SPACE – TIME CODING USING MODULATION WITH MEMORY

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Abstract. *This paper addresses the space-time coding for Rayleigh–fading using modulation with memory. GSM' modulation format, GMSK with BT = 0.3, is a constant envelope modulation which allows transmission with a greater bandwidth efficiency and using an inexpensive Class C amplifiers. GSM' new EDGE modulation promises to increase data rates by up to 3 times and is also viewed as a migration path from existing IS–136 systems to 3G systems. The* $3\pi/8$*-shifted 8–PSK modulation scheme triple the on-air data rate while meeting the same bandwidth occupancy of the original 0.3–GMSK signal. Both of these modulation format have memory and demonstrates compelling advantages for wireless aplications. This paper presents how we can use space-time coding with this kind of modulation.*
Keywords: *space-time codes, modulation with memory, quasi-static Rayleigh fading, Laurent decomposition.*

Introduction

As the mobile communications market develops, interest is building for data applications and higher data rate operation. Modulation with memory have a beneficial effect on spectrum occupancy, highly desirabile for applications like wireless systems.
One of the major difficulties in wireless communications is the multipath fading. A very effective techniques to overcome the fading is diversity. An usual technique is interleaved coded modulation, where the interleaving separates the code symbols so that the fading statistics is independent on each of the codeword symbols. In a quasi-static fading, the interleaver depth must be very long, that affect dramatically the delay of decoding, undesirable effect with vocal or video wireless communications. We can solve this problem using multiple transmit antennas (and, if possible, multiple receive antennas) to provide transmission diversity.

System model

In fig. 1 is represented a model of the transmitter diversity system using a modulation with memory, and only one antenna at the receiver side (the case of multiple receiving antennas is straightforward) for simplicity.

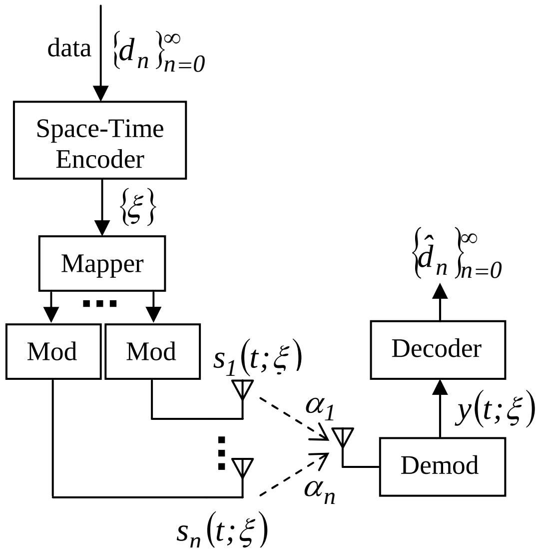

Fig. 1. System model

The received signal (the complex envelope) is

$$y(t;\xi) = \sum_{i=1}^{n} \alpha_i s_i(t;\xi) + w(t) \qquad (1)$$

where $w(t)$ is an independent zero-mean complex white Gaussian noise (AWGN) with

double-sided power spectral density $N_0/2$ per dimension, $\{\alpha_i\}_{i=1}^{n}$ are the complex-valued fading coefficients between the ith transmitter antenna and the receiving antenna, modeled as independent zero-mean complex Gaussian random variables with variance E_s (we include here the energy of the emitted signal).
Assume synchronism and negligible intersymbol interference, and $\{\alpha_i\}_{i=1}^{n}$ to be constant over the duration of one codeword (N complex symbols periods), but change independent from one codeword to another.
Assume also the channel state information is known at the receiver side.

Performance analysis

The receiver is to estimate $\{\xi\}$ from the received signal $y(t;\xi)$. The observation time of $y(t;\xi)$ is supposed to be as long as an coded frame of NT length is transmitted (T - coded symbol period).
The maximum-likelihood receiver determines as the best estimate of $\{\xi\}$, the sequence $\{\hat{\xi}\}$ that maximize the likelihood function $p\left[y(t;\xi)\,|\,\{\alpha_i\}_{i=1}^{n}\right]$.
We have the expresion[1] [4]

