7th International Conference on DEVELOPMENT AND APPLICATION SYSTEMS
S u c e a v a, R o m a n i a, M a y 27 – 29, 2 0 0 4

NA-VCS AND MA-VVS METHODS: NEW POWERFUL APPROACHES

G. E. CHATZARAKIS [1], V. S. PAPANIKOLAOU [1], P. B. MALATESTAS [2], D P. E. SINIOROS [2]

[1])*Department of Electrical Engineering,*
School of Pedagogical and Technological Education (ASPETE),
Athens, Greece,
geaxatz@otenet.gr, gea.xatz@aspete.gr
[2])*Department of Electrical Technology,*
Technological Educational Institute of Piraeus (TEI),
Piraeus, Greece

Abstract. *This paper presents two systematic methods for solving dc Linear Electric Circuits (LECs): Nodal Analysis with Virtual Current Sources (NA-VCS), applicable to planar or nonplanar LECs and Mesh Analysis with Virtual Voltage Sources (MA-VVS), applicable to planar LECs only.*
To apply these methods, the nonconvertible voltage or current sources (independent and dependent) are replaced by Virtual Current or Voltage Sources (VCS, VVS), respectively. Solving any dc LEC in this way is extremely systematic and straightforward, since most of the work is done by inspection and some of the matrix manipulations required are easily implemented. Since the proposed methods are well algorithmized, they can by used in most modern simulators of analog networks.
Keywords: *dc LEC, inspection, mesh analysis, nodal analysis, nonconvertible current source, nonconvertible voltage source, virtual current source, virtual voltage source.*

I. Introduction

Many introductory electric circuit textbooks [1]–[10] provide two powerful methods for solving electric circuits: nodal analysis, which is based on a systematic application of Kirchhoff's Current Law (KCL), and mesh analysis, which is based on a systematic application of Kirchhoff's Voltage Law (KVL).These methods are easy and systematic for circuits that contain only independent current (voltage) sources (ICS, IVS).
The difficulty of these methods starts when the circuit contains also dependent voltage (current) sources (DVS, DCS) and when there are voltage (current) sources (IVS, DVS, ICS, DCS) which are not transformable to current (voltage) sources. In Section II, these sources are referred to as Nonconvertible Independent Voltage Sources (NCIVS), Nonconvertible Dependent Voltage Sources (NCDVS), Nonconvertible Independent Current Sources (NCICS) and Nonconvertible Dependent Current Sources (NCDCS), respectively.
These difficulties are removed by the method presented by J. G. Gottling [11], who shows how to write nodal (mesh) analysis matrix equations for a linear circuit by inspection and derives a general matrix solution for the node voltage (mesh current) vector.
Also, these difficulties and known limitations of classical nodal and mesh methods are removed by the MNA method presented by C. W. Ho et al [12], which is well suited both to symbolic and numerical analysis of complex circuits using modern matrix-based software.
This paper presents systematic methods for solving dc LECs by the NA-VCS or MA-VVS, which as the Gottling method and MNA method, remove the above mentioned limitations.

II. Methods Description

A. *NA-VCS method*
The building elements of a dc LEC are given in Table 1.
To apply Nodal Analysis, where the necessary condition is that all sources must be current

sources, the concept of the *Virtual Current Source* (VCS) is introduced. That is, in place of Nonconvertible Independent Voltage Sources (NCIVS) or of Nonconvertible Dependent Voltage Sources (NCDVS), *Virtual Current Sources* (VCS) are considered with current values equal to the currents through these voltage sources.

The NCIVSs and the NCDVSs, are replaced by the VCSs with the notation $(\text{ncivs})_i^*,\ i=1,\dots,r_2$, and $(\text{ncdvs})_i^*,\ i=1,\dots,s_2$, respectively.

Next, defining the reference node and labeling the rest nodes, by inspection nodal analysis gives:

$$\mathbf{G}_{k\times k}\cdot\mathbf{v}_{k\times 1}=\mathbf{i}_{k\times 1}=\mathbf{W}_{k\times m}\cdot\mathbf{S}^{(1)}_{m\times 1} \quad (1)$$

where $\mathbf{G}_{k\times k}$ is the conductance matrix and $\mathbf{v}_{k\times 1}$ the node voltage vector. Matrix $\mathbf{W}_{k\times m}$ and vector $\mathbf{S}^{(1)}_{m\times 1}$ are given in Appendix A.

Table 1. Building elements of a dc LEC

SOURCES		
Kind	**No.**	**Notation**
ICS	r_1	$(\text{ics})_1,\dots,(\text{ics})_{r_1}$
DCS	s_1	$(\text{dcs})_1,\dots,(\text{dcs})_{s_1}$
NCIVS	r_2	$(\text{ncivs})_1,\dots,(\text{ncivs})_{r_2}$
NCDVS	s_2	$(\text{ncdvs})_1,\dots,(\text{ncdvs})_{s_2}$
OTHER ELEMENTS		
Resistances		
OTHER DETAILS		
• $r=r_1+r_2$: Total No. of indep. sources • $s=s_1+s_2$: Total No. of dep. sources • $m=r+s$: Total No. of sources • k : No. of nodes (besides the ref. node)		

But all voltage sources replaced by VCSs can be expressed as a linear combination of the node voltages through the matrix equation

$$\mathbf{F}_{(r_2+s_2)\times k}\cdot\mathbf{v}_{k\times 1}=\mathbf{Z}_{(r_2+s_2)\times m}\cdot\mathbf{S}^{(2)}_{m\times 1} \quad (2)$$

where, each row of the **F** matrix describes one of the voltage sources as a function of the node voltages. Therefore, the **F** matrix elements are -1, 1 or 0. Matrix $\mathbf{Z}_{(r_2+s_2)\times m}$ and vector $\mathbf{S}^{(2)}_{m\times 1}$ are given in Appendix A.

