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THE STATISTICAL PROPERTIES OF PERTURBATION ANALYSIS IN SYSTEM IDENTIFICATION

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Abstract: *The restricted total least Squares (RTLS) problem is devised for solving over determined set of parametering sensitivities for discrete event dynamic* $Ax \approx B$ *in which the date [A;B] are perturbed by errors of the form* $E^* = DEC$ *. These consider nonlinear system that can be described by differential polynomials with the sense of max-algebra D and C are known E is arbitrary but bounded. In this contribution all parameterization of such nonlinear system, can be transformed to linear regression. The method of proof gives an algorithm that can transform the original equation to get linear regression relationship explicitly, in a finite number of steps. This algorithm also gives an excitation for the system.*
Keywords: *Identification, parameterization, nonlinear system, linear regression, differential polynomials, excitation, restricted total least squares.*

Introduction in model set parameterization.

Consider a dynamical system with input $u(n)$ (an r – dimensional vector function of time), and output $y(n)$ (m - dimensional). The dynamics of the systems is not known to the user, but it is supposed – that it belongs to a set of candidate description or models. If each is described in state space form we have [1,2,3]:

$$x(n+1) = f\left(x(n); u(n), \theta\right), x(0) = x_0; \qquad (1)$$

$$y(n) = h\left(x(n), u(n), \theta\right). \qquad (2)$$

Here θ is a d – dimensional parameter vector.
For each given value of θ gives a conventional state space description of the dynamics between the input $u(n)$ and the output $y(n)$.
The sate variation or internal variables are thus $x(n+1)$. The parameterization interns of θ may be of "black box" character, in which case θ is just a vehicle to describe a sufficiently broad set of models in (1),(2). More often in the nonlinear case, corresponds to parameters with physical significance whose numerical values and need to be estimated.

For each given value of θ, and for the given input sequence $u(n)$, $0 \le t \le T$, where $t = n - \Delta t, T = N\Delta t$, the state equation (1) is solved giving the states $x(n,\theta)$. These are inserted into (2) giving [4]:

$$\hat{y}(n/\theta) = h\left((x/n,\theta), u(n), \theta\right), \ \ldots\ldots \qquad (3)$$

then

$$\varepsilon(n,\theta) = y(n) - \hat{y}(n/\theta) \qquad (4)$$

is formed, where $y(n)$ is actually measured output, $0 \le t \le T$ and

$$V(\theta) = \sum_{k=1}^{n} \varepsilon^2(k,\theta) \qquad (5)$$

is minimized with respect to θ.

Problem formulation and main results

The parameters θ are assumed to be constant so that the equations $\theta(k+1) = 0$ are included $g_k\left(y(n), u(n), x(n), \theta\right) = 0,\ k = \overline{1,n}$.
This means that time varying parameters have to be modeled as x - variables.
Let

$$u = r,\ y = \eta,\ w = \xi \qquad (6)$$

be a generic solution. The inputs $u_k(n)=r_k(n)$, form a differential transcendence basis for $F(r,\eta,\xi)$ [5,6], i.e. the inputs are independent and suffice to determine the internal variables and the output, $\prod$ is prime and w denote (θ,x).

Definition 1. The variable w_k is globally identifiable with excitation polynomial P, if P is a nonzero differential polynomial in u and y, not belonging to $\prod$, such that for any two solution $u=r,\ y=\eta,\ w=\xi'$;

$u=r,\ y=\eta,\ w=\xi'';\ \xi'\neq\xi''\Rightarrow P(r,\eta)=0$.

Theorem 1. Let the parameter θ_k fee globally identifiable with some excitation polynomial P. Then there exist differential polynomials p and q in u, y with $p\notin\prod$, such that $pQ_x-q\in\prod$.