$$p\left[y(t;\xi)\,|\,\{\alpha_i\}_{i=1}^{n}\right] \approx exp\left[-\int_{\Gamma}\int_{\Gamma} \begin{matrix} x(t_1;\xi)^* \times \\ \times R_w^{-1}(t_1-t_2)\times \\ \times x(t_2;\xi)dt_1dt_2 \end{matrix}\right] \quad (2)$$

where Γ is the domain of integration and

$$x(t;\xi) = y(t;\xi) - \sum_{i=1}^{n} \alpha_i s_i(t;\xi) \quad (3)$$

and

$$R_w(t_1-t_2) = N_0\delta(t_1-t_2) \quad (4)$$

so that

$$p\left[y(t;\xi)\,|\,\{\alpha_i\}_{i=1}^{n}\right] \approx exp\left[-\frac{1}{N_0}\int_{\Gamma} |x(t;\xi)|^2 dt\right] \quad (5)$$

The probability that the receiver decodes codeword e when c was actually transmitted (*pairwise error event*) is given by

$$Pr\left\{c \to e\,|\,\{\alpha_i\}_{i=1}^{n}\right\} = Pr\{m(y(t;c),e) \geq m(y(t;c),c)\} \quad (6)$$

where

$$m\left(y(t;\xi),\hat{\xi}\right) = -\int_{\Gamma}\left|y(t;\xi) - \sum_{i=1}^{n}\alpha_i s_i\left(t;\hat{\xi}\right)\right|^2 dt \quad (7)$$

is the metric of the decoding algorithm.
The pairwise error event probability can be evaluated in the form

$$Pr\left\{c \to e\,|\,\{\alpha_i\}_{i=1}^{n}\right\} = Pr\left\{\begin{matrix}\int_{\Gamma}\left|\sum_{i=1}^{n}\alpha_i D_{ce}^{i}(t)\right|^2 dt + \\ + 2\,Re\left\{\sum_{i=1}^{n}\alpha_i^* \int_{\Gamma} w(t) D_{ce}^{i}(t)^* dt\right\} \leq 0\end{matrix}\right\} \quad (8)$$

where

$$D_{ce}^{i}(t) = s_i(t;c) - s_i(t;e) \quad (9)$$

We denote

$$\alpha = [\alpha_1 \quad \dots \quad \alpha_n]^T \quad (10)$$

$$D_{ce}(t) = \left[D_{ce}^{1}(t) \quad \dots \quad D_{ce}^{n}(t)\right]^T \quad (11)$$

and we have the form of the paiwise error event probability as

[1] In the following, x^*, x^T and x^H will denote the complex conjugate, transpose and Hermitian transpose of x.

$$
\begin{aligned}
&Pr\{c \to e \mid \{\alpha_i\}_{i=1}^{n}\} = \\
&= Pr\left\{ \begin{aligned} &\alpha^H \left[\int_{\Gamma} D_{ce}^{*}(t) D_{ce}^{T}(t)\, dt \right] \alpha + \\ &+ 2Re\left\{ \alpha^H \left[\int_{\Gamma} w(t) D_{ce}^{*}(t)\, dt \right] \right\} \le 0 \end{aligned} \right\} \\
&= Pr\{V(t) \le 0\}
\end{aligned} \qquad (12)
$$

We notice that the expresion $V(t)$ is a realisation of a random proccess with a Gaussian distribution, with mean $\mu\left(t; \{\alpha_i\}_{i=1}^{n}\right)$ and variance $var\left(t; \{\alpha_i\}_{i=1}^{n}\right)$ given by

$$
\mu\left(t; \{\alpha_i\}_{i=1}^{n}\right) = \alpha^H \left[\int_{\Gamma} D_{ce}^{*}(t) D_{ce}^{T}(t)\, dt \right] \alpha \qquad (13)
$$

$$
var\left(t; \{\alpha_i\}_{i=1}^{n}\right) = 2N_0 \alpha^H \left[\int_{\Gamma} D_{ce}^{*}(t) D_{ce}^{T}(t)\, dt \right] \alpha \qquad (14)
$$

We can claim that

$$
\begin{aligned}
Pr\{c \to e \mid \{\alpha_i\}_{i=1}^{n}\} &= Pr\{V(t) \le 0\} \\
&= Q\left(\sqrt{ \frac{\mu^2\left(t; \{\alpha_i\}_{i=1}^{n}\right)}{var\left(t; \{\alpha_i\}_{i=1}^{n}\right)} } \right)
\end{aligned} \qquad (15)
$$

where

$$
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{x^2}{2}}\, dx \qquad (16)
$$

Design criterion

In the sequel we focus on the modulation format used in GSM systems: 0.3-GMSK and EDGE. Both of them have a very interesting property for our problem: they can be described as quadrature-amplitude modulation.