Combining equations (1) and (2), a new matrix equation comes up, where the first r_2+s_2 equations are the equations given by (2) and the rest $k-(r_2+s_2)$ equations are obtained from (1). These equations are obtained following one of the next two cases:

- case a) unchanged, if all the VCS coefficients in matrix **W** are zero, or
- case b) after appropriate additions or subtractions of the equations of (1) aiming to the elimination of all the VCSs, if the conditions of case a) are not valid.

Thus, an equivalent set of equations of the following form is obtained

$$\mathbf{D}_{k\times k}\cdot\mathbf{v}_{k\times 1}=\mathbf{T}_{k\times m}\cdot\mathbf{S}^{(2)}_{m\times 1} \quad (3)$$

where $\mathbf{D}_{k\times k}$ and $\mathbf{T}_{k\times m}$ are matrices given in Appendix A.

However, since the dependent sources are expressed as functions of the node voltages, one may write

$$\mathbf{S}^{(3)}_{s\times 1}=\mathbf{X}_{s\times k}\cdot\mathbf{v}_{k\times 1} \quad (4)$$

where $\mathbf{X}_{s\times k}$ is a matrix whose elements describe the values of the dependent sources as functions of the node voltages of the whole circuit. Vector $\mathbf{S}^{(3)}_{s\times 1}$ is given in Appendix A.

Based on (4), matrix equation (3) is rearranged as follows:

$$\mathbf{DD}_{k\times k}\cdot\mathbf{v}_{k\times 1}=\mathbf{TT}_{k\times r}\cdot\mathbf{S}^{(4)}_{r\times 1} \quad (5)$$

Vector $\mathbf{S}^{(4)}_{r\times 1}$, and matrices $\mathbf{DD}_{k\times k}$, $\mathbf{TT}_{k\times r}$ are given in Appendix A.

Finally, based on (5), the node voltage vector is

$$\mathbf{v}_{k\times 1}=\mathbf{DD}^{-1}_{k\times k}\cdot\mathbf{TT}_{k\times r}\cdot\mathbf{S}^{(4)}_{r\times 1} \quad (6)$$

The voltages of all branches are calculated combining the node voltages. As a consequence, the currents of all circuit elements are known, except those flowing through the NCVSs. These currents are calculated from the proper set of equations contained in (1). These equations are obtained following one of the next two cases:

- case i) unchanged, if the coefficients $w_{i(r_1+s_1+j)}$, $j=1,...r_2$ and $w_{i(r+s_1+j)}$, $j=1,...s_2$ for $i=\text{const}$, are all zero except one, or
- case ii) after appropriate additions or subtractions of the equations of (1) aiming to the elimination of all the VCSs except one, if the conditions of case i) are not valid.

In other words, the solution of the dc LEC is completed and then the dc power conservation is easy to prove, if required.

➢ **Example**

As an example, we proceed to determine the node voltages and the currents flowing through the NCVSs for the dc LEC shown in Figure 1.

Applying the NA-VCS method, we replace the nonconvertible voltage sources $v_{s_4}, v_{s_5}, 3v_\varphi$ by virtual current sources i_1^*, i_2^*, i_3^*. Next, defining the reference node and labeling the rest of the nodes as a, b, c, d, e, f, the equivalent circuit takes the form shown in Figure 2.

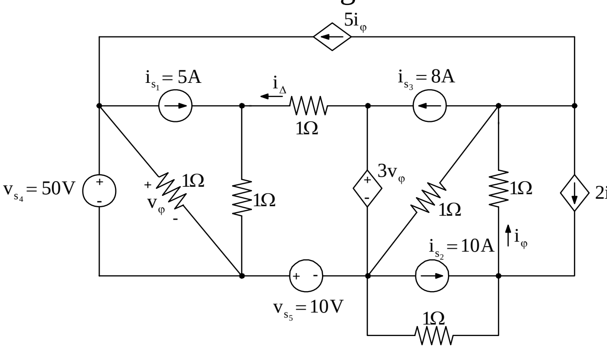

Fig.1. dc LEC for the example

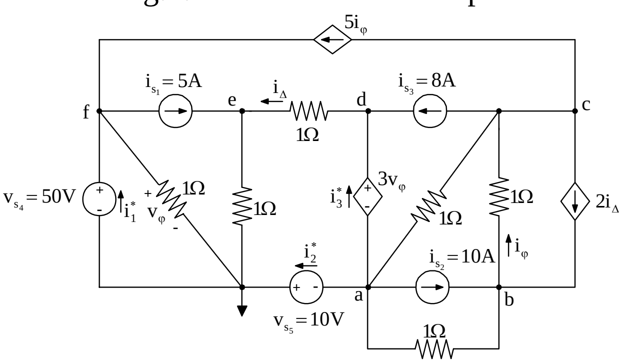

Fig.2. Equivalent circuit for the dc LEC of Fig.1

Since $r_1 = 3$, $r_2 = 2$, $s_1 = 2$, $s_2 = 1$, $k = 6$ and

$$\mathbf{S}^{(1)} = \left[i_{s_1} \quad i_{s_2} \quad i_{s_3} \;\vdots\; 2i_\Delta \quad 5i_\varphi \;\vdots\; i_1^* \quad i_2^* \;\vdots\; i_3^* \right]^T$$