Prof. Use a ranking of the variables such that

$$u_i(k)<y_j(k)<\theta(k)<\dot{\theta}(k)<\ldots \qquad (7)$$

The characteristic set then has the form [1]

$$A_1(u,y),\ldots,A_m(u,y),B(u,y,\theta),\ldots$$

The differential polynomial B contains $\theta(k)$ effectively but no derivatives of $\theta(k)$. Now suppose that B has degree r >1 in $\theta(k)$. Let (6) be a generic solution in some extension $\hat{F}\ of\ F$. Let B be the differential polynomial in $F(\theta(k))$ obtained by substituting $u=r\ and\ y=\eta\ in\ B$. Since the coefficient of θ_k' is not in $\prod$, B also has degree r in $\theta(k)$. Now take one of the irreducible factors of B and construct a generic solution ξ. Since P can not vanish for a generic solution $(P\notin\prod)$ and $\xi=\xi(k)$.

It follows that B must be of the form $B=a(\theta(k)-\rho(k))^r$.

Letting b denote the coefficient of $(\theta(k))^{r-1}$ we have the relationship $b+ra\xi(k)=0$. Substituting back to $u,y,\theta(k)$ to be get a deferential polynomial $Q\theta(k)+R\notin\prod$ with Q nonzero. This contradicts B is belonging to the characteristic set and must be of first order in $\theta(k)$. Furthermore, p, being the coefficient of $\theta(k)$ can not belong to $\prod$, since it is reduced with respect to the A_i [8].

Corollary 1. A characteristic set of $\prod$ for the ordering given by (7) has the form

$$A_1(u,y),\ldots,A_m(u,y),p(u,y)\theta_k-q(u,y)\ldots \qquad (8)$$

i.e. the linear regression relationship can be found by forming the characteristic set. Also the excitation condition is found from $p(u,y)$. The importance of this corollary lies in the fact that characteristic sets can be computed explicitly. The first differential polynomials of the characteristic sets (8) are input output relationships that are independent of the parameters and interval variables. In principally they could be used to test the variability of the model structure, since they should be satisfied by the measured inputs and outputs, no matter what the parameters are.

The restricted total least squares (rtls) problem in system identification

Every linear parameter estimation problem arising in signal processing, system identification, automatic converter, statistics, medicine, [9÷15] gives rise to an over determined set of linear equations $AX\approx B$ which are usually solved with the ordinary Least Squares method. Let $AX\approx B$ with

$$A\in R^{m\times n},\ B\in R^{m\times d}\text{ and }X\in R^{n\times n} \qquad (9)$$

Is given where $[A;B]=[A_0;B_0]+E^*$. $A_0;B_0$- are error – free and E^* is a matrix of the form $E^*=DEC$ with

$$D\in R^{m\times p},C\in R^{q+(n+d)},E\in R^{p\times q} \qquad (10)$$

D,C are known, E is in known and arbitrary but bounded. The RTLS of (9) (10) is any solution of the set

$$\left(A-\Delta\hat{A}\right)X=B-\Delta\hat{B} \qquad (11)$$

where $\left\lfloor\Delta\hat{A},\Delta\hat{B}\right\rfloor$ is a matrix on the form $\left\lfloor\Delta\hat{A},\Delta\hat{B}\right\rfloor=D\hat{E}C$;

$$Range\left(B-\Delta\hat{B}\right)\leq Range\left(A-\Delta\hat{A}\right) \qquad (12)$$

and

$$\left\|\hat{E}\right\|F=\min. \qquad (13)$$

The problem of finding $\lfloor \Delta\hat{A}, \Delta\hat{B} \rfloor$ satisfying (12), (13) is called the RTLS problem [8,9].

Theorem 2. The RTLS problem $A_{m\times n}X \approx B_{m+d}$ with given $D_{m+p}, C_{q\times(n+d)}$ is solvable and $\sigma_n([A;B], D, C) > \sigma_{n+1}([A;B], D, C)$, then the restricted singular value decomposition (RSVD) of the RTLS approximation $([A,B] - \lfloor \Delta\hat{A}, \Delta\hat{B} \rfloor, D, C)$ is given by the RVSD of $([A,B], D, C)$ in which the d smallest restricted singular values are equal to zero.

Where

$\sigma_i(T,D,C) = \min_{E\in R^{p\times q}} \{\|E\| = |rank(T+DEC) \le i-1\}$, $i = 1,2,\ldots,n,n+1,\ldots$, $T \in R^{m\times n}$, $T = [A_{m\times n}; B_{m\times d}]$; $D_{m\times p}, C_{q\times(n+d)}$, as defined in the RTLS formulation. The algorithm RTLS is given.

