THE PRACTICAL STABILITY OF THE LINEAR SYSTEMS WITH THE PHASE SPACE VARIABLE MEASURABILITY

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Abstract. *For the linear transitional systems with the variable measurability of the phase space it was formulated and solved the problem about the practical stability. It was proved the theorem about the criteria of the practical stability, on the base of which it was developed the algorithm of the digital method of the search of the quality criteria of the practical stability*
Keywords: *practical stability, phase space, variable measurability, criteria of the practical stability, vector of the optimization parameters, quality criteria, Liapunova's function, digital algorithm*

Introduction

In given paper the practical stability (1) of the linear systems with the variable measurability of phase space (2) is investigating. Here the theorems about the practical stability are proved and it was indicate the criteria, which are suitable for direct use as algorithms of the digital stability area determination. The obtained results are proved on the base of A.M. Liapunova's general theory of stability and N.G. Chetaev's investigation problems of the stability on the finishing time interval. The considered problems of the practical stability of the dynamic systems with the variable measurability of phase space with permanently acting agitations.

The mathematical model of practical stability systems with the variable measurability

Let we suppose τ_1, τ_2, …, τ_N – is some part of the segment $[T_0, T_1]$, $\tau_j = \{t : t \in [t_{j-1}, t_j]\}$, $j = 1,2,...,N-1$, $\tau_N = \{t : t \in [t_{N-1}, t_N]\}$, $t_0 = T_0 < t_1 < ... < t_{N-1} < \ < t_N = T_1$. Let we suppose that on this segment the system dynamics is set as follows:

$$\frac{dx^{(j)}(t)}{dt} = A_j(t)x^{(j)}(t), t \in \tau_j, \qquad (1)$$

on condition of variable measurability of the phase space

$$x^{(j)}(t_{j-1}) = C_j x^{(j-1)}(t_{j-1} -), \qquad (2)$$

where $A_j(t)$ – are the square matrix of order n_j with such elements that the decision of the system (1) exists and is the single with $t \in \tau_j$, $x^{(j)}(t)$ – is the measuring vector of the phase space, C_j – are the rectangular constant matrixes of dimensionality $n_j \times n_{j-1}$, $j = \overline{1,N}$, with $j = 1$ we'll consider that $C_1 = E_1$ – is the single matrix of order n_1 , $x^{(0)}(t_0 -) = x_0^{(1)}$ – is the initial condition of the system (1).

Definition 1 The non-agitated movement $x^{(j)}(t) = 0$, $t \in \tau_j$, $j = \overline{1,N}$, of the system (1) on conditions (2) let we call $\{\lambda, \Gamma_t^{(1)}, \Gamma_t^{(2)},...,\Gamma_t^{(N)}, T_0, T_1\}$ -stable, if $x^{(j)}(t) \in \Gamma_t^{(j)}$ for all $t \in \tau_j$, $j = \overline{1,N}$, when $x^{(1)}(t_0) \in G_0 = \{x^{(1)} : x^{(1)T} x^{(1)} < \lambda^2\}$, where $\Gamma_t^{(j)} = \{x^{(j)} : |l_{s_j}^{(j)T} x^{(j)}| \leq 1, s_j = 1,2,...,M_j\}$, where $l_{s_j}^{(j)}(t)$ – uninterrupted vectors-functions of the dimensionality n_j , $t \in \tau_j$, $j = \overline{1,N}$.

Definition 2 The non-agitated movement $x^{(j)}(t) = 0$, $t \in \tau_j$, $j = \overline{1,N}$, of the system (1) on conditions (2) let we call $\left\{ c, B, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$ -stable, if $x^{(j)}(t) \in \Gamma_t^{(j)}$ for all $t \in \tau_j$, $j = \overline{1,N}$, when $x^{(1)}(t_0) \in G_{10} = \left\{ x^{(1)} : x^{(1)T} B x^{(1)} < c^2 \right\}$, where c – is the constant, B – is a known additionally set matrix,

$\Gamma_t^{(j)} = \left\{ x^{(j)} : \left| l_{s_j}^{(j)T} x^{(j)} \right| \leq 1, s_j = 1,2,...,M_j \right\}$, $l_{s_j}^{(j)}(t)$
– are the set uninterrupted vectors-functions of the dimensionality n_j, $t \in \tau_j$, $j = \overline{1,N}$.

Problem 1 To find the conditions when the decision of the system (1) on conditions of the change of phase space measurability (2), is $\left\{ \lambda, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$-stable.

Problem 2 To find the conditions when the decision of the system (1) on conditions of the change of phase space measurability (2), is $\left\{ c, B, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$-stable.

