2a_2 J - a_3) + 2k_d J = \frac{1}{k}\left(\frac{a_1^3}{16a_0^2} - a_3\right); \quad (19)$$

$$k_i = \frac{1}{k}(-a_0 J^4 + a_1 J^3 - a_2 J^2 a_3 J) - \\ - k_d J^2 + k_p J = \frac{1}{k}\frac{a_1^4}{256 a_0^3}. \quad (20)$$

For automatic control system with object's model (9) and with:

- P regulator:

$$4a_0 J^3 - 3a_1 J^2 + 2a_2 J - a_3 = 0; \quad (21)$$

$$k_p = (1/k)(-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4); \quad (22)$$

- PI regulator:

$$-10a_0 J^3 + 6a_1 J^2 - 3a_2 J + a_3 = 0; \quad (23)$$

$$k_p = (1/k)(-5a_0 J^4 + 4a_1 J^3 - \\ - 3a_2 J^2 + 2a_3 J - a_4); \quad (24)$$

$$k_i = (J^2/k)(-4a_0 J^3 + 3a_1 J^2 - 2a_2 J + a_3); \quad (25)$$

- PID regulator:

$$10a_0 J^2 - 4a_1 J + a_2 = 0; \quad (26)$$

$$k_p = (1/k)(15a_0 J^4 - 8a_1 J^3 + 3a_2 J^2 - a_4); \quad (27)$$

$$k_i = (J^3/k)(6a_0 J^2 - 3a_1 J + a_2); \quad (28)$$

$$k_d = (1/k)(10a_0 J^3 - 6a_1 J^2 + 3a_2 J - a_3). \quad (29)$$

For the control system with object's model (11) with the respectively regulators P, PI, PID have the same expressions for the tuning parameters as for the control system with object's model (9).

For tuning of regulators P, PI, PID to the models (8) and (10) have been used the results obtained in other article of authors. Further the algebraic expressions of tuning parameters are represented.

For automatic control system with object's model (8) and with:

- P regulator:

$$-c_0 J^3 + c_1 J^2 + c_2 J - c_3 = 0; \quad (30)$$

$$k_p = \frac{a_0 J^3 - a_1 J^2 + a_2 J - a_3}{b_1 + b_2 J}; \quad (31)$$

- PI regulator:

$$d_0 J^5 + d_1 J^4 + d_2 J^3 + d_3 J^4 + d_4 J + d_5 = 0; \quad (32)$$

$$k_p = \frac{d_0 J^4 + d_1 J^3 + d_2 J^2 + d_3 J - d_4}{(b_1 + b_0 J)^2}; \quad (33)$$

$$k_i = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J}{b_1 + b_0 J} + k_p J; \quad (34)$$

- PID regulator:

$$c_0 J^8 + c_1 J^7 + c_2 J^6 + c_3 J^5 + c_4 J^4 + \\ + c_5 J^3 + c_6 J^2 + c_7 J - c_8 = 0; \quad (35)$$

$$k_d = \\ = \frac{-d_0 J^5 - d_1 J^4 - d_2 J^3 - d_3 J^2 - d_4 J - d_5}{2(b_1 + b_0 J)^4}; \quad (36)$$

$$k_p = \frac{d_0 J^4 + d_1 J^3 + d_2 J^2 + d_3 J - d_4}{(b_1 + b_0 J)^2} + \\ + 2k_d J; \quad (37)$$

$$k_i = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J}{b_1 + b_0 J} - \\ - k_d J^2 + k_p J. \quad (38)$$

For automatic control system with object's model (10) and with:

- P regulator:

$$c_0 J^4 + c_1 J^3 + c_2 J^2 + c_3 J - c_4 = 0; \quad (39)$$

$$k_p = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_1 + b_0 J}; \quad (40)$$

- PI regulator:

$$-c_0 J^6 + c_1 J^5 + c_2 J^4 + \\ + c_3 J^3 + c_4 J^2 + c_5 J + c_6 = 0; \quad (41)$$

$$k_p = \frac{-d_0 J^5 + d_1 J^4 + d_2 J^3 + d_3 J^2 + d_4 J - d_5}{(b_1 + b_0 J)^2}; \quad (42)$$

$$k_i = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 + b_0 J} + k_p J; \quad (43)$$

- PID regulator:

$$-c_0 J^9 + c_1 J^8 + c_2 J^7 + c_3 J^6 + c_4 J^5 + \\ + c_5 J^4 + c_6 J^3 + c_7 J^2 + c_8 J - c_9 = 0; \quad (44)$$

$$k_d = \frac{d_0 J^6 - d_1 J^5 - d_2 J^4 - d_3 J^3 - d_4 J^2 - d_5 J - d_6}{2(b_1 + b_0 J)^4}; \quad (45)$$

$$k_p = \frac{-d_0 J^5 + d_1 J^4 + d_2 J^3 + d_3 J^2 + d_4 J - d_5}{(b_1 + b_0 J)^2} + \\ + 2k_d J; \quad (46)$$

$$k_i = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 + b_0 J} - \qquad (47)$$
$$-k_d J^2 + k_p J .$$

For analysis of the modifications which appear in the approximate models (7)…(11) in comparison with the initial model (1) the respective expressions of models' parameters are presented in the table 1.