Using Laurent decomposition to the GMSK waveform, in [1] the GMSK signal is represented as

$$
s(t;\xi) = exp[j\varphi(t;\xi)] = \sum_{l}^{\left\lfloor \frac{t}{T} \right\rfloor} I(l) g(t - lT) \qquad (17)
$$

where

$$
I(l) = exp\left[j \frac{\pi}{2} \sum_{k=1}^{l} \xi(k) \right] = j^l \prod_{k=1}^{l} \xi(k) \qquad (18)
$$

, $\xi(l)$ is the binary information in the bipolar format, $\{\pm 1\}$, and $g(t)$ is the principal pulse in the Laurent decomposition of the 0.3-GMSK modulation,

$$
g(t) = \begin{cases} \prod_{i=0}^{3} f(t + iT) & 0 \le t \le 5T \\ 0 & elsewhere \end{cases} \qquad (19)
$$

$$
f(t) = \begin{cases} sin\left(\frac{\pi}{2} \int_{0}^{t} q(u)\, du \right) & 0 \le t \le 4T \\ cos\left(\frac{\pi}{2} \int_{0}^{t-4T} q(u)\, du \right) & 4T \le t \le 8T \\ 0 & elsewhere \end{cases} \qquad (20)
$$

$$
q(t) = \frac{1}{2T} \left[erfc\left(\beta \left(\frac{t}{T} - \frac{1}{2} \right) \right) - erfc\left(\beta \left(\frac{t}{T} + \frac{1}{2} \right) \right) \right] \qquad (21)
$$

where

$$
\beta = \frac{2\pi}{\sqrt{2\ln 2}} \times 0.3 \qquad (22)
$$

It can be made some simplifications. In [2] it was shown that an aproximation to the pulse $g(t)$ that is significantly less complicated to define, is

$$q\left(t+\frac{5}{2}T\right)\approx exp\left[-1.045\left(\frac{t}{T}\right)^2-0.218\left(\frac{t}{T}\right)^4\right] \tag{23}$$

where $-\frac{5}{2}T \le t \le \frac{5}{2}T$

Now assume the data has been differentially encoded, so that

$$\xi(l)=b(l)\times b(l-1) \tag{24}$$

Substituting this into expression (17) gives

$$I(l)=j^l b(l) \tag{25}$$

In the case of EDGE modulation, the signal can be described as

$$s(t;\xi)=\sum_{l}^{\left\lfloor \frac{t}{T}\right\rfloor} I(l)p(t-lT) \tag{26}$$

where

$$I(l)=exp\left[j\left(\frac{2\pi}{8}\xi(l)+\frac{3\pi}{8}l\right)\right] \tag{27}$$

with $\xi(l)\in\{0, \ \ldots \ 7\}$ and

$$p(t)=g\left(t+\frac{5}{2}T\right) \tag{28}$$

defined below.

With these observations, we are able to simplify the pairwise error event probability in this case (see Apendix)

$$Pr\left\{c\to e\,|\,\{\alpha_i\}_{i=1}^{n}\right\}=Q\left(\sqrt{\frac{1}{2N_0}\left(\alpha^H \Delta_{ce}^H R\Delta_{ce}\alpha\right)}\right) \tag{29}$$

where we can apply the Chernoff bound and

$$Pr\left\{c\to e\,|\,\{\alpha_i\}_{i=1}^{n}\right\}\le exp\left[-\frac{1}{4N_0}\left(\alpha^H \Delta_{ce}^H R\Delta_{ce}\alpha\right)\right] \tag{30}$$

We can make some remarks. The matrix

$$M=\Delta_{ce}^H R\Delta_{ce} \tag{31}$$

is a Hermitian matrix, so we can write this matrix like

$$M=V\Lambda V^H \tag{32}$$

where V is a unitary complex matrix with dimensions $[n\times n]$ (the number of transmitter antennas) having as columns the eigenvectors of M, and Λ a diagonal matrix with elements on the principal diagonals be the nonzero eigenvalues $\{\lambda_i\}_{i=1}^{r}$ of the matrix M. The number of nonzero eigenvalues of the matrix M, denoted with r, is also the rank of the matrix M.

