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A TUNING ALGORITHM OF DIGITAL CONTROLLERS TO THE OBJECTS' MODELS WITH INERTIA AND TIME DELAY

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Abstract. *A tuning algorithm of digital controllers to the objects' models with inertia and time delay using the maximal stability degree method of the continuous control system is proposed in this paper. The object's model is a model with inertia (second or third order) and time delay. The controller represents a digital algorithm – PI, PID quasicontinuous and it is tuning in two stages. On first stage the PI, PID controllers are tuning in conformity with the maximal stability degree method of continuous control system. On second stage the discretized PI, PID controllers are tuning as the quasicontinuous controllers choosing initial the discretization period in conformity with the constant of inertia and time delay of object's model.*
Keywords: *the object's model with inertia and time delay, controller, the tuning of controllers, the maximal stability degree method.*

Introduction

At the automation of many slow technological processes the mathematical objects' models of control process are represented as the models with respectively order inertia and time delay [1, 2].
In the paper is admitting that the object's model is a model with second order inertia and time delay:

$$H(s)=\frac{ke^{-\tau s}}{(T_1 s+1)(T_2 s+1)}=\frac{ke^{-\tau s}}{a_0 s^2+a_1 s+a_2}, \tag{1}$$

where the parameters of object $k, T_1, T_2, \tau, a_0, a_1, a_2$ are known. For the object's model (1) is proposed to tune the discretized PI, PID quasicontinuous controllers (digital controllers).
For design of numerical control algorithm are used many methods [1, 3]. Digital controllers frequently implement typical control laws PI, PID used in analog controllers. At the tuning of these types of controllers is used the calculating of parameters of digital controller in dependence of parameters of continuous controller choosing initial the discretization time.

The tuning algorithm of digital controllers

Assume that the control system is formed of object with transfer function H(s), which has forms (1), and the digital controller $H_R(z)$ with typical control laws PI, PID respectively (Figure 1).

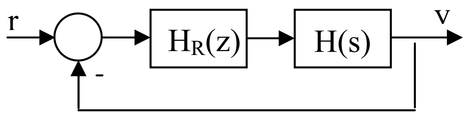


Figure 1. The structure scheme of control system.

The tuning procedure of continuous controllers PI, PID to the object's model (1) in conformity with the maximal stability degree method is presented in [4]. In order to illustrate the discretization method of the continuous control laws, the case of a PI, PID controllers with the following transfer functions is used:

$$H_{PI}(s)=k_R(1+\frac{1}{T_i s}); \tag{2}$$

$$H_{PID}(s) = k_R(1 + \frac{1}{T_i s} + \frac{T_d s}{T_f s + 1}); \qquad (3)$$

where k_R, T_i, T_d, T_f are the tuning parameters of controllers P, I, D respectively.

For tuning of digital PI, PID controllers to the object's model (1) the following procedure is proposed:

- The continuous PI, PID controllers are tuning to the object's model (1) in conformity with the maximal stability degree method:
  - for tuning of continuous PI controller to the object's model (1) the following procedure is using:
    a) the algebraic expression is solving and the numerical value of the optimal stability degree (J_{min}) is determining:

$$-c_0 J^3 + c_1 J^2 - c_2 J + c_3 = 0; \qquad (4)$$

where

$$c_0 = \tau^2 a_0; c_1 = a_1 \tau^2 + 6\tau a_0;$$
$$c_2 = a_2 \tau^2 + 4\tau a_1 + 6a_0; c_3 = 2\tau a_2 + 2a_1;$$

b) the optimal numerical values of tuning parameters of PI controller are determined from expressions:

$$k_R = \frac{\exp(-\tau J)}{k}(a_0 \tau J^3 - (\tau a_1 + 3a_0)J^2 + (\tau a_2 + 2a_1)J - a_2); \qquad (5)$$

$$k_i = \frac{1}{T_i} = \frac{\exp(-\tau J)}{k}(a_0 J^3 - a_1 J^2 + a_2 J) - k_R J; \qquad (6)$$