$$\mathbf{S}^{(2)} = \left[i_{s_1} \quad i_{s_2} \quad i_{s_3} \;\vdots\; 2i_\Delta \quad 5i_\varphi \;\vdots\; v_{s_4} \quad v_{s_5} \;\vdots\; 3v_\varphi \right]^T$$

$$\mathbf{S}^{(3)} = \left[2i_\Delta \quad 5i_\varphi \;\vdots\; 3v_\varphi \right]^T$$

$$\mathbf{S}^{(4)} = \left[i_{s_1} \quad i_{s_2} \quad i_{s_3} \;\vdots\; v_{s_4} \quad v_{s_5} \right]^T = \left[5A \quad 10A \quad 8A \;\vdots\; 50V \quad 10V \right]^T$$

the following matrices are determined by inspection

$$\mathbf{G} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{W} = \left[\begin{array}{ccc:cc:cc:c} 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & -1 & -1 & -1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ ** \\ ** \\ *** \\ ** \\ * \end{array}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{Z} = \left[\begin{array}{ccc:cc:cc:c} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Next, the matrices **D** and **T** involved in (3) are obtained by inspection as follows:

- The first three rows of **D** and **T** are the rows of **F** and **Z** respectively, and
- The last three rows of **D** and **T** are the 2nd, 3rd and 5th rows of **G** and **W** respectively, because the VCS coefficients in matrix **W** are zero (Sec. II, A, case a), indicated by ** sign in matrix **W.**

$$\mathbf{D} = \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ \hdashline -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \end{array}\right] \quad \mathbf{T} = \left[\begin{array}{ccc:cc:cc:c} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hdashline 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Next, the matrix $\mathbf{X}$ involved in (4), is obtained by inspection

$$\mathbf{X} = \begin{bmatrix} 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 5 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Thus, matrices **DD** and **TT** are obtained

$$\mathbf{DD}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & -3 \\ -1 & 2 & -1 & -2 & 2 & 0 \\ -1 & 4 & -3 & 2 & -2 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \end{bmatrix} \quad \mathbf{TT}=\left[\begin{array}{ccc|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

Next, by simple matrix manipulations (multiplication and inversion) the node voltage vector, associated with eq. (6), comes up

$$\mathbf{v}=\begin{bmatrix}-10\text{V} & 279\text{V} & 423\text{V} & 140\text{V} & 72{,}5\text{V} & 50\text{V}\end{bmatrix}^T$$

In order to find the currents flowing through the nonconvertible voltage sources, the following are considered:

Since the coefficients $w_{6,7}=0$, $w_{6,8}=0$, and $w_{6,6}\neq 0$ (Sec. II, A, case i), indicated by * sign in the **W** matrix of the example, from the 6th row of **G** and **W**, it is derived

$$1\cdot v_f=-1\cdot i_{s_1}+1\cdot 5i_\varphi+1\cdot i_1^*\Rightarrow i_1^*=775\,\text{A}$$

Since the coefficients $w_{4,6}=0$, $w_{4,7}=0$, and $w_{4,8}\neq 0$ (Sec. II, A, case i), indicated by *** sign in the **W** matrix of the example, from the 4th row of **G** and **W**, it is derived

$$1\cdot v_d-1\cdot v_e=1\cdot i_{s_3}+1\cdot i_3^*\Rightarrow i_3^*=59{,}5\,\text{A}$$

By adding the 1st and 4th row of **G** and **W** respectively, since in this way all VCSs are eliminated except one (Sec. II, A, case ii)), it is derived

$$2\cdot v_a-1\cdot v_b-1\cdot v_c+1\cdot v_d-1\cdot v_e=$$
$$=-1\cdot i_{s_2}+1\cdot i_{s_3}-1\cdot i_2^*\Rightarrow i_2^*=652{,}5\,\text{A}$$

B. *MA-VVS method*

The building elements of a planar dc LEC are given in Table 2.

To apply Mesh Analysis, where the necessary condition is that all sources must be voltage sources, the concept of the *Virtual Voltage Source* (VVS) is introduced. That is, in place of NCICSs or of NCDCSs, VVSs are considered with voltage values equal to the voltages across these current sources.

The NCICSs and the NCDCSs, are replaced by the VVSs with the notation $(\text{ncics})_i^*$, $i=1,\dots,r_2$, and $(\text{ncdcs})_i^*$, $i=1,\dots,s_2$, respectively.