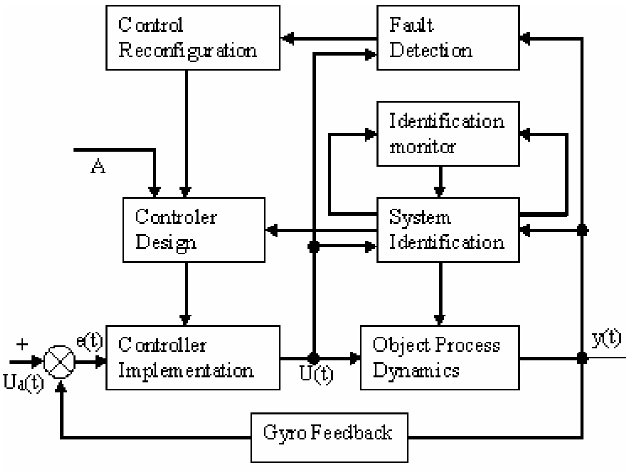

Fig1. Block diagram of a simple reconfigurable adaptive flight control system. A – desired pole locations

Let $T = [A_{m\times n}; B_{m\times d}]$, $D_{m\times p}, C_{q\times(n+d)}$, as defined in the RTLS formulation; nonsingular weighting matrices $(F_1)_{n\times n}$ and $(F_2)_{d\times d}$ such, in case of nonuniqueness, the RTLS solution $\hat{X}$ with minimal $\|F_1\hat{X}F_2\|_F$ is singlet out.

1. Reduced by orthogonal transformation $T - DEC$ to

$$\hat{T} - \hat{D}\hat{E}\hat{C} = \begin{bmatrix} \hat{T}_{11} & \hat{T}_{12} - \hat{D}_1\hat{E}\hat{C}_2 & \hat{T}_{12} - \hat{D}_1\hat{E}[\hat{C}_3;0]^T \\ 0 & \hat{T}_{22} & 0 \\ 0 & \hat{T}_{32} - [\hat{D}_3;0]\hat{E}\hat{C}_2 & \hat{T}_{32} - \hat{D}_3\hat{E}_{11}\hat{C}_3 \end{bmatrix}$$

such that sub triplet $(\hat{T}_{33}, \hat{D}_3, \hat{C}_3)$ and

$(\hat{T}_{33}, \hat{D}_3, \hat{C}_3) = \left\{\sigma_i(T,D,C) | \sigma_i < \infty, \sigma_i \ne \frac{0}{0}\right\}$

2. Compute the implicit singular value decomposition $U^T\hat{D}_3^{-1}\hat{T}_{33}\hat{C}_{33}^{-1}V = diag(\hat{\sigma}_1\hat{\sigma}_2\ldots\hat{\sigma}_s)$.

3. Compute $rank\, rf\hat{T}_{33} - \hat{D}_3\hat{E}_{11}\hat{C}_3(T - DEC) \le n$, then such that rank $E_{11} = Udiag(0,\ldots,0,\sigma_{r+1},\ldots,\sigma_s)V^T$.

4. Compute a basis $\hat{Z}_3$ of $Null(\hat{T}_{33} - \hat{D}_3\hat{E}_{11}\hat{C})$.

5. Compute a basis $\hat{Z}$ of $Null(T - DEC)$, $E = \begin{bmatrix} \hat{E}_{11} & 0 \\ 0 & 0 \end{bmatrix}$.

6. Compute a basis $Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}_d^n$ of $null(T - D\hat{E}C)$.

7. If $\sigma_n(T,D,C) > \sigma_{n+1}(T,D,C)$ and Z_2 nonsingular, then

$\hat{X} = -Z_1Z_2^{-1}$ RTLS solution

else begin

if $\sigma_n(T,D,C) > \sigma_{n+1}(T,D,C)$ then compute weighted minimum norm solution:

$\min_{\hat{X}} \min \|F_1\hat{X}F_2\|_F$

if Z_2 is singular then compute nongeneric RTLS solution.

