ELECTRIC PERMITTIVITY BEHAVIOR OF THEORETICAL MODELED DIELECTRICS, USING THE EQUIVALENT CIRCUIT METHOD

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Abstract. *Theoretical models for dielectric materials were used to obtain their equivalent electrical circuits having frequency responses similar with the electric permittivity frequency behavior. A graduate evolution of the equivalent circuits was included here, considering that theoretical models for dielectric material were describing more of the inside processes. The time response of the equivalent circuit, ended on a load impedance, can be used for studying the dielectric filled capacitor behavior in an electronic circuit.*
Keywords: *Debye dielectric, dielectric with resonant absorptions, model, equivalent circuit, dielectric permittivity, frequency and time response.*

Introduction

For some categories of dielectric materials, one can model the frequency behavior of the complex electric permittivity, $\varepsilon(\omega)$, using an equivalent electric RLC circuit. Determination of equivalent circuit parameters was done considering the expression of the complex permittivity for each dielectric category.
We will start with the Debye dielectric category, for which we have the complex permittivity given by ([1, 4]):

$$\begin{aligned}\varepsilon(\omega) &= \varepsilon' - j\varepsilon'' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau_D} = \\ &= \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2\tau_D{}^2} - j\frac{\omega\tau_D(\varepsilon_s - \varepsilon_\infty)}{1+\omega^2\tau_D{}^2}\end{aligned} \tag{1}$$

where $\varepsilon_\infty = \varepsilon(\omega \to \infty)$, $\varepsilon_s = \varepsilon(\omega = 0)$, also the stationary field value and τ_D is the Debye dielectric relaxation time.
An ideal capacitor, with C = 1 F, filled with a Debye dielectric, has an impedance of $\frac{1}{\omega\varepsilon(\omega)}\Omega$. Because $\varepsilon(\omega)$ is a complex quantity, this impedance can be modeled by a capacitance $C = \varepsilon'(\omega)\mathrm{F}$ parallel with a resistance $R = \frac{1}{\omega\varepsilon''(\omega)}\Omega$. Of course, this is not the only possibility and other equivalent circuits, with the same equivalent impedance can be found.
If we work in the same manner, we can find the equivalent circuit modeling the relation (1), as is shown in figure 1 ([4]):

$$C_1 = \varepsilon_\infty C_0,\ C_2 = (\varepsilon_s - \varepsilon_\infty)C_0,\ R = \frac{\tau_D}{(\varepsilon_s - \varepsilon_\infty)C_0}.$$

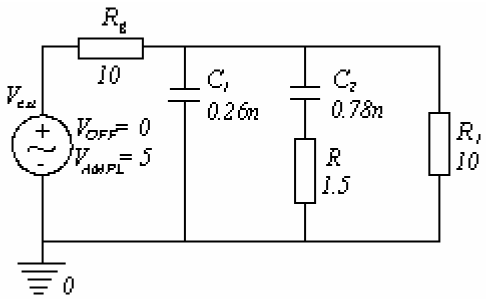

Fig. 1. The equivalent circuit modeling the relation (1), excited by a signal source and ended on a load impedance.

These circuit impedances are computed for distilled water, at normal pressure and temperature, (which can be well approximated as a Debye dielectric), with a proper value for C_0. The equivalent circuit is excited by a signal source, with variable frequency and ends on a proper load impedance. This simple circuit allows us to analyze the $\varepsilon(\omega)$ frequency behavior, which is the same as the circuit frequency response.

The real dielectrics behave more complicated, especially at frequencies from optical range, where the resonant absorptions appear ([2, 3, 5]). In these cases, we can write the complex electric permittivity as (Born and Wolf):

$$\varepsilon(\omega)=\varepsilon_\infty+\frac{\varepsilon_s-\varepsilon_\infty}{1-\left(\frac{\omega}{\omega_0}\right)^2+j\omega\gamma}=$$

$$=\varepsilon_\infty+\frac{(\varepsilon_s-\varepsilon_\infty)\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]}{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2+\omega^2\gamma^2}-j\frac{\omega\gamma(\varepsilon_s-\varepsilon_\infty)}{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2+\omega^2\gamma^2} \qquad (2)$$

where ω_0 is the resonance frequency and γ is attenuation constant of an electromagnetic field in the material.
The equivalent circuit for relation (2) is given in figure 2, ([4]), and consists of the group of impedances:

$$C_1=\varepsilon_\infty C_0,\, C_2=(\varepsilon_s-\varepsilon_\infty)C_0,$$

$$L=\frac{1}{\omega_0^2(\varepsilon_s-\varepsilon_\infty)C_0},\; R=\frac{\gamma}{(\varepsilon_s-\varepsilon_\infty)C_0}.$$