We can write

$$Pr\left\{c\to e\,|\,\{\alpha_i\}_{i=1}^{n}\right\}=Q\left(\sqrt{\frac{E_s}{2N_0}\sum_{i=1}^{r}\lambda_i|\boldsymbol{u}_i|^2}\right) \tag{33}$$

where

$$\sqrt{E_s}\boldsymbol{u}_i=\alpha^H column_i(V)=\sqrt{E_s}\alpha'^H\, column_i(V)$$

with α' having zero-mean complex Gaussian distribution with unitary variance and, because V is a unitary complex matrix, the statistics of $\boldsymbol{u}_i$ is also Gaussian with zero-mean and unitary variance,so that $|\boldsymbol{u}_i|^2$ is chi-square with two degree of freedom variable.

If $\{\lambda_i\}_{i=1}^{r}$ are all equal we have

$$Pr\{c\to e\}=\left[\frac{1}{2}(1-\mu)\right]^r\sum_{i=0}^{r-1}\binom{r-1+i}{i}\left[\frac{1}{2}(1+\mu)\right]^i \tag{34}$$

where

$$\mu = \sqrt{\frac{\lambda E_s/(4N_0)}{1+\lambda E_s/(4N_0)}} \qquad (35)$$

For a signal-to-noise ratio high enough, we can aproximate

$$Pr\{c \to e\} \approx \binom{2r-1}{r}\left(\frac{\lambda E_s}{N_0}\right)^{-r} \qquad (36)$$

If $\{\lambda_i\}_{i=1}^{r}$ are all different, then

$$Pr\{c \to e\} = \frac{1}{2}\sum_{i=1}^{r}\frac{\lambda_i^{r-1}}{\prod_{\substack{k=1\\k\neq i}}^{r}(\lambda_i-\lambda_k)}(1-\mu_i) \qquad (37)$$

where

$$\mu_i = \sqrt{\frac{\lambda_i E_s/(4N_0)}{1+\lambda_i E_s/(4N_0)}} \qquad (38)$$

For high SNR (35) has the asymptotic form

$$Pr\{c \to e\} \approx \binom{2r-1}{r}\prod_{i=1}^{r}\left(\frac{1}{\frac{\lambda_i E_s}{N_0}}\right) \qquad (39)$$

In the sequel we can apply the criterions for space-time codes design derived in [5], [6] or [7], with only difference that the *rank criterion* and *determinant criterion* [5], or the *"equal eigenvalues" criterion* [7] are applied on the matrix $M = \Delta_{ce}^{H} R \Delta_{ce}$ and not on the matrix $A = \Delta_{ce}^{H}\Delta_{ce}$.

In [5] Tarokh et al. presented the space-time coding criteria and which can be transformed for this case in:

- Rank criterion: *Maximize the rank, r, of the matrix* $M = \Delta_{ce}^{H} R \Delta_{ce}$, *for all possible codewords* ***c*** *and* ***e***. *Note that* $r \le n$ *(the number of the transmitter antennas).*
- Determinant criterion: *Maximize the minimum of the products of all the non-zero eigenvalues of* $M = \Delta_{ce}^{H} R \Delta_{ce}$ *taken over all the distinct codewords* ***c*** *and* ***e***.

The *"equal eigenvalues criterion"* found by Ionescu [7] can be put, in this case, into the form:

- ESV criterion: *Codes designed over N time epochs and* ***n*** *transmitter antennas, must satisfy the property that all the eigenvalues of* $M = \Delta_{ce}^{H} R \Delta_{ce}$ *are equal for every pair of codewords in the code set.*

We notice that *R* is a real, square (dimensions $[N \times N]$) and symetric matrix, and the matrix

$$R_{GMSK} = diag\left(j,\ldots,j^N\right)^H \times R \times diag\left(j,\ldots,j^N\right) \qquad (40)$$

in the case of GMSK, or

$$R_{EDGE} = diag\left(e^{j\frac{3\pi}{8}},\ldots,e^{j\frac{3\pi}{8}N}\right)^H \times R \times diag\left(e^{j\frac{3\pi}{8}},\ldots,e^{j\frac{3\pi}{8}N}\right) \qquad (41)$$

in the case of EDGE, are also square, symetric and complex matrix. Any of the two resulting matrix can be write in the form

$$\tilde{R} = LL^H \qquad (42)$$

where *L* is a complex lower triangular matrix which is inversable in this case.