Table 1. The parameters of models

Parameters of models	(1)	(7)	(8)	(9)	(10)	(11)
				Models of objects		
a_0	a_0'	$\tau a_0'$	$0{,}5\tau a_0'$	$0{,}5\tau^2 a_0'$	$(\tau^2/6)a_0'$	$0{,}25\tau^2 a_0'$
a_1	a_1'	$a_0' + \tau a_1'$	$a_0' + 0{,}5\tau a_1'$	$\tau a_0' + 0{,}5\tau^2 a_1'$	$\frac{\tau}{3}a_0' + \frac{\tau^2}{6}a_1'$	$\tau a_0' + 0{,}25\tau^2 a_1'$
a_2	1	$a_1' + 0{,}5\tau$	$a_1' + 0{,}5\tau$	$a_0' + \tau a_1' + 0{,}5\tau^2$	$a_0' + \frac{\tau}{3}a_1' + \frac{\tau^2}{6}$	$a_0' + \tau a_1' + 0{,}25\tau^2$
a_3	-	1	1	$a_1' + \tau$	$a_1' + \frac{\tau}{3}$	$a_1' + \tau$
a_4	-	-	-	1	1	1
b_0	-	-	$0{,}5\tau k$	-	$\tau k/3$	-
b_1	k	k	k	k	k	k

Application and computer simulation

Admit the parameters of object's model (1) are known: $a_0' = 10;\ a_1' = 7;\ a_2' = 1;\ k = 0{,}5;\ \tau = 1s.$

In conformity with the expressions (7)…(11) have been determinate the parameters' values of the approximated models (7)…(11) which are presented in the table 2.

The results of tuning of typical regulators P, PI, PID to the objects' models (1), (7)…(11) in conformity with the maximal stability degree method are presented in the table 3.

For verification of obtained results the transient responses of control systems with P, PI, PID regulators and objects' models (1), (7)…(11) have been build up (fig.1, 2). The curves 1, 2, 3, 4, 5, 6 correspond with the numbers of respectively expressions (1), (7)…(11).

Table 2. The values of objects' parameters

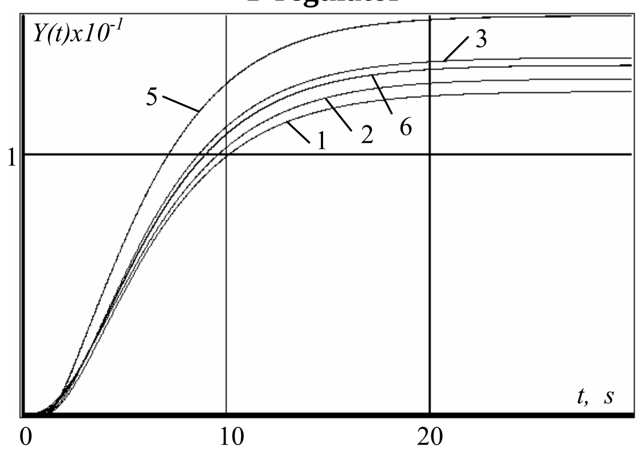
Parame-ters of models	(1)	(7)	(8)	(9)	(10)	(11)
			Models of objects			
a_0	10	10	5	5	1,67	2,5
a_1	7	17	13,5	13,5	4,5	11,8
a_2	1	8	7,5	17,5	12,5	17,3
a_3	-	1	1	8	7,33	8
a_4	-	-	-	1	1	1
b_0	-	-	0,25	-	0,17	-
b_1	0,5	0,5	0,5	0,5	0,5	0,5