The criteria of the practical stability

Let we enter the signification: $X_j(t,\tau)$ as the matrix solution of Koshi's task

$$\frac{dX^{(j)}(t,\tau)}{dt} = A_j(t) X^{(j)}(t,\tau) \qquad (3)$$

$X_j(\tau,\tau) = E_j$, $t < \tau_j$, $\tau \in \tau_j$, $t \in \tau_j$, E_j – are the single matrixes of the order n_j;
$\bar{l}_{s_j}^{(j)}(t) = C_2^T X_2^T(t_2-,t_1)...C_j^T X_j^T(t_j-,t_{j-1}) l_{s_j}^{(j)}(t)$;
$\mu_1^2 = \rho_{1\max} \lambda^2$, $\mu_2^2 = \rho_{2\max} \lambda^2$, ...,
$\mu_N^2 = \rho_{N\max} \lambda^2$, where $\rho_{j\max}$ – the most personal value of matrix $\psi_j^T \psi_j$,
$\psi_j = C_j X_{j-1}(t_{j-1}-,t_{j-2}) C_{j-1}...X_1(t_1-,t_0) C_1$;
$\bar{\mu}_1^2 = \bar{\rho}_{1\max} \lambda^2$, $\bar{\mu}_2^2 = \bar{\rho}_{2\max} \lambda^2$, ...,
$\bar{\mu}_N^2 = \bar{\rho}_{N\max} \lambda^2$, where $\bar{\rho}_{j\max}$ – the most personal value of matrix $\psi_{1j}^T \psi_{1j}$, $\psi_{1j} = \psi_j B^{-\frac{1}{2}}$,

$Q_j(t_{j-1},t) = X_j^T(t_{j-1},t) X_j(t_{j-1},t)$, $t \in \tau_j$, $j = \overline{1,N}$, $\overline{Q}_1(t_0,t) = X_1^T(t_0,t) B X_1(t_0,t)$, $t \in \tau_j$.

Theorem 1 For $\left\{ \lambda, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$ - stability of the system (1) on conditions (2) is enough to satisfy the following conditions:

$$\lambda^2 \leq \min(\lambda_1, \lambda_2, ..., \lambda_N), \qquad (4)$$

$$x^{(j-1)}(t_{j-1}-)(E_{j-1} - C_j^T C_j) x^{(j-1)}(t_{j-1}-) \geq 0, \quad (5)$$
where

$$\lambda_j = \min_{1 \leq s_j \leq M_j} \left(\inf_{t \in \tau_j} \left(\bar{l}_{s_j}^{(j)T}(t) Q_1^{-1}(t_0, t_1-) \bar{l}_{s_j}^{(j)}(t) \right)^{-1} \right)$$
$$(6)$$

$x^{(j-1)}(t)$, $t \in \tau_j$, $j = \overline{1,N}$, is the decision (1) with the conditions (2), that satisfies an initial condition $x^{(1)}(t_0) = x_0^{(1)} \in G_0$.

Proof Let we made the proof by means of Liapunov's functions. Let we choose the following Liapunov's functions $V_j(x^{(j)}(t),t) =$
$$= \frac{1}{\mu^2} x^{(j)T} Q_j(t_{j-1},t) x^{(j)}, \quad t \in \tau_j, \quad j = \overline{1,N},$$

where $\mu^2 = \max(\mu_1^2, \mu_2^2, ..., \mu_N^2)$.

Let suppose $x^{(j)}(t)$, $t \in \tau_j$, $j = \overline{1,N}$, is the decision of the system (1) with the conditions (2), which satisfies the initial condition $x^{(1)}(t_0) = x_0^{(1)}$, $x_0^{(1)} \in G_0$. If the conditions (4) and (5) fulfill it mean that for the functions $V_j(x^{(j)}(t),t)$ on the decisions of the system (1) the following non-equations are realized $V_j(x^{(j)}(t),t) < 1$, $t \in \tau_j$, $j = \overline{1,N}$.

Besides, it is easy to check that $\left\{ x^{(j)}(t) : V_j(x^{(j)}(t),t) < 1 \right\} \subset \Gamma_t^{(j)}$. Really,

$$\left| l_{s_j}^{(j)T}(t) x^{(j)}(t) \right|^2 \leq$$
$$\leq \left(\bar{l}_{s_j}^{(j)T}(t) Q_1^{-1}(t_0,t_1-) \bar{l}_{s_j}^{(j)}(t) \right) x_0^{(1)T} x_0^{(1)} \leq$$
$$\leq \left(\bar{l}_{s_j}^{(j)T}(t) Q_1^{-1}(t_0,t_1-) \bar{l}_{s_j}^{(j)}(t) \right) \lambda^2 .$$

From this, taking into account the condition (4) we receive with a inequality $\left|l_{s_j}^{(j)T}(t)x^{(j)}(t)\right|^2 \leq 1$, $t \in \tau_j$, $s_j = 1,2,...,M_j$, $j = \overline{1,N}$. So far as

$$\frac{dV_j\left(x^{(j)}(t),t\right)}{dt} = 0$$

with $x^{(j)}(t) \in \left\{x^{(j)}(t) : V_j\left(x^{(j)}(t),t\right) < 1\right\}$, $t \in \tau_j$, $j = \overline{1,N}$, all the conditions of the theorem 1 (3) realize that finish the theorem proof.