Next, defining the mesh currents in the same direction (clockwise or counter clockwise) for symmetry reasons, by inspection mesh analysis gives:

Table 2. Building elements of a planar dc LEC

SOURCES		
Kind	**No.**	**Notation**
IVS	r_1	$(\text{ivs})_1,\dots,(\text{ivs})_{r_1}$
DVS	s_1	$(\text{dvs})_1,\dots,(\text{dvs})_{s_1}$
NCICS	r_2	$(\text{ncics})_1,\dots,(\text{ncics})_{r_2}$
NCDCS	s_2	$(\text{ncdcs})_1,\dots,(\text{ncdcs})_{s_2}$
OTHER ELEMENTS		
Resistances		
OTHER DETAILS		
• $r=r_1+r_2$: Total No. of indep. sources • $s=s_1+s_2$: Total No.of dep. sources • $m=r+s$: Total No. of sources • k : No. of meshes		

$$\mathbf{R}_{k\times k}\cdot\mathbf{i}_{k\times 1}=\mathbf{v}_{k\times 1}=\mathbf{Q}_{k\times m}\cdot\mathbf{P}^{(1)}_{m\times 1} \quad (7)$$

where $\mathbf{R}_{k\times k}$ is the resistance matrix and $\mathbf{i}_{k\times 1}$ the mesh current vector. Matrix $\mathbf{Q}_{k\times m}$ and vector $\mathbf{P}^{(1)}_{m\times 1}$ are given in Appendix B.

But all current sources replaced by VVSs can be expressed as a linear combination of the mesh currents through the matrix equation

$$\mathbf{N}_{(r_2+s_2)\times k}\cdot\mathbf{i}_{k\times 1}=\mathbf{L}_{(r_2+s_2)\times m}\cdot\mathbf{P}^{(2)}_{m\times 1} \quad (8)$$

where, each row of the **N** matrix describes one of the current sources as a function of the mesh currents. Therefore, the **N** matrix elements are -1, 1 or 0. Matrix $\mathbf{L}_{(r_2+s_2)\times m}$ and vector $\mathbf{P}^{(2)}_{m\times 1}$ are given in Appendix B.

Combining equations (7) and (8), a new matrix equation comes up, where the first r_2+s_2 equations are the equations given by (8) and the rest $k-(r_2+s_2)$ equations are obtained from (7). These equations are obtained following one of the next two cases

- case a) unchanged, if all the VVS coefficients in matrix **Q** are zero, or

- case b) after appropriate additions or subtractions of the equations of (7) aiming to the elimination of all the VVSs, if the conditions of case a) are not valid.

Thus, an equivalent set of equations of the following form is obtained

$$\mathbf{C}_{k\times k}\cdot\mathbf{i}_{k\times 1}=\mathbf{Y}_{k\times m}\cdot\mathbf{P}^{(2)}_{m\times 1} \qquad (9)$$

where $\mathbf{C}_{k\times k}$ and $\mathbf{Y}_{k\times m}$ are matrices given in Appendix B.

However, since the dependent sources are expressed as functions of the mesh currents, one may write

$$\mathbf{P}^{(3)}_{s\times 1}=\mathbf{A}_{s\times k}\cdot\mathbf{i}_{k\times 1} \qquad (10)$$

where $\mathbf{A}_{s\times k}$ is a matrix whose elements describe the values of the dependent sources as functions of the mesh currents of the whole circuit. Vector $\mathbf{P}^{(3)}_{s\times 1}$ is given in Appendix B.

Based on (10), the matrix equation (9) is rearranged as follows:

$$\mathbf{CC}_{k\times k}\cdot\mathbf{i}_{k\times 1}=\mathbf{YY}_{k\times r}\cdot\mathbf{P}^{(4)}_{r\times 1} \qquad (11)$$

Vector $\mathbf{P}^{(4)}_{r\times 1}$, and matrices $\mathbf{CC}_{k\times k}$ $\mathbf{YY}_{k\times r}$ are given in Appendix B.

Finally, based on (11), the mesh current vector is

$$\mathbf{i}_{k\times 1}=\mathbf{CC}^{-1}_{k\times k}\cdot\mathbf{YY}_{k\times r}\cdot\mathbf{P}^{(4)}_{r\times 1} \qquad (12)$$

The currents of all branches are calculated combining the mesh currents. As a consequence, the voltages of all circuit elements are known, except those at the terminals of the Nonconvertible Current Sources (NCCS). These voltages are calculated from the proper set of equations contained in (7). These equations are obtained following one of the next two cases:

- case i) unchanged, if the coefficients $q_{i(r_1+s_1+j)}$, $j=1,...r_2$ and $q_{i(r+s_1+j)}$, $j=1,...s_2$ for $i=\text{const}$, are all zero except one, or
- case ii) after appropriate additions or subtractions of the equations of (7) aiming to the elimination of all the VVSs except one, if the conditions of case i) are not valid.

In other words, the solution of the planar dc LEC is completed and then the dc power conservation is easy to prove, if required.

Example

As an example, we proceed to determine the mesh currents and the voltages across the NCCSs for the planar dc LEC shown in Figure 3.