We can say, under these observations, that

$$M = \Delta_{ce}^{H} R \Delta_{ce} = \Delta_{ce}^{1\,H} L L^H \Delta_{ce}^{1} \qquad (43)$$

and

$$rank(M) = rank\left(L^H \Delta_{ce}^{1}\right) = rank\left(\Delta_{ce}^{1}\right) \qquad (44)$$

where

$$\Delta_{ce}^{1} = \begin{cases} \Delta_{ce}^{binary} & 0.3-GMSK \\ \Delta_{ce}^{8-PSK} & EDGE \end{cases} \qquad (45)$$

So, we can apply the rank criterion directly to Δ_{ce}^{1}, and we must take into account the effect of the autocorrelation matrix *R* over the eigenvalues of the matrix *M* relative at the coding gain.

Simulation results

The simplest form of space-time coding is *delay diversity coding*. In this kind of coding the coded symbols are repeated, in succesive complex symbols periods, on succesive antennas. We can see the combination of the encoder and the mapper like in fig. 2.

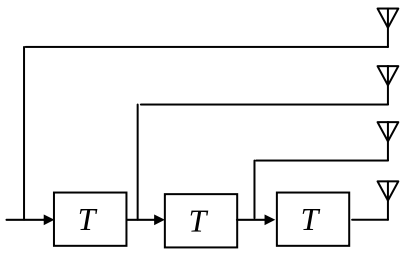

Fig. 2. Delay diversity transmitter

For delay diversity, the dominant error event has a single error, so that we have

$$\Delta_{ce} = \begin{cases} 2I_n & 0.3-GMSK \\ \sqrt{2-\sqrt{2}}\,I_n & EDGE \end{cases} \qquad (46)$$

where $rank(\Delta_{ce}) = n$, the number of transmiter antennas (maximum rank).

This sort of coding agree only with the rank criterion, and not with the determinant criterion or ESV criterion. For the other two criterions, one must pay attention to the effect of the matrix *R* over the eigenvalues of the matrix *M*.

In the delay diversity coding case, the BER is upper-bounded by

$$P_b \le \begin{cases} \binom{2n-1}{n}\left(\frac{E_b}{N_0}\right)^{-n}\frac{1}{det(R)} & GMSK \\ \frac{1}{3}\binom{2n-1}{n}\left(\frac{3E_b}{N_0}\right)^{-n}\frac{1}{det(R)} & EDGE \end{cases} \qquad (47)$$

where it is taken into account the relation $det(ABC) = det(B) \times det(CA)$ (in this case all matrices are square $[N \times N]$), so that

$$det(R_{GMSK}) = det(R_{EDGE}) = det(R) \qquad (48)$$

and the fact that EDGE' 8-PSK symbols are Gray coded.

The simulated performance are presented in fig. no.3 and no. 4. We have considered the cases with the 2, 3 and 4 antennas at the transmitter side and only one antenna at the receiver side.