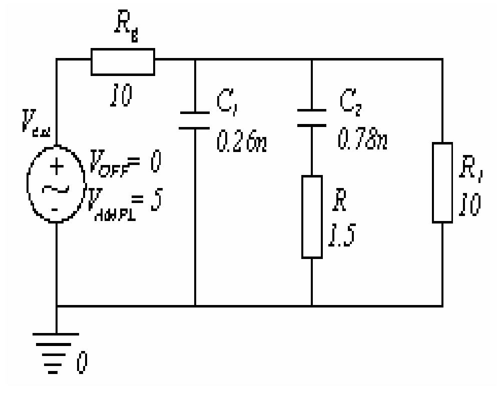

Fig. 2. The equivalent circuit modeling the relation (2), excited by a signal source and ended on a load impedance.

At the resonance frequency, $(\varepsilon'(\omega_0)-\varepsilon_\infty)$ vanishes and the losses are maximal in the material: $\varepsilon''=\frac{\varepsilon_s-\varepsilon_\infty}{\omega_0\gamma}$. The inductance L models the inertial effects of the elementary dipoles orientation in the applied filed. All impedances are computed here for paraffin ($C_{25}H_{52(s)}$), with a proper C_0.

More complicated resonant absorptions can be described by (van Vleck and Weisskopf):

$$\varepsilon(\omega)=\varepsilon_\infty+\frac{1}{2}(\varepsilon_s-\varepsilon_\infty)\left[\frac{1+j\omega_0\tau}{1+j(\omega+\omega_0)\tau}+\frac{1-j\omega_0\tau}{1+j(\omega-\omega_0)\tau}\right] \qquad (3)$$

which can be modeled by equivalent circuit ([4]) consisting of the group of impedances:

$$C_1=\varepsilon_\infty C_0,\; C_2=\frac{1}{2}(\varepsilon_s-\varepsilon_\infty),\; C_3=\frac{1}{2}(\varepsilon_s-\varepsilon_\infty),$$

$$L_1=\frac{2\tau^2}{(\varepsilon_s-\varepsilon_\infty)(1+\omega_0^2\tau^2)C_0},$$

$$L_2=\frac{2\tau^2}{(\varepsilon_s-\varepsilon_\infty)(1+\omega_0^2\tau^2)C_0},$$

$$R_1=\frac{4\tau}{(\varepsilon_s-\varepsilon_\infty)(1+\omega_0^2\tau^2)C_0},\; R_2=\frac{\tau}{(\varepsilon_s-\varepsilon_\infty)C_0},$$

given in figure 3, computed here for glycerin (1,2,3 propantryol – $C_3H_8O_3$).

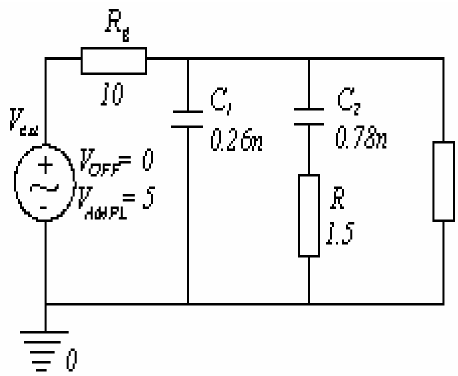

Fig. 3. The equivalent circuit modeling the relation (3), excited by a signal source and ended on a load impedance.

One remarks that the relation (3) reduces to relation Debye, (1), for very high frequencies, beyond the resonances range, $\omega \gg \omega_0$.

Frequency and time responses of the equivalent circuit

For the dielectrics in the categories considered above, frequency evolution of the electric permittivity is given by the frequency response of the equivalent circuit.