Where $\hat{E}$ satisfying $\|\hat{E}\|_F = \min$, $Range(B - \Delta\hat{B}) \le Range(A - \Delta\hat{A})$ can now be computed and also a basis of $Null(\lfloor A - \Delta\hat{A}; B - \Delta\hat{B} \rfloor)$, the null space of $\lfloor A - \Delta\hat{A}; B - \Delta\hat{B} \rfloor$ dimension at least d $(step 3 \div 6)$. If the RTLS solution is not unique, the solution $\hat{X}$ with minimal $\|F_1\hat{X}F_2\|_F$ is singled out (step7).

Stochastic complexity for parametric linear stochastic systems

Let as consider a state space equation [10]:

$$x(n,\theta) = A(\theta)X((n-1),\theta) + B(\theta)e(n) \qquad (14)$$

where the dimensionality of $x(n,\theta)$ and $e(n)$ are s and m respectively, $A(\theta), B(\theta)$ are defined and smooth in an open domain $D_\theta \in R^F$. Moreover the matrix $A(\theta), \theta \in D_\theta$ are jointly stable in the sense that there exist a positive $S \times S$ matrix V such that for all $\theta \in D_\theta$, $A^T(\theta)VA(\theta) \le \lambda V$, with some $0 < \lambda < 1$. The $(Q(x))$ is a smooth function defined on R^S such that its first order partial derivatives grow at most linearly in X, and all other derivatives increase at most polynomials. If $Q(x)$ quadratic then it is sufficient to assume that e(t) is a wide sense stationary process,

$$G(\theta) = \lim_{n\to\infty} M\left[Q\left(\widetilde{X}_n(\theta)\right)\right] \qquad (15)$$

The recursive methods can be used to define an off-line estimation the empirical function

$$F_N(\theta) = \sum_{m=1}^{N} Q(X(n,\theta)) \qquad (16)$$

Then the off-line estimation $\hat{\theta}_N$ of θ^* can be defined as the solution of the equation

$$F_N(\theta) = 0 \qquad (17)$$

More exactly we define $\hat{\theta}_N$ as the solution (17) if it has a unique solution then

$$Sup\left|\frac{1}{N}F_N(\theta) - G(\theta)\right| = O_M\left(N^{-1/2}\right)$$

$$\hat{\theta} \in D_\theta$$

$$\eta_N = \frac{1}{N^{1/2}}\sum_{n=1}^{N} Q(x(n,\theta)) \qquad (18)$$

$$\hat{\theta}_N - \theta^* = -\frac{1}{N^{1/2}}\left(G_\theta^*\right)^{-1}\eta_N + O_M\left(N^{-1}\right).$$

The model

To illustrate the computational tasks involved in reconfigurable flight control it is examine a simplified model of a pitch displacement control loop (fig.1):

$$G(z) = -(b_1 z + b_2)/\left(z^2 + a_1 z + a_2\right).$$

The digital controller will have the transfer function: $U(z)/E(z) = -S(z)/R(z)$, where $S(z) = S_0 z^2 + S_1 z + S_2$; $R(z) = (z-1)(z + r_1)$; $E(z)$ is the z-transform of the error $e(t)$, and $e(t) = r(t) - y(t)$, where $r(t)$ is the desired reference value for the pitch angle. A damping ratio X, a frequency W, and a coefficient a can be chosen so that the closed-loop continuous system has two dominant poles corresponding to X, wand two real poles at-aw. The control design algorithms to find the controller parameters form the model parameters and the desired pole location is as follows:

$p_1 = -2\exp(-xwT)\cos\left(wt\sqrt{1-x^2}\right)$;

$p_2 = \exp(-2xwT)$; $c = \exp(-awT)$;

$m_1 = \left(p_2 - 2p_1 c + c^2 a_2 + a_1 - \right.$
$\left. - \left(b_2 / b_1^2\right)(p_1 - 2ca_1 + 1)\right)$;

$m_2 = p_2\left(-2c - (b_1/b_2)c^2\right)$; $m_3 = p_1 c^2 + a_2$;

$m_4 = a_2 a_1 + (b_2/b_1)(-a_1 + 1) +$
$+ (b_2/b_1)^2 + b_1 a_2 / b_2$;

$r_1 = (m_2 + m_3 - b_2 m_1)/m_4$;

$S_0 = (p_1 - 2ca_1 + 1 - r_1)/b_1$;

$S_1 = r_1\left((-a_1 + 1)/b_1 + b_2/b_1^2\right) + m_1$;

$S_2 = \left(p_1 c^2 + a_2 r_1\right)/b_2$.