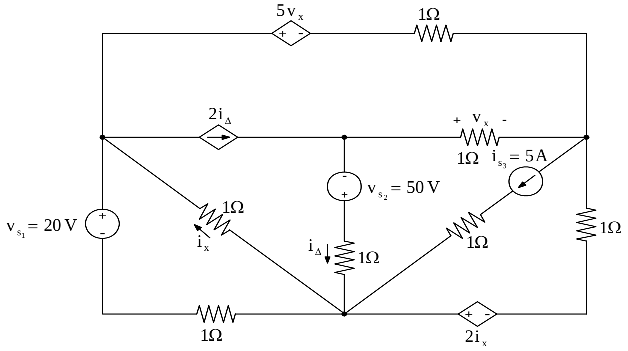

Fig.3. Planar dc LEC for the example

Applying MA-VVS, we replace the nonconvertible current sources $i_{s_3}, 2i_\Delta$ by virtual voltage sources v_1^*, v_2^*. Next, defining the mesh currents in the same direction (clockwise or counter clockwise), the equivalent circuit takes the form shown in Figure 4.

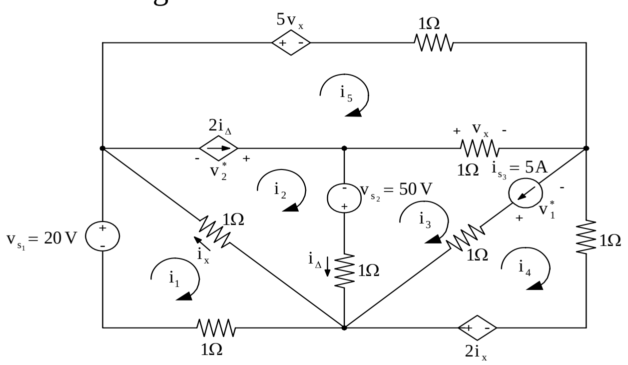

Fig.4. Equivalent circuit for the planar dc LEC of Fig. 3

Since $r_1=2$, $r_2=1$, $s_1=2$, $s_2=1$, $k=5$ and

$$\mathbf{P}^{(1)}=\left[v_{s_1}\quad v_{s_2}\ \vdots\ 2i_x\quad 5v_x\ \vdots\ v_1^*\ \vdots\ v_2^*\right]^T$$

$$\mathbf{P}^{(2)}=\left[v_{s_1}\quad v_{s_2}\ \vdots\ 2i_x\quad 5v_x\ \vdots\ i_{s_3}\ \vdots\ 2i_\Delta\right]^T$$

$$\mathbf{P}^{(3)}=\left[2i_x\quad 5v_x\ \vdots\ 2i_\Delta\right]^T$$

$$\mathbf{P}^{(3)}=\left[v_{s_1}\quad v_{s_2}\ \vdots\ i_{s_3}\right]^T=\left[20V\quad 50V\ \vdots\ 5A\right]^T$$

the following matrices are determined by inspection

$$\mathbf{R}=\begin{bmatrix}2&-1&0&0&0\\-1&2&-1&0&0\\0&-1&3&-1&-1\\0&0&-1&2&0\\0&0&-1&0&2\end{bmatrix}\qquad \mathbf{Q}=\left[\begin{array}{cc:cc:c:c}1&0&0&0&\mathbf{0}&\mathbf{0}\\0&1&0&0&0&\mathbf{1}\\0&-1&0&0&\mathbf{1}&0\\0&0&1&0&-\mathbf{1}&0\\0&0&0&-1&0&-\mathbf{1}\end{array}\right]\begin{matrix}**\\*\\***\\***\\*\end{matrix}$$

$$\mathbf{N}=\begin{bmatrix}0 & 0 & 1 & -1 & 0\\ 0 & 1 & 0 & 0 & -1\end{bmatrix} \quad \mathbf{L}=\left[\begin{array}{cc:cc:c:c}0 & 0 & 0 & 0 & 1 & 0\\ \hdashline 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$$

Next, matrices **C** and **Y** involved in (9) are obtained by inspection as follows:

- The first two rows of **C** and **Y** are the rows of **N** and **L** respectively, and
- The 3rd row of **C** and **Y** is the 1st row of **N** and **L** respectively, because the VVS coefficients in matrix **Q** are zero (Sec. II, B, case a), indicated by ** sign in matrix **Q.**
- The 4th row of **C** and **Y** comes up by adding the 2nd and 5th row of **N** and **L** respectively, because this way the VVSs are eliminated (Sec. II, B, case b), indicated by * sign in matrix **Q.**
- The 5th row of **C** and **Y** comes up by adding the 3rd and 4th row of **N** and **L** respectively, because this way the VVSs are eliminated (Sec. II, B, case b), indicated by***sign in matrix **Q.**

$$\mathbf{C}=\left[\begin{array}{ccccc}0 & 0 & 1 & -1 & 0\\ 0 & 1 & 0 & 0 & -1\\ \hdashline 2 & -1 & 0 & 0 & 0\\ -1 & 2 & -2 & 0 & 2\\ 0 & -1 & 2 & 1 & -1\end{array}\right] \quad \mathbf{Y}=\left[\begin{array}{cc:cc:c:c}0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ \hdashline 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & -1 & 1 & 0 & 0 & 0\end{array}\right]$$

Next, matrix **A** involved in (10), is obtained by inspection

$$\mathbf{A}=\begin{bmatrix}-2 & 2 & 0 & 0 & 0\\ 0 & 0 & 5 & 0 & -5\\ 0 & 2 & -2 & 0 & 0\end{bmatrix}$$