We will give, for comparison, the frequency response of the equivalent circuit for ideal dielectric. Permittivity evolution is a classical one, as can be observed in figure 4 (the curve falls down with constant slope). The permittivity values are given in arbitrary units.

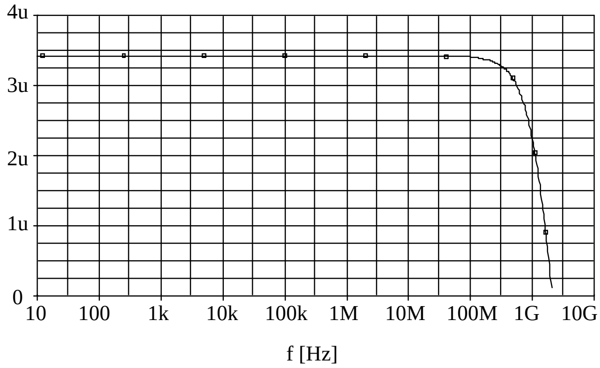

Fig.4. Frequency evolution of an ideal dielectric permittivity, obtained with the equivalent circuit.

Permittivity for the Debye dielectric has the frequency evolution presented in figure 5.

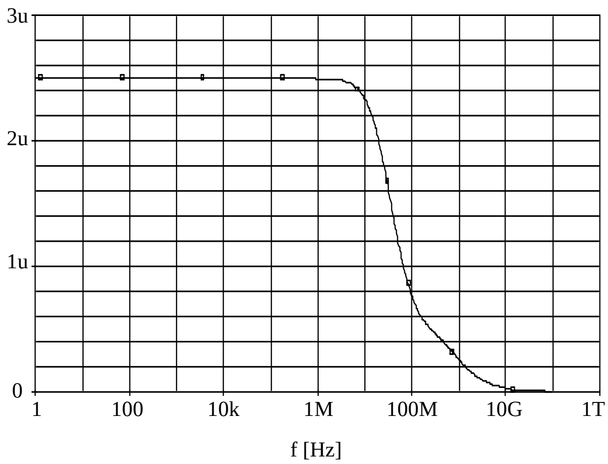

Fig. 5. Electric permittivity versus frequency, for a Debye dielectric, obtained with the equivalent circuit.

For dielectrics with resonant absorptions, frequency behavior of permittivities described by relations (2) and (3) are obtained, using the equivalent circuits, as follows (figures 6, respectively 7).

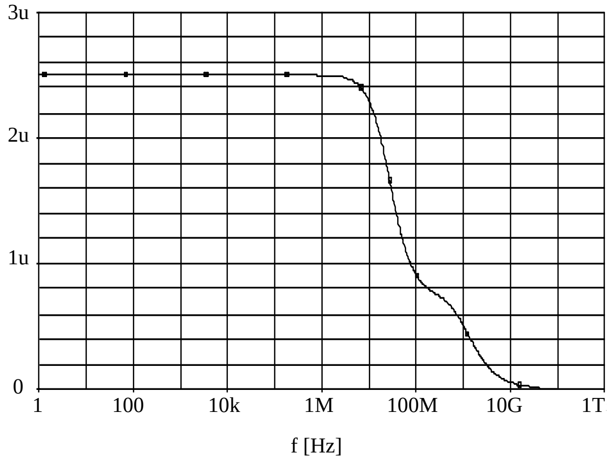

Fig. 6. Electric permittivity (Born and Wolf) versus frequency, for a dielectric with resonant absorptions, obtained with the equivalent circuit.

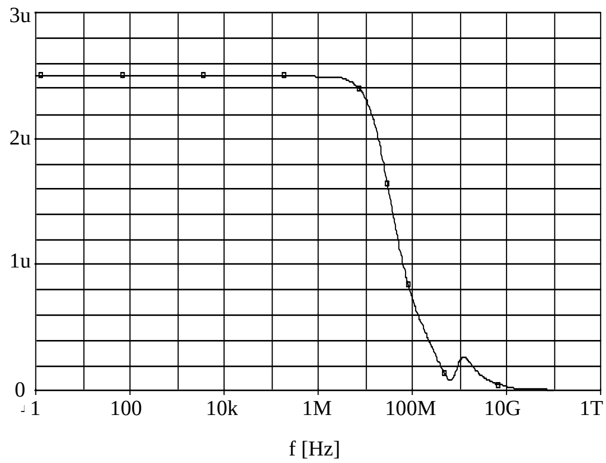

Fig. 7. Electric permittivity (van Vleck and Weisskopf) versus frequency, for a dielectric with resonant absorptions, obtained with the equivalent circuit.