The implementation of this controller is in the form of a difference equation

$U(k) = (1 - r_1)(u(k-1)) + r_1 u(k-2) +$
$+ S_0 e(k) + S_1 e(k-1) + S_2 e(k-2)$

and the error calculation equation

$e(k) = r(k) - u(k)$.

The model structure is assumed know and a simple recursive least squares on line identification scheme with variable forgetting factor is used. The algorithm requires values of the input $u(k)$, $u(k-1)$, $u(k-2)$ and the output $y(k)$, $y(k-1)$, $y(k-2)$, where k refers to the most recent sampling interval. Let the vector $Z = [a_1 a_2 b_1 b_2]$ contain the unknown model parameters, and $Z(k)$ be the estimate of Z at time k.

Define

$H(k) = [-y(k-1) - y(k-2) u(k-1) u(k-2)]$, then $y(k)=H(k)z$, and the identification algorithm consists of interactive computation of $Z(k)$.

The following statements are based on a Matlab simulation of the identification algorithm:

$$\lambda(k)=(1-\alpha)\lambda_1+\alpha\lambda(k);$$
$$K(k)=p(k-1)H(k)*$$
$$*(H(k)p(k+1)H(k)+\lambda(k))^{-1};$$
$$e(k)=(y(1)-H(k)Z(k));$$
$$Z(k)=Z(k-1)+K(k)e(k);$$
$$p(k)=(e(k)y(k)e(k)2n-K(k)H(k))*$$
$$*p(k-1)/\lambda(k);$$
$$p(k-1)=p(k),\ z(k-1)=z(k),$$

where $\lambda(k)$ is a time – varying forgetting factor, λ_1 is the steady state value of the forgetting factor, α controls the speed, $K(k)$ is a gain matrix, $p(k)$ and $p(k-1)$ are $2n\times 2n$ matrices, where n is the order of the process, $y(1)$ is the current output value.

Hawing described the different algorithmic components of the adaptive control system, shows in block diagram form in Fig.1.

The control consists of repetitive execution of the following cycles [15÷17]:

1. In the beginning of each cycle, the processor associated with each control loop reads the log reference input $r(k)$, which describes desired output as well as sensor measurement of the actual process output $y(k)$ and controller input $U(k)$.
2. The processor reads the output of the reconfiguration supervisor to determine which control law implementation algorithm should be used. It also reads the output of the identification monitor to determine whether the identification is turned on or off.
3. Is the identification is on then an identification iteration is executed. This results in updated values for the model parameters. A controller design step is also executed. Otherwise, if the identification is off then old parameter values are used.
4. Values of the current estimated of model parameters are fed into the control law implementation algorithm to produce a new controller output value $u(k)$.
5. The controller output is applied as input to the object process and to the fault detection predictor filters.
6. The fault detection filters operate concurrently with the control loop. They use the most recent readings of $u(k)$ and stored previous values of y and u to produce estimates for the new output $y(k)$ under different failure / fault assumptions.
7. The outputs of the predictors are compared with actual output as described above.
8. A decision making procedure uses the distance between actual output and predictor outputs, as well as other available information to conclude whether a change in object process model due to a failure has taken place.
9. If it has been determined that a change took place then the identification monitor is signaled to turn identification on and to initialize it with values based on the conclusion reached.
10. An identification system monitor runs concurrently with the control loop. It the identification is on the monitor compares new and old estimates of the model parameters to determine if convergence took place and hence to turn identification off. It also monitors the matrix P(k) to detect if it is getting to be too large. If the identification is off then the monitor looks for a signal from the fault detector to determine when to turn it an again. It also obtains initial values for the identification from the fault detector.
11. There may be several control law algorithm. For example a minimum variance controller may be used for some missions instead of the PID. An a some points perhaps only a PI would be used, and soon. To avoid sudden jumps in controller output when a new controller is switched in some techniques for bump less transfer are typically used. In this case the decoupled control can presented

above may be replaced by a multivariable control law combining two loops.