Thus, matrices **CC** and **YY** are obtained.

$$\mathbf{CC}=\begin{bmatrix}0 & 0 & 1 & -1 & 0\\ 0 & -1 & 2 & 0 & -1\\ 2 & -1 & 0 & 0 & 0\\ -1 & 2 & 3 & 0 & -3\\ 2 & -3 & 2 & 1 & -1\end{bmatrix} \quad \mathbf{YY}=\left[\begin{array}{cc:c}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -1 & 0\end{array}\right]$$

Next, by simple matrix manipulations (multiplication and inversion) the mesh current vector, associated with eq. (12), comes up

$$\mathbf{i}=\begin{bmatrix}-35\text{A} & -90\text{A} & -155\text{A} & -160\text{A} & -220\text{A}\end{bmatrix}^{T}$$

In order to find the voltages across the nonconvertible current sources, the following are considered:

Since the coefficients $q_{3,5} \neq 0$, and $q_{3,6} = 0$ (as described in Sec. II, B, case i)), from the 3rd row of **R** and **Q**, it is derived

$$-1\cdot i_2 + 3\cdot i_3 - 1\cdot i_4 - 1\cdot i_5 =$$
$$= -1\cdot v_{s_2} + 1\cdot v_1^* \Rightarrow v_1^* = 55\text{V}$$

Since the coefficients $q_{2,5} = 0$, and $q_{2,6} \neq 0$ (as described in Sec. II, B, case i)), from the 2nd row of **R** and **Q**, it is derived

$$-1\cdot i_1 + 2\cdot i_2 - 1\cdot i_3 =$$
$$= 1\cdot v_{s_2} + 1\cdot v_2^* \Rightarrow v_2^* = -40\text{V}$$

Conclusions

Systematic methods for obtaining the node voltage (mesh current) vector for dc LECs are presented. These methods (NA-VCS, MA-VVS) make it possible to treat any dc LEC (planar or nonplanar) in a similar straightforward way, regardless of the circuit complexity.

The NA-VCS (MA-VVS) has none of the limitations of the basic nodal (mesh) analysis and it is well suited both to symbolic and numerical analysis of complex circuits. Furthermore, it minimizes significantly the work needed to obtain the node voltage (mesh current) vector, since most of the matrices involved are found by inspection due to the use of virtual current (voltage) sources. Some matrix manipulations required are easily implemented using either calculators that can treat large matrices or economically reasonable math programs for personal computers. Also, the algorithm is easily formulated in the computer.

Another equally important advantage of the use of *virtual current (voltage) sources* is the immediate finding of the currents (voltages) through (across) the non-convertible voltage (current) sources. This is because the node voltages (mesh currents) are known, since their currents (voltages) are already expressed by the way the equations are written in matrix form. So the power developed by these sources is easily calculated and therefore the proof of the power balance does not present any difficulties.

NA-VCS and MA-VVS methods can be obviously used for non-dc LECs (i.e. Laplace domain, sinusoidal steady state). Especially, for ac LECs, the necessary condition is that all circuit sources are of the same frequency (otherwise the principle of superposition is used). Under this condition, the NA-VCS and MA-VVS methods are applicable after the circuit transformation to the frequency domain.

Finally, since the proposed methods are well algorithmized, they can by used in most modern simulators of analog networks.

References

[1] Nilsson, J.W. & Riedel, S.A (1996) *Electric Circuits,* Addison Wesley.

[2] Alexander, C.K. & Sadiku, M.N.O. (2000) *Fundamentals of Electric Circuit*, McGraw – Hill.

[3] DeCarlo, R.A. & Lin, P.-M. (2001) *Linear Circuit Analysis*, Oxford University Press.

[4] Chatzarakis, G.E. (2000) *Electric Circuits*, Vol. II, Thessaloniki: Tziolas publications.

[5] Hayt, W.H & Kemmerly, J.E. (1993) *Engineering Circuit Analysis*, New York: McGraw – Hill.

[6] Desoer, C.A & Kuh, E.S. (1969) *Basic Circuit Theory*, McGraw – Hill.

[7] Davis, A. (1998) *Linear Circuit Analysis,* Boston, MA: PWS.

[8] Johnson, D.E., Johnson, J.R. & Hilburn, J.L. (1997) *Electric Circuit Analysis,* Englewood Cliffs, NJ: Prentice Hall.

[9] Irwin, J.D. (1993) *Basic Engineering Circuit Analysis,* 4th ed. NY: Macmillan.

[10] Dorf, R.C. (1993*) Introduction to Electric Circuits*, 2nd ed. NY: Willey.

[11] Gottling, J.G. (1995) *Node and Mesh Analysis by Inspection,* IEEE Transaction on Education, Vol. 38, pp. 312-316, Nov.

[12] Ho, C.W., Ruehli, A.E. & Brennan, P.A. (1975) *The Modified Nodal Approach to Network Analysis,* IEEE Transaction on Circuits and Systems, Vol. CAS-22, pp. 504-509.