One observes that the permittivity evolution differs of that for the ideal dielectric and a very high frequency evolution starts to appear. If we extract from literature the frequency of an resonant permittivity peak for our epoxy resin, the resonance appears clearly in the last model (van Vleck and Weisskopf) equivalent circuit response.

If we consider now the equivalent circuits excited by a pulse generator, we can study the pulse deformation on an R, C parallel circuit, where the capacitor is dielectric filled. For the categories of dielectrics modeled above, we obtained the time responses given in figure 8.

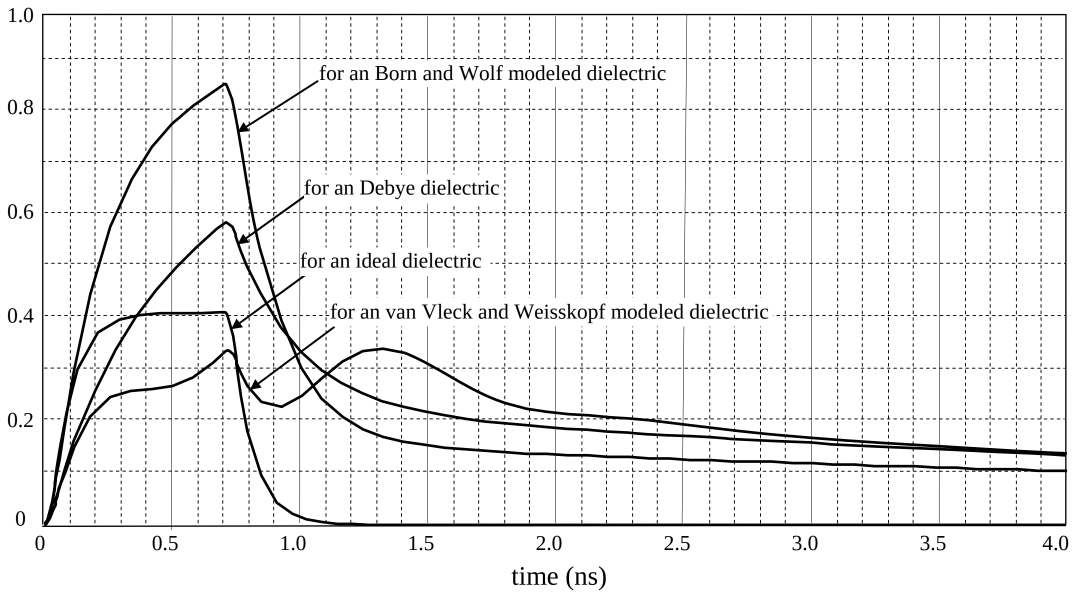

Fig. 8. Time response of an R, C parallel circuit, with the capacitor filled with dielectric differently modeled.

One can observe that the classical R, C filter behavior is modified by the nature of dielectric which fills the capacitor. The inner real dielectric determinates an increase of the rising time of the circuit response and a slow decreasing of it. The equivalent circuit for Born and Wolf modeled dielectric gives a time response which approximates better the reality than the circuit for van Vleck and Weisskopf modeling, even the curve amplitude is to high. A van Vleck and Weisskopf modeling gives a time response of the dielectric filled capacitor in which pulse is strongly flattened and the decrease imposed by the capacitor is no illustrated.

Conclusions

A real dielectric which fills a capacitor can be differently modeled, pursuing a more fare description of its complex electric permittivity. The Debye model gives circuit responses close to these corresponding to the ideal dielectric model. It is not relevant.

The equivalent circuit for Born and Wolf modeled dielectric gives a not so fare frequency response and it is preferable a van Vleck and Weisskopf modeling for a better resonant absorption representation. But, a van Vleck and Weisskopf modeling gives an inadequate time response of the dielectric filled capacitor and consequently it is no indicated in time analysis.

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