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ABOUT ONE PROBLEM OF MULTICRITERIA OPTIMIZATION

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Abstract. *We continue to study the application of the methods of multicriteria optimization for the decision of the task of building farms designing. The task is to find such farm parameters which allow the farm to stay stable and steady while being maximally loaded on the condition that the farm weight is minimal.*
Keywords: *optimization, multicriteria optimization, maximum, minimum, variables of projection, purpose functions, methods of optimization, farm, the force arising in rods.*

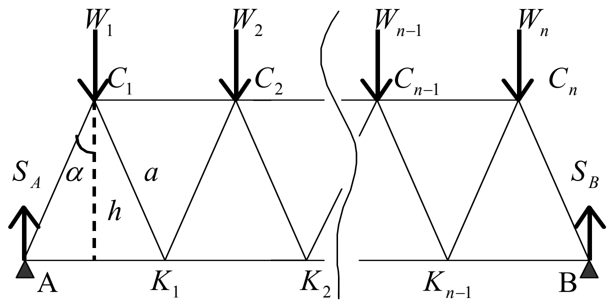
Introduction

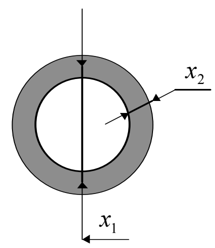
The generalization of the task, that was described in the article [1], is being considered in this paper.
In this problem the geometry of a farm is supposed to be known and the load is affixed only in the upper knots. The purpose functions are the gross total farm weight and the sum of the load affixed. The variables of projection are parameters of rods cross-section.

Statement of the problem

Let the building farm is fixed on supporting two points (A and B). The height h and the length l of the lower foundation of the farm are known. There are n knots (C_1, $C_2,\ldots,$ C_n) on the upper farm foundation and there are $n-1$ knots (K_1, $K_2,\ldots,$ K_{n-1}) between points A and B. They are connected by rods to the upper knots of a farm. The rods are made of rigid empty pipes. The thickness of a wall is x_2 and the average diameter is x_1. The forces W_1, $W_2,\ldots,$ W_n are applied to the knots C_1, $C_2,\ldots,$ C_n of this construction. The problem of projection is to find the thickness of the walls x_2 and the average diameter x_1 of the pipes so that the farm could withstand maximum summarized load, the farm weight G to be minimum.

At the projection two basic conditions should be fulfilled. First, the voltage of the compression in rods should not exceed the possibilities of a material the rod is made of. Second, rods should be not deformed by the pressure applied.





Apart from basic conditions some additional conditions should be fulfilled: the average diameter of pipes should not be smaller than the minimum diameter $x_{\min}$ and the ratio of the thickness of rods wall to their average diameter should not be smaller than the minimum ratio $x^{\min}$.

Mathematical model

The area of the cross section of rods equals to $F = \pi x_1 x_2$. The voltage of the compression in rods is $\sigma = \frac{P^{(i)}}{F}$ where $P^{(i)}$ designates the effort in the rod number i. This must satisfy to the inequality $\sigma \le R$ where R is the resistance of a material. The condition of the durability of this rod follows the formula $P^{(i)} \le \pi R x_1 x_2$.

The stability condition can be received in the next way. The main moment of inertia of rod cross section is $I = \frac{\pi (x_1 + x_2)^4}{64}(1-\beta^4)$ where $\beta = \frac{x_1 - x_2}{x_1 + x_2}$. The critical force of the rod number i of length l_i equals to

$$P_* = \frac{\pi^2 EI}{l_i^2} = \pi^2 E \frac{\pi (x_1 + x_2)^4 (1-\beta^4)}{64 l_i^2} \ge P^{(i)}$$

where E is the module of material elasticity. The condition of the stability for this rod takes the form

$$P^{(i)} l_i^2 \le \frac{\pi^3}{8} E x_1 x_2 (x_1^2 + x_2^2).$$

Then the lengths of rods are equal l/n for the upper and lower foundation and

$$a = \tfrac{1}{2n}\sqrt{4n^2h^2 + l^2}$$

for cross rods.

Let's define the force $P^{(i)}$ that arises in rods. It is necessary to determine the meanings of such values $P^{(C_1C_2)}$, $P^{(C_2C_3)}, \ldots,$ $P^{(C_{n-1}C_n)}$, $P^{(AK_1)}$, $P^{(K_1K_2)}, \ldots,$ $P^{(K_{n-1}B)}$, $P^{(AC_1)}$, $P^{(K_1C_2)}, \ldots,$ $P^{(K_{n-1}C_n)}$, $P^{(K_1C_1)}$, $P^{(K_2C_2)}, \ldots,$ $P^{(BC_n)}$.

The reaction of the support S_A is determined from the condition that the following formula is valid:

$$0 = \sum M_B = l \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{n-i} - S_A l.$$

This implies that $S_A = \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{n-i}$.

