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COLLISION DETECTION OF CONVEX POLYTOPES USING PSEUDOINVERSE MATRIX

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Abstract*: This paper presents an efficient algorithm for collision detection between two convex polytopes. It proposes a new method for collision detection which use pseudoinverse matrix(CDPM). It also describes multi level algorithms to realize CDPM method.*
Keywords: *collision detection, convex polytopes, matrix inequalities.*

Introduction

Collision detection has been a fundamental problem in computer animation, geometric modeling, molecular modeling, CAD/CAM and robotics. In the literature this term also is known as interference detection, contact determination, clash detection, intersection tests, etc. The names of technical terms depend on the content of applications. The problems concern the fact that two impenetrable objects cannot share a common region. The problem defined between two or more objects. In all these fields, prompt recognition of possible impacts is necessary for computing responses as an effect of collision.
The problem collision detection has been well studied in the literature[1-9]. There are many techniques in the literature to solve this problem and numerous solutions have been proposed. The simplest algorithms for collision detection are based on using bounding volumes and spatial decomposition techniques in a hierarchical manner. Typical examples of bounding volumes include spheres, Axis Aligned Bounding Boxes (AABBs)[1,7], Oriented Bounding boxes (OBBs)[3, 4] and they are chosen for the simplicity of finding collision between two such volumes. Another possibility is using spatial decomposition[1,2,5] into a set of convex pieces or voxelised containers. Typically, in those cases where the broad phase of the algorithm is not able to determine the collision status, the narrow phase takes over in order to do more detailed intersection calculations.
Therefore fast and effective algorithm should consist of two levels [1,2]. The top level should be very fast, for example algorithm AABB, and on bottom at a level more an accurate. This paper proposes a new accurate method for collision detection which use pseudoinverse matrix (CDPM) for convex polytopes.

Object representation

In modelling are two major representations schemata exist: boundary representation and constructive solid geometry (CSG)[1]. Each has its own advantages and inherent problems. For object describing it is use boundary representation and CSG operations include rotation and translation.
Object representation by surface, parameters needs to describe a face, an edge, and a vertex.
All objects are defined with respect to a global coordinate system, the world coordinate frame. The initial configuration is specified in terms of the origin of the system. And each object has a local coordinate frame associated with it. The configuration of each object is expressed by the basic operations (rotation and translation) to

place each object with respect to a global world coordinate frame.

Object description

This part will describe some basic concepts used in the development of algorithms. The objects which are under construction of convex polytopes, are considered

Convex Polytopes. Convex Polytopes in affine space E^n is represented by a compact set $\mathrm{P}_c \subset \mathrm{E}^n$. The P_c is defined by set of points $\mathrm{P} = \{P_1, P_2, ..., P_r\}$ in E^n. A point in $X \in \mathrm{E}^n$ is inside the convex polytopes P_c, if real numbers $\lambda_1 \geq 0, ..., \lambda_r \geq 0$ and $\lambda_1 + \lambda_2 + ... + \lambda_r = 1$ such as $X = \lambda_1 P_1 + \lambda_2 P_2 + ... + \lambda_r P_r$ exist.

Hyperplanes. A hyperplane H is an affine subspace of E^n of dimension $n-1$. It is defined by

$$\{X \in \mathrm{E}^n \mid S \cdot X = \beta\},$$

or

$$s_1 x_1 + s_2 x_2 + ... + s_n x_n = \beta \tag{1}$$

where $S = (s_1, s_2, ..., s_n)$ is a some vector in E^n, β is a const.

The two half-closed spaces determined by a hyperplane H are defined as

$$\{X \in \mathrm{E}^n \mid S \cdot X \leq \beta\}$$

and

$$\{X \in \mathrm{E}^n \mid S \cdot X \geq \beta\}$$

or

$$s_1 x_1 + s_2 x_2 + ... + s_n x_n \leq \beta$$

and

$$s_1 x_1 + s_2 x_2 + ... + s_n x_n \geq \beta \tag{2}$$

Using (1)-(2) a convex polytopes in affine space E^n we can represent a system linear inequalities

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq \beta_1$$
$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq \beta_2$$
$$....$$
$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq \beta_m$$

or matrix inequalities $Ax <= b$

where m - quantity of sides of polytopes, $b = (\beta_1, \beta_2, ...\beta_m)$.