  - for tuning of continuous PID controller to the object's model (1) the following procedure is using:
    a) the algebraic expression is solving and the numerical value of the optimal stability degree (J_{min}) is determining:

$$c_0 J^3 - c_1 J^2 + c_2 J - c_3 = 0; \qquad (7)$$

where

$$c_0 = \tau^3 a_0; c_1 = a_1 \tau^3 + 9\tau^2 a_0;$$
$$c_2 = a_2 \tau^3 + 6\tau^2 a_1 + 18\tau a_0;$$
$$c_3 = 3a_2 \tau^2 + 6\tau a_1 + 6a_0;$$

b) the optimal numerical values of tuning parameters of PID controller are determined from expressions:

$$k_R = \frac{\exp(-\tau J)}{k}(a_0 \tau J^3 - (\tau a_1 + 3a_0)J^2 + (\tau a_2 + 2a_1)J - a_2) + 2k_d J; \qquad (8)$$

$$k_i = \frac{1}{T_i} = \frac{\exp(-\tau J)}{k}(a_0 J^3 - a_1 J^2 + a_2 J) - k_d J^2 + k_R J; \qquad (9)$$

$$k_d = \frac{\exp(-\tau J)}{2k}(a_0 \tau^2 J^3 - (\tau^2 a_1 + 6\tau a_0)J^2 + (\tau^2 + 4\tau a_1 + 6a_0)J - 2\tau - 2a_1) \qquad (10)$$

- For the object's model (1) with known parameters the discretization period is determined in conformity with the expression [3]:

$$T \approx 0{,}1(\tau + \min(T_1, T_2)); \qquad (11)$$

- The continuous controllers (2), (3) are discretized in conformity of the Tustin method and the transfer function in transform z [3]:
  - For the PI controller:

$$H_{PI}(z^{-1}) = \frac{q_0 + q_1 z^{-1}}{1 + p_1 z^{-1}}; \qquad (12)$$

where $q_0 = k_R(\frac{T}{2T_i} + 1)$,

$q_1 = k_R(\frac{T}{2T_i} - 1)$; $p = 1$.

- For the PID controller:

$$H_{PID}(z^{-1}) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1} + p_2 z^{-2}}; \qquad (13)$$

where $q_0 = k_R \dfrac{4T_i(T_d + T_f) + 2T(T_i + T_f) + T^2}{2T_i(2T_f + T)}$,

$$q_1 = -k_R \frac{4T_i(T_d + T_f) - T^2}{2T_i(2T_f + T)};$$

$$q_2 = k_R \frac{4T_i(T_d + T_f) - 2T(T_i + T_f) + T^2}{2T_i(2T_f + T)}$$

;

$$p_1 = \frac{4T_f}{2T_f + T}; \; p_2 = -\frac{2T_f - T}{2T_f + T}.$$

Application and computer simulation

To show the efficiency to the proposed algorithms for tuning of PI, PID digital controllers we'll study an example with the model of object (1) which has the following parameters:

$$k = 0.5; \; \tau = 1; \; T_1 = 2; \; T_2 = 5;$$

$$a_0 = 10; \; a_1 = 7; \; a_2 = 1.$$

- Is required to tune the following continuous controllers PI, PID:

Doing the respectively calculations in conformity with the elaborated algorithm for the given object are obtained the following results:

- for the control system with PI controller:

$$J = 0{,}2127; \; k_R = 0{,}9906;$$

$$k_i = 0{,}198; \; (T_i = 5{,}05s).$$

- for the control system with PID controller:

$$J = 0{,}635; \; k_R = 6{,}745;$$

$$k_i = 1{,}245; \; (T_i = 0{,}8s);$$

$$k_d = 8{,}511s.$$

- In conformity with the expression (11) the discretization time period is T=0,3s.
- The continuous controllers are discretized in conformity of the Tustin method. .
  - For the PI controller the tuning parameters are determined from (12):

$$q_0 = 1{,}049; \; q_1 = 0{,}9906; \; p_1 = 1.$$

  - For the PID controller the tuning parameters are determined from (13):

$$q_0 = 16{,}64; \; q_1 = 31{,}11;$$

$$q_2 = 14{,}48; \; p_1 = 1{,}703; p_2 = 0{,}703..$$

The computer simulation have been made in MATLAB and the simulation diagram of control systems for continuous and digital controllers are presented in figure 2.