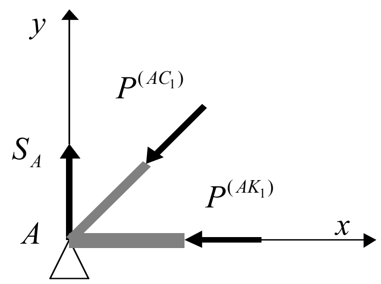
The reaction of the support S_B is determined from the condition that the following formula is valid:

$$0 = \sum M_A = l \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{i+1} - S_B l.$$

This implies that

$$S_B = \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{i+1}.$$

Let's determine $P^{(AC_1)}$. For this we shall take the advantage of projections method: the sum of the



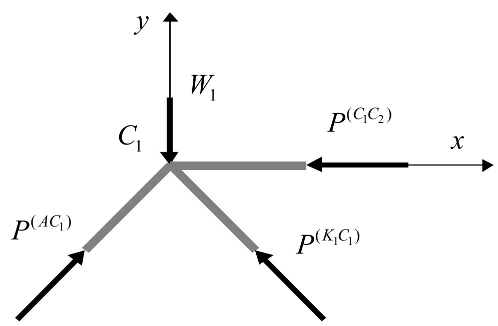
projections the forces on the axis of ordinates gives the equality $S_A - P^{(AC_1)} \cos\alpha = 0$. This implies that

$$P^{(AC_1)} = \frac{a}{h} \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{n-i} = \frac{a}{h} S_A.$$

Let's determine $P^{(AK_1)}$. The sum of the projections the forces on the abscissa axis gives the equality $-P^{(AC_1)} \sin\alpha - P^{(AK_1)} = 0$. This implies that

$$P^{(AK_1)} = -\frac{l}{2nh} \sum_{i=0}^{n-1} \frac{2i+1}{2n} W_{n-i}.$$

Let's determine $P^{(K_1C_1)}$. Here and futher we shall



use the method of knots cutting. Imagine that we cut out the knot C_1. The sum of the projections of the forces on the axis of ordinates gives

$$-W_1 + P^{(K_1C_1)}\cos\alpha + P^{(AC_1)}\cos\alpha = 0 .$$

This implies that

$$P^{(K_1C_1)} = \frac{a}{h}\left(W_1 - \sum_{i=0}^{n-1}\frac{2i+1}{2n}W_{n-i}\right) = \frac{a}{h}\left(W_1 - S_A\right).$$

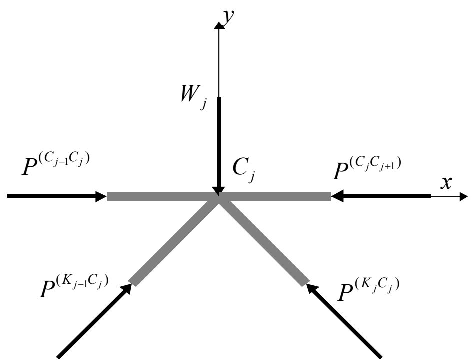
Let's determine $P^{(C_1C_2)}$. The sum of the projections of the forces on the abscissa axis results in the equality

$$-P^{(C_1C_2)} - P^{(K_1C_1)}\sin\alpha + P^{(AC_1)}\sin\alpha = 0 .$$

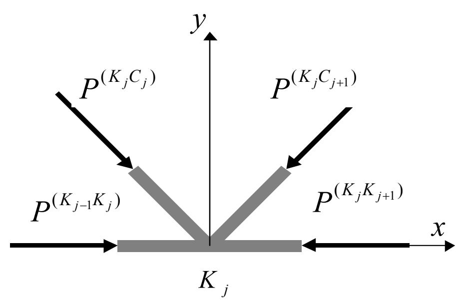
This implies that

$$P^{(C_1C_2)} = \frac{l}{2nh}\left(2S_A - W_1\right).$$

Similarly we shall take the advantage of the method of the projections while determining the meanings for the rest of the rods. By generalizing the received formulas we come to the conclusion that



$$P^{(K_jC_j)} = \frac{a}{h}\left(\sum_{i=1}^{j}W_i - S_A\right), \quad j = \overline{1, n}$$



$$P^{(K_jC_{j+1})} = \frac{a}{h}\left(S_A - \sum_{i=1}^{j}W_i\right), \quad j = \overline{0, n}$$

$$P^{(K_jK_{j+1})} = \frac{l}{2nh}\left(2\sum_{i=1}^{j}(j-i+1)W_i - (2j+1)S_A\right),$$

$$j = \overline{0, n}$$

$$P^{(C_jC_{j+1})} = \frac{l}{2nh}\left(2jS_A - \sum_{i=1}^{j}W_i - 2\sum_{i=1}^{j-1}(j-i)W_i\right),$$

$$j = \overline{1, n-1} .$$

The gross weight of the farm is equal to such value:
$G = \pi x_1 x_2 \rho(\frac{3}{2}l + 4a)$ where ρ is the density of a material.

Thus, we can outline the multicriteria problem of the optimization. The problem goal is to find such x_1 and x_2 at which

$$\min G ,$$

$$\max W = \sum_{i=1}^{n} W_i$$

are reached and the conditions

$$P^{(i)} \le \pi R x_1 x_2 ,$$

$$P^{(i)} l_i^2 \le \frac{\pi^3}{8} E x_1 x_2 (x_1^2 + x_2^2) ,$$

$$x_1 \ge x_{\min} , \ \frac{x_2}{x_1} \ge x^{\min}$$

are fulfilled.

We can solve this problem by one of the multicriteria optimization methods [2] application.

References

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[2] Жадан В.Г., Кушнирчук В.И. (1987) *Пакет методов многокритериальной оптимизации в системе ДИСО* // Пакеты прикладных программ: Программное обеспечение оптимизационных задач.– М.: Наука,. С.17-26.