APPENDIX A

$$\mathbf{W}=\left[\begin{array}{ccc|ccc|ccc|ccc} w_{11} & \cdots & w_{1r_1} & w_{1(r_1+1)} & \cdots & w_{1(r_1+s_1)} & w_{1(r_1+s_1+1)} & \cdots & w_{1(r+s_1)} & w_{1(r+s_1+1)} & \cdots & w_{1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{k1} & \cdots & w_{kr_1} & w_{k(r_1+1)} & \cdots & w_{k(r_1+s_1)} & w_{k(r_1+s_1+1)} & \cdots & w_{k(r+s_1)} & w_{k(r+s_1+1)} & \cdots & w_{km} \end{array}\right]$$

$$\mathbf{S}^{(1)}_{m\times 1}=\left[(ics)_1\cdots(ics)_{r_1} \;\vdots\; (dcs)_1\cdots(dcs)_{s_1} \;\vdots\; (ncivs)^*_1\cdots(ncivs)^*_{r_2} \;\vdots\; (ncdvs)^*_1\cdots(ncdvs)^*_{s_2}\right]^T$$

$$\mathbf{S}^{(2)}_{m\times 1}=\left[(ics)_1\cdots(ics)_{r_1} \;\vdots\; (dcs)_1\cdots(dcs)_{s_1} \;\vdots\; (ncivs)_1\cdots(ncivs)_{r_2} \;\vdots\; (ncdvs)_1\cdots(ncdvs)_{s_2}\right]^T$$

$$\mathbf{S}^{(3)}=\left[(dcs)_1 \;\ldots\; (dcs)_{s_1} \;\vdots\; (ncdvs)_1 \;\cdots\; (ncdvs)_{s_2}\right]^T \qquad \mathbf{S}^{(4)}=\left[(ics)_1 \;\cdots\; (ics)_{r_1} \;\vdots\; (ncivs)_1 \;\cdots\; (ncivs)_{r_2}\right]^T$$

$$\mathbf{Z}=\left[\begin{array}{c|ccc|ccc|cccc|cccc} & \multicolumn{3}{c|}{\overbrace{\qquad}^{r_1\text{ colums}}} & \multicolumn{3}{c|}{\overbrace{\qquad}^{s_1\text{ colums}}} & \multicolumn{4}{c|}{\overbrace{\qquad}^{r_2\text{ colums}}} & \multicolumn{4}{c}{\overbrace{\qquad}^{s_2\text{ colums}}} \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ r_2\text{ rows} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \hline & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ s_2\text{ rows} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{array}\right]$$

$$\mathbf{D}=\left[\begin{array}{cl} r_2+s_2\text{ rows} & \left\{\begin{array}{cccc} f_{11} & f_{12} & \cdots & f_{1k} \\ f_{21} & f_{22} & \cdots & f_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ f_{(r_2+s_2)1} & f_{(r_2+s_2)2} & \cdots & f_{(r_2+s_2)k} \end{array}\right. \\ \hline k-(r_2+s_2)\text{ rows} & \left\{\begin{array}{l} \text{from the } \mathbf{G} \text{ matrix, either as they are} \\ \text{or by row additions or subtractions} \end{array}\right. \end{array}\right] \qquad \mathbf{T}=\left[\begin{array}{cl} r_2+s_2\text{ rows} & \left\{\begin{array}{cccc} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ z_{(r_2+s_2)1} & z_{(r_2+s_2)2} & \cdots & z_{(r_2+s_2)m} \end{array}\right. \\ \hline k-(r_2+s_2)\text{ rows} & \left\{\begin{array}{l} \text{from the } \mathbf{W} \text{ matrix, either as they are} \\ \text{or by row additions or subtractions} \end{array}\right. \end{array}\right]$$

$$\mathbf{DD}=\begin{bmatrix} d_{11}-\sum_{j=1}^{s_1} t_{1(r_1+j)}x_{j1}-\sum_{j=1}^{s_2} t_{1(r_1+s_1+r_2+j)}x_{(s_1+j)1} & \cdot\ \cdot & d_{1k}-\sum_{j=1}^{s_1} t_{1(r_1+j)}x_{jk}-\sum_{j=1}^{s_2} t_{1(r_1+s_1+r_2+j)}x_{(s_1+j)k} \\ \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot\ \cdot & \cdot \\ d_{k1}-\sum_{j=1}^{s_1} t_{k(r_1+j)}x_{j1}-\sum_{j=1}^{s_2} t_{k(r_1+s_1+r_2+j)}x_{(s_1+j)1} & \cdot\ \cdot & d_{kk}-\sum_{j=1}^{s_1} t_{k(r_1+j)}x_{jk}-\sum_{j=1}^{s_2} t_{k(r_1+s_1+r_2+j)}x_{(s_1+j)k} \end{bmatrix}$$

$$\mathbf{TT}=\left[\begin{array}{ccc|ccc} t_{11} & \cdots & t_{1r_1} & t_{1(r_1+s_1+1)} & \cdots & t_{1(r_1+s_1+r_2)} \\ t_{21} & \cdots & t_{2r_1} & t_{2(r_1+s_1+1)} & \cdots & t_{2(r_1+s_1+r_2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{k1} & \cdots & t_{kr_1} & t_{k(r_1+s_1+1)} & \cdots & t_{k(r_1+s_1+r_2)} \end{array}\right]$$