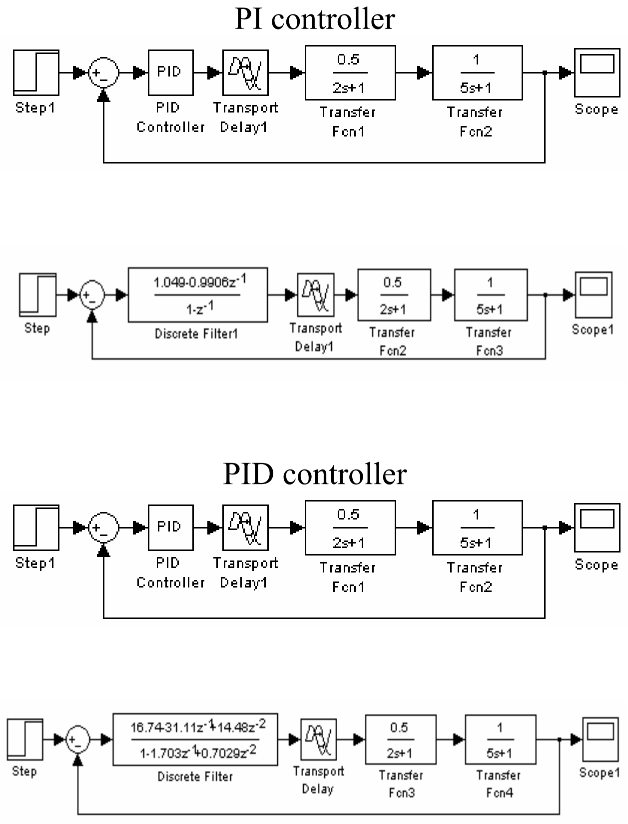
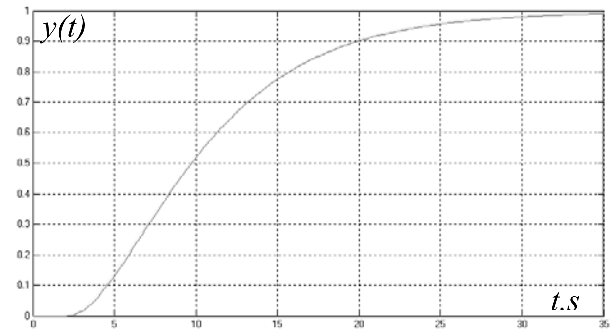


Figure 2. Simulation diagrams of the control systems.

On the computer was simulated the control system with the given objects' models forms (1), and respectively continuous and digital PI, PID controllers. The results of computer simulations are represented in the figure 3 and 4. Curves 1 for the control system with continuous controller, curves 2 for the control system with digital controller for the discretization period T=0,3s, curves 3 for the control system with digital controller for the discretization period T=0,2s and curves 4 for the control system with digital controller for the discretization period T=0,7s.

PI controller



PID controller

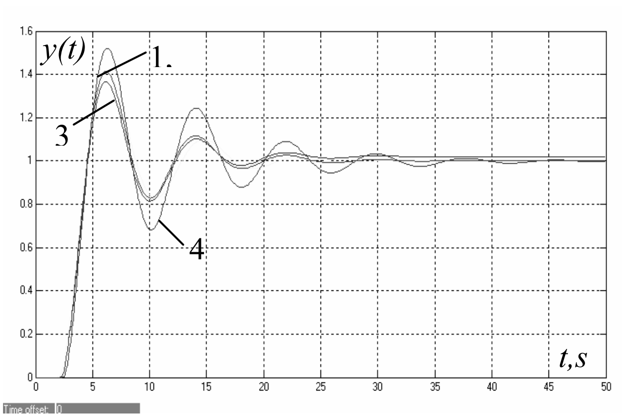


Figure 3. The transient responses of control systems.

For the control system with PI controller the transient responses 1, 2, 3 and 4 for the different discretization period coincide.

Conclusions

As a result of the study, which was made for given class of objects' models, the following conclusion can be made:

1. The proposed tuning algorithm for linear regulators PI, PID to given object's model (1) represents an algebraic method.

2. The proposed tuning algorithm represents a simple procedure which consists of following stages:

- the value of optimum stability degree of the designed continuous system with respectively type of controllers is determined and the tuning parameters of respectively regulators are determined from algebraic expressions (4)…(10);
- the discretization time period is determining in conformity with the constant of inertia and time delay of object's model (11).
- the tuning parameters of respectively digital controller are determining from algebraic expressions (12), (13).

References

[1] Dumitrache, I. şi al. (1993) *Automatizări electronice*, Bucureşti: Ed. Didactică şi Pedagigică.

[2] Tertişco, M. şi al. (1991) *Automatizări industriale continue*, Bucureşti: Ed. Didactică şi Pedagigică.

[3] Preitl, Ş., Precup, R.-E. (2001) *Introducere în ingineria reglării automate,* Timişoara: Editura Politehnica.

[4] Izvoreanu, B. (1998) *The Synthesis of Regulators to the with Advance Delay. Proceedings of the 6th International Symposium on Automatic Control and Computer Science (SACCA'98),* Iaşi, V.1.

[5] Izvoreanu, B., Fiodorov, I. (1997) *The Synthesis of Linear Regulators for Aperiodic Objects with Time Delay According to the Maximal Stability Degree Method. Preprints the Fourth IFAC Conference on System Structure and Control*, Vol. 1, Bucureşti: Editura Tehnică.