Collision detection for two convex polytopes

Let's have two convex polytopes: Φ_1 and Φ_2. Each convex polytopes can be described at system of linear inequalities:

Φ_1:

$$a_{11}{}^1 x_1 + a_{12}{}^1 x_2 + ... + a_{1n}{}^1 x_n \leq \beta_1{}^1$$
$$a_{21}{}^1 x_1 + a_{22}{}^1 x_2 + ... + a_{2n}{}^1 x_n \leq \beta_2{}^1$$
$$....$$
$$a_{m1}{}^1 x_1 + a_{m2}{}^1 x_2 + ... + a_{mn}{}^1 x_n \leq \beta_m{}^1$$

or matrix inequalities $A^1 x <= b^1$

Φ_2:

$$a_{11}{}^2 x_1 + a_{12}{}^2 x_2 + ... + a_{1n}{}^2 x_n \leq \beta_1{}^2$$
$$a_{21}{}^2 x_1 + a_{22}{}^2 x_2 + ... + a_{2n}{}^2 x_n \leq \beta_2{}^2$$
$$....$$
$$a_{r1}{}^2 x_1 + a_{r2}{}^2 x_2 + ... + a_{rn}{}^2 x_n \leq \beta_r{}^2$$

or matrix inequalities $A^2 x <= b^2$

The problem collision detection can be presented a task of existence of the solving of the incorporated system of linear inequalities:

$$Ax <= b \tag{3}$$

where $A = \begin{pmatrix} A^1 \\ A^2 \end{pmatrix}$, $b = \begin{pmatrix} b^1 \\ b^2 \end{pmatrix}$, $x \in \mathrm{E}^n$, $b \in \mathrm{E}^p$, $p = m + r$.

The problem of existence of the solving of system of linear inequalities is more fully described in [10-12].

Theorem 1 [11]. *It is enough for existence of the solving of an inequality (3), that pair of indexes i, j would be found: $1 \leq i, j \leq p$ with $e_i^T P_A e_j$ so that the matrix inequality was carried out*:

$$\{E_p - (e_i^T P_A e_j) \cdot P_A e_j \cdot e_i^T\} Z(A^T) b \geq 0 \tag{4}$$

where P_A - orthogonal projector on subspace ($P_A = A \cdot A^+ = (A^T)^+ \cdot A^T$); A^+- pseudo-inverse matrix of matrix A ; $Z(A^T)$ -operator of orthogonal designing on orthogonal addition to L_A; L_A - is the space of meanings of the operator with a matrix A; e_i - vector with one i - by that and zero by others E^m [10].

Algorithm Overview

In general cases the CDPM method, considered in this paper, slow enough, is especial when the objects are placed in the different parties. At the same time at direct contact it gives good result in comparison with iterative algorithms.

The basic idea of construction is effective of algorithm in combinations with fast simple and slow one, but exacter. As simple it is possible to choose algorithms AABB or OBB, but without iterative specification. In that cases if the simple algorithm does not give positive result then exacter algorithm is caused, at overlapping areas of contact.

The algorithm, submitted in this paper, is based on the *theorem 1*. The essence of algorithm consists in checks of correctness of an inequality (4) for pairs of indexes i and j. That is, in consecutive testing of pairs indexes i and j, where $1 \le i, j \le p$. The convex polytopes Φ_1 an d Φ_2 are crossed if there is such pair indexes at what inequality (4) is carried out. In opposite cases the Φ_1 and Φ_2 are not crossed.

Conclusions

In this presents an new method of collision detection which use psevdoinverse matrix where convex polyhera defined in E^n. The method is based on conditions of existence of the solving of system of linear inequalities. Using this method multi-level algorithm is created. This paper focused on search of more effective method of definition of collisions where convex polytopes is in immediate proximity.

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