Table 3. The values of tuning parameters

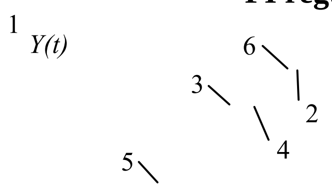
Models	Type of regulator

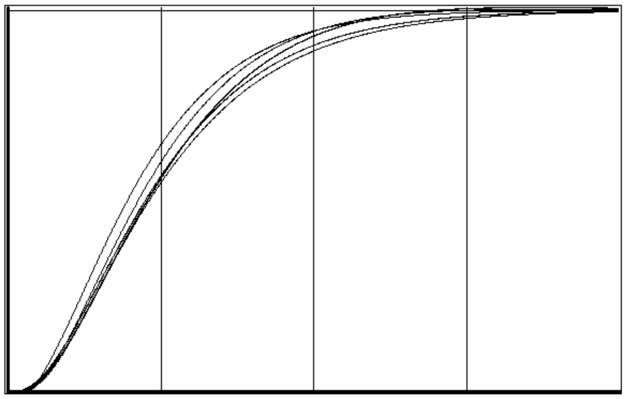
of objects	P	PI	PID
(1)	$J=0{,}291$ $k_p=0{,}284$	$J=0{,}213$ $k_p=0{,}647$ $k_i=0{,}125$	$J=0{,}635$ $k_p=6{,}745$ $k_i=1{,}246$ $k_d=8{,}51$
(7)	$J=0{,}333$ $k_p=0{,}296$	$J=0{,}2$ $k_p=0{,}96$ $k_i=0{,}192$	$J=0{,}425$ $k_p=4{,}141$ $k_i=0{,}652$ $k_d=5{,}675$
(8)	$J=0{,}34$ $k_p=0{,}318$	$J=0{,}21$ $k_p=0{,}99$ $k_i=0{,}21$	$J=0{,}576$ $k_p=6{,}17$ $k_i=1{,}096$ $k_d=7{,}957$
(9)	$J=0{,}166$ $k_p<0$	$J=0{,}2$ $k_p=0{,}904$ $k_i=0{,}184$	$J=0{,}54$ $k_p=7{,}36$ $k_i=1{,}379$ $k_d=9{,}207$
(10)	$J=0{,}33$ $k_p=0{,}362$	$J=0{,}221$ $k_p=1{,}1$ $k_i=0{,}221$	$J=0{,}627$ $k_p=13{,}14$ $k_i=1{,}882$ $k_d=14{,}86$
(11)	$J=0{,}336$ $k_p=0{,}31$	$J=0{,}21$ $k_p=0{,}864$ $k_i=0{,}2$	$J=0{,}5$ $k_p=5{,}062$ $k_i=0{,}844$ $k_d=6{,}75$

P regulator



PI regulator





PID-regulator

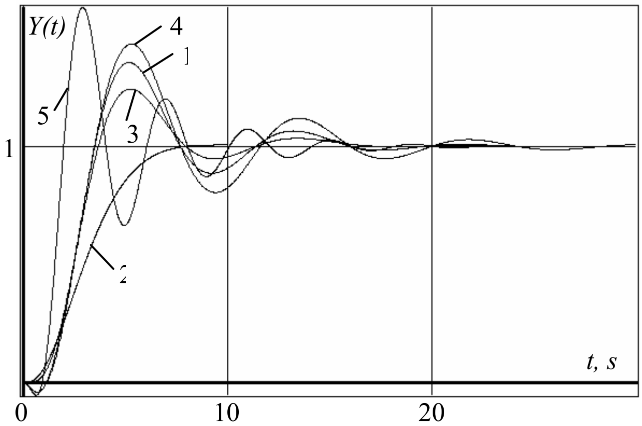


Fig.1. The transient responses of automatic control systems

PID regulator

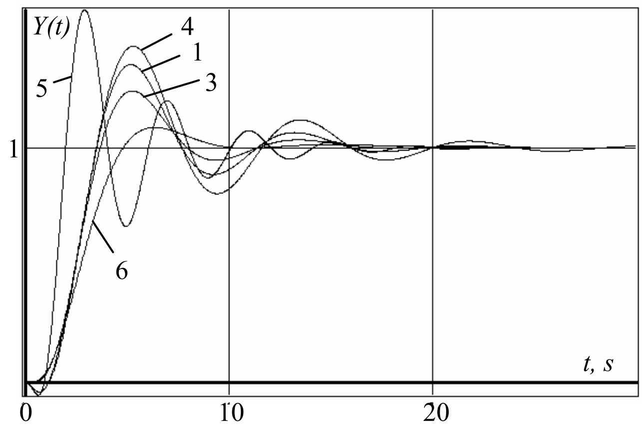


Fig.2. The transient responses of automatic control systems

Analysis of transient responses of control systems with objects' models (1), (7), …, (11) and P, PI, PID regulators have been appraise its performances which are represented in the table 4 (error $\varepsilon = \pm 5\%$).

Table 4. The performances of control systems

Models of objects	Perfor-mances	Type of regulator	
		PI	PID
(1)	t_t, s	40	14
	$\sigma, \%$	-	35
(7)	t_t, s	23	6
	$\sigma, \%$	-	-
(8)	t_t, s	20	10
	$\sigma, \%$	-	26
(9)	t_t, s	24	19
	$\sigma, \%$	-	43
(10)	t_t, s	20	8
	$\sigma, \%$	-	50
(11)	t_t, s	22	8
	$\sigma, \%$	-	9

Conclusions

In the result approximate of time delay of models (1) by approximants (2), …, (6) the analysis of transient responses of the automatic control systems with P, PI, PID regulators the following conclusions can be made:

1. For control system with P regulator the best performances have been obtained for approximant (5) (approximate model (10)). For control system with model (9) and P regulator have been obtain a negative result (see the table 3).
2. For control system with PI regulator the transient responses have an aperiodic form. The best performances have been obtained for approximants (3) and (5) (approximate models (8) and (10)).
3. For control system with PID regulator the best performances have been obtained for approximants (2) and (6) (approximate models (7) and (11)).

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