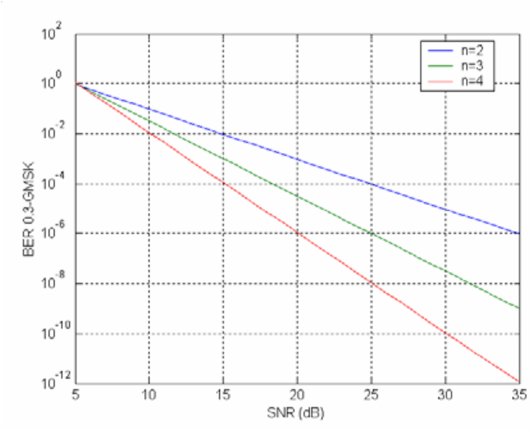

Fig. 3 - BER ***0.3-GMSK***

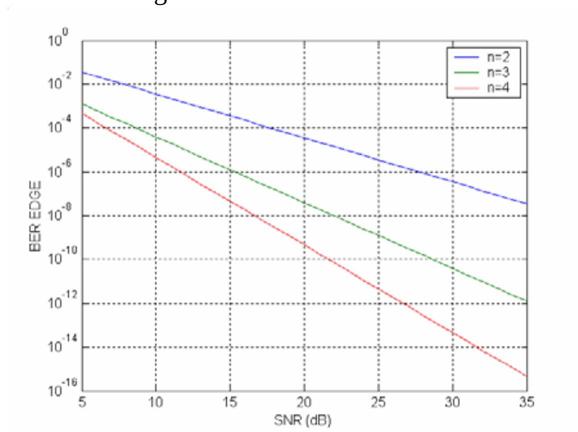

Fig. 4 - BER ***EDGE***

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Appendix A

We assume $NT \le t < (N+1)T$. We have

$$\int_\Gamma D^*_{ce}(t) D_{ce}(t)\,dt =$$
$$= \begin{bmatrix} \int_\Gamma \left|D^1_{ce}(t)\right|^2 dt & \dots & \int_\Gamma D^1_{ce}(t)^* D^n_{ce}(t)\,dt \\ \vdots & \ddots & \vdots \\ \int_\Gamma D^n_{ce}(t)^* D^1_{ce}(t)\,dt & \dots & \int_\Gamma \left|D^n_{ce}(t)\right|^2 dt \end{bmatrix}$$

where, using the finite time duration of the pulse $g(t)$, we can write

$$\int_\Gamma D^{i\,*}_{ce}(t) D^j_{ce}(t)\,d\tau =$$
$$= \int_\Gamma \left[\begin{array}{c} \left(\sum_{l=1}^{N} \left(I^i_c(l) - I^i_e(l)\right) g(t-lT) \right)^* \times \\ \times \left(\sum_{k=1}^{N} \left(I^j_c(k) - I^j_e(k)\right) g(t-kT) \right) \end{array} \right] dt$$
$$= \sum_{l=1}^{N} \sum_{k=1}^{N} \left\{ \begin{array}{c} \left(I^i_c(l) - I^i_e(l)\right)^* \left(I^j_c(k) - I^j_e(k)\right) \times \\ \times \int_\Gamma g(t-lT) g(t-kT)\,dt \end{array} \right\}$$

We can put this expression into the form

$$\int_\Gamma D^*_{ce}(t) D_{ce}(t)\,dt = \Delta^H_{ce} R \Delta_{ce}$$

where

$$\Delta_{ce} = i \begin{pmatrix} & \overset{j}{\vdots} & \\ \dots & I^j_c(i) - I^j_e(i) & \dots \\ & \vdots & \end{pmatrix}$$

and

$$R = i \begin{pmatrix} & \overset{j}{\vdots} & \\ \dots & \int_\Gamma g(t-iT) g(t-jT)\,dt & \dots \\ & \vdots & \end{pmatrix}$$

with i – the complex symbol time period, $1 \le i \le N$, and j – the jth transmitter antena. For 0.3-GMSK differentially encoded we have

$$\Delta_{ce} = \begin{pmatrix} j & 0 & \dots & 0 \\ 0 & j^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & j^N \end{pmatrix} \begin{pmatrix} & \vdots & \\ \dots & b^j_c(i) - b^j_e(i) & \dots \\ & \vdots & \end{pmatrix}$$
$$= diag\left(j, \dots, j^N\right) \Delta^{binar}_{ce}$$

and for EDGE case we have

$$\Delta_{ce} = \begin{pmatrix} e^{j\frac{3\pi}{8}} & 0 & \dots & 0 \\ 0 & e^{j\frac{3\pi}{8}2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{j\frac{3\pi}{8}N} \end{pmatrix} \times$$
$$\times \begin{pmatrix} & \vdots & \\ \dots & e^{j\frac{2\pi}{8}c^j_i} - e^{j\frac{2\pi}{8}e^j_i} & \dots \\ & \vdots & \end{pmatrix}$$
$$= diag\left(e^{j\frac{3\pi}{8}}, \dots, e^{j\frac{3\pi}{8}N}\right) \Delta^{8-PSK}_{ce}$$