Conclusion

We have shown that globally identifiable parameters can be written as linear regression relationships. Also we have demonstrated that the actual computation can be done by forming certain characteristic sets. These computations also give the excitation conditions. The RTLS algorithm, which solves the RTLS problem is based on the restricted singular value decomposition (SVD), a generalization of the SVD for triple matrix products. It is presented an algorithm for a reconfigurable adaptive flight control system. The use more robust schemes is explored and a more accurate estimate of the computational requirement is being evaluated.

References

[1]. Glad S.T. (June 1988) *Nonlinear state space and input output descriptions using differential polynomials*, to appear in Proceedings of Collaqua International Automatique Nonlinear, Nantes.

[2]. Diop S.A. (June 1988) *State elimination procedure for nonlinear systems*, to appear in Proceedings of Collaqua International Automatique Nonlinear, Nantes.

[3]. Cao X.R., Ho Y.C. (1987) *Sensitivity analysis and optimization of throughput in a production line with block.* – ing. IEEE+RANS. Automat.Control., 32. p.959-967.

[4]. Suri R., Zazanis M.A. (1988) *Perturbation analysis gives strongly consistent sensivity estimates for the MIG I queue.* – Management Science, 34. – p39-64.

[5]. De Moor B.L.R., Golub G.H. (1989) *The restricted singular value decomposition: proprieties and application*, Technical Report, NA-89-03, Dept. of Computer Science, Stanford University, Stanford.

[6].Nicula D. (2000) *Proiectarea sistemelor digitale Implementate cu dispozitive programabile*. – Bucureşti: Editura Tehnică.

[7]. Shumvay R.H. (1984) *Some applications of the EM algorithm to analyzing incomplete time series data.* In D. Brillinger and other. Time series Analysis of irregularly observed Data. – New York: Springer. – pp. 290-324.

[8]. Sano A., Ohmori M., Furuya M. (1989) *Least squares type of adaptive filter with optimally stabilized convergence based on eigen–structured approach* Proc. IFAC Symp. Adaptive Systems and Sygnal Processing. Glasgow U.K. – pp. 473 – 476.

[9]. Pushner P.P., Schedl A.V. (1997) *Computing Maximum Task Execution Times – A Grof-Based Approach. – Real-Time Systems, N13*, - Boston: Kluwer Academic Publishers, – p.67-91.

[10]. Manabe Y., Aoyagi S. A. (1998) *Feasibility Decision Algorithm for Rate Monotonic and Deadline Scheduling Publisher*, – p.171-181.

[11]. Nisanke N. (1998) *Real-time Systems*. – Boston: Ed. Prentice Hall, – p.20-35.

[12]. Altenbernd P. Hansson H. (1998) *The Slack Method: A New Method for static Allocation of Hard Real-Time Tasks*. Real-Time Systems N.15. – Boston: Kluwer Academic Publishers, – p.103-130.

[13]. Bohanastiuc V., Kobyliatzky N., (Iaşi 2001) *Anestiadi V., Vascan A., Lorcencov A., Rîcu V. Identification and Adaptive Control of Stochastic Systems*. // In Proceedings: The 7th International Symposium on Automatic Control and Coputer Science. SACCS 2001.

[14]. Bohanastiuc V. (Chişinău 2002) *The Hopfield Model as a Content-Addressable Memory in Dynamic Systems*, II International Conference on Information Technologies, Vol. I

[15]. Bohanastiuc V. (22-23 mai, Chişinău 2001) *The Information Models for Self-organising biological systems*// Materiale a VII-a Confernță de Ştiințe aplicative, Informatică şi aplicații, pag.21-24.

[16]. Bohanastiuc V. (Chişinău 2002) *The Hopfield Model as a Content-Addressable Memory in Dynamic Systems*, II International Conference on Information Technologies, Vol. I

[17]. Bohanastiuc V. N.Kobyliatsky, V. Rîcu, V. Anestiady, V. Scorpan. (2000) *The combined Method of optimal control and divisible maximal approximate Systhesis.*// Proceedings

of 5th International Conference on Development and Aplication Szstems, DAS 2000, Suceava, România - pag. 340-345