APPENDIX B

$$\mathbf{Q}=\left[\begin{array}{ccc|ccc|ccc|ccc} q_{11} & \cdots & q_{1r_1} & q_{1(r_1+1)} & \cdots & q_{1(r_1+s_1)} & q_{1(r_1+s_1+1)} & \cdots & q_{1(r+s_1)} & q_{1(r+s_1+1)} & \cdots & q_{1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{k1} & \cdots & q_{kr_1} & q_{k(r_1+1)} & \cdots & q_{k(r_1+s_1)} & q_{k(r_1+s_1+1)} & \cdots & q_{k(r+s_1)} & q_{k(r+s_1+1)} & \cdots & q_{km} \end{array}\right]$$

$$\mathbf{P}^{(1)}_{m\times 1}=\left[(ivs)_1\cdots(ivs)_{r_1}\ \vdots\ (dvs)_1\cdots(dvs)_{s_1}\ \vdots\ (ncics)^*_1\cdots(ncics)^*_{r_2}\ \vdots\ (ncdcs)^*_1\cdots(ncdcs)^*_{s_2}\right]^T$$

$$\mathbf{P}^{(2)}_{m\times 1}=\left[(ivs)_1\cdots(ivs)_{r_1}\ \vdots\ (dvs)_1\cdots(dvs)_{s_1}\ \vdots\ (ncics)_1\cdots(ncics)_{r_2}\ \vdots\ (ncdcs)_1\cdots(ncdcs)_{s_2}\right]^T$$

$$\mathbf{P}^{(3)}=\left[(dvs)_1\ \ \ldots\ \ (dvs)_{s_1}\ \vdots\ (ncdcs)_1\ \ \cdots\ \ (ncdcs)_{s_2}\right]^T \quad \mathbf{P}^{(4)}=\left[(ivs)_1\ \ \cdots\ \ (ivs)_{r_1}\ \vdots\ (ncics)_1\ \ \cdots\ \ (ncics)_{r_2}\right]^T$$

$$\mathbf{L}=\left[\begin{array}{c|ccc|ccc|cccc|cccc} & \multicolumn{3}{c|}{r_1 \text{ colums}} & \multicolumn{3}{c|}{s_1 \text{ colums}} & \multicolumn{4}{c|}{r_2 \text{ colums}} & \multicolumn{4}{c}{s_2 \text{ colums}} \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ r_2 \text{ rows} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \hline & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ s_2 \text{ rows} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{array}\right]$$

$$\mathbf{C}=\left[\begin{array}{cc} r_2+s_2 \text{ rows} & \begin{cases} n_{11} & n_{12} & \cdots & n_{1k} \\ n_{21} & n_{22} & \cdots & n_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ n_{(r_2+s_2)1} & n_{(r_2+s_2)2} & \cdots & n_{(r_2+s_2)k} \end{cases} \\ \hline k-(r_2+s_2) \text{ rows} & \begin{cases} \text{from the } \mathbf{R} \text{ matrix, either as they are} \\ \text{or by row additions or subtractions} \end{cases} \end{array}\right]$$

$$\mathbf{Y}=\left[\begin{array}{cc} r_2+s_2 \text{ rows} & \begin{cases} \ell_{11} & \ell_{12} & \cdots & \ell_{1m} \\ \ell_{21} & \ell_{22} & \cdots & \ell_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \ell_{(r_2+s_2)1} & \ell_{(r_2+s_2)2} & \cdots & \ell_{(r_2+s_2)m} \end{cases} \\ \hline k-(r_2+s_2) \text{ rows} & \begin{cases} \text{from the } \mathbf{Q} \text{ matrix, either as they are} \\ \text{or by row additions or subtractions} \end{cases} \end{array}\right]$$

$$\mathbf{YY}=\left[\begin{array}{ccc|ccc} y_{11} & \cdots & y_{1r_1} & y_{1(r_1+s_1+1)} & \cdots & y_{1(r_1+s_1+r_2)} \\ y_{21} & \cdots & y_{2r_1} & y_{2(r_1+s_1+1)} & \cdots & y_{2(r_1+s_1+r_2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{k1} & \cdots & y_{kr_1} & y_{k(r_1+s_1+1)} & \cdots & y_{k(r_1+s_1+r_2)} \end{array}\right]$$

$$\mathbf{CC}=\begin{bmatrix} c_{11}-\sum_{j=1}^{s_1} y_{1(r_1+j)}a_{j1}-\sum_{j=1}^{s_2} y_{1(r_1+s_1+r_2+j)}a_{(s_1+j)1} & \cdot\ \cdot & c_{1k}-\sum_{j=1}^{s_1} y_{1(r_1+j)}a_{jk}-\sum_{j=1}^{s_2} y_{1(r_1+s_1+r_2+j)}a_{(s_1+j)k} \\ \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot\ \cdot & \cdot \\ c_{k1}-\sum_{j=1}^{s_1} y_{k(r_1+j)}a_{j1}-\sum_{j=1}^{s_2} y_{k(r_1+s_1+r_2+j)}a_{(s_1+j)1} & \cdot\ \cdot & c_{kk}-\sum_{j=1}^{s_1} y_{k(r_1+j)}a_{jk}-\sum_{j=1}^{s_2} y_{k(r_1+s_1+r_2+j)}a_{(s_1+j)k} \end{bmatrix}$$