Remark 1 The matrix $Q_1^{-1}(t_0,t_1-)$ can be found using the formula $Q_1^{-1}(t_0,t_1-) = X_1(t_1-,t_0)X_1^T(t_1-,t_0)$.

Remark 2. If in the system of differential equations (1) the matrix A_j is the stable, then in the condition of stability (4) λ_j can be discovered using the formula

$$\lambda_j = \min_{1 \leq s_j \leq M_j} \inf_{t \in \tau_j}\left(\overline{l}_{s_j}^{(j)T}(t)e^{\left(A_j+A_j^T\right)(t_1-t_0)}\overline{l}_{s_j}^{(j)}(t)\right)^{-1}. \quad (7)$$

Theorem 2 For $\left\{c, B, \Gamma_t^{(1)},..., \Gamma_t^{(N)}, T_0, T_1\right\}$ stability of the system (1) on conditions (2) is sufficient that realizes the condition

$$c^2 \leq \min\left(\overline{\lambda}_1, \overline{\lambda}_2,..., \overline{\lambda}_N\right), \quad (8)$$

and the condition (4), where

$$\overline{\lambda}_j = \min_{1 \leq s_j \leq M_j}\left(\inf_{t \in \tau_j}\left(\overline{l}_{s_j}^{(j)T}(t)\overline{Q}_1^{-1}(t_0,t_1-)\overline{l}_{s_j}^{(j)}(t)\right)^{-1}\right), \quad (9)$$

$x^{(j-1)}(t)$, $t \in \tau_j$, $j = \overline{1,N}$, is the decision (1) on conditions (2) which satisfies the initial condition $x^{(1)}(t_0) = x_0^{(1)} \in G_{10}$.

Proof Let we chose the Liapunova's functions as follows:

$$V_1\left(x^{(1)}(t),t\right) = \frac{1}{\overline{\mu}^2}x^{(1)T}(t)\overline{Q}_1(t_0,t)x^{(1)}(t),$$

$$V_j\left(x^{(j)}(t),t\right) = \frac{1}{\overline{\mu}^2}x^{(j)T}(t)\overline{Q}_j(t_{j-1},t)x^{(j)}(t),$$

$$j = \overline{2,N},$$

where $\overline{\mu}^2 = \max\left(\overline{\mu}_1^2, \overline{\mu}_2^2,..., \overline{\mu}_N^2\right)$.

Now is possible to make the proof of the theorem 2 follow the scheme of theorem 1. The theorem is proved.

Remark 3 The additionally denoted symmetrical matrix $\overline{Q}_1(t_0,t)$ with the dimensionality $n_1 \times n_1$ can be found as matrix decision of the differential equation

$$\frac{d\overline{Q}_1(t_0,t)}{dt} = -A_1^T(t)\overline{Q}_1(t_0,t) - \overline{Q}_1(t_0,t)A_1(t),$$

$$\overline{Q}_1(t_0,t_0) = B, \, t \in \tau_1.$$

Remark 4 For the calculation $\overline{\lambda}_j$ using the formula (9) the reverse matrix $\overline{Q}_1^{-1}(t_0,t)$ can be found solving the Koshi's task

$$\frac{d\overline{Q}_1^{-1}(t_0,t)}{dt} = \overline{Q}_1^{-1}(t_0,t)A_1^T(t) + A_1(t)\overline{Q}_1^{-1}(t,t_0),$$

$$\overline{Q}_1^{-1}(t_0,t) = B^{-1}.$$

Then let we examine on the segment $[T_0,T_1]$ with the division τ_1, τ_2, ..., τ_N the system with the permanently acting agitation

$$\frac{dx^{(j)}(t)}{dt} = A_j(t)x^{(j)}(t) + f^{(j)}(t), \, t \in \tau_j, \, j = \overline{1,N}, \quad (10)$$

provided the variable measurability of phase space (2), where $A_j(t)$ – are the square matrixes, which satisfy above mentioned conditions, $f^{(j)}(t)$ – some n_j-dimension function which satisfies the conditions of the theorem about the

existence and unity of the system (10) decision when $t \in \tau_j$. Let we suppose that the functions $f^{(j)}(t)$, $t \in \tau_j$, either are known vector functions of the time or they are unknown but satisfy the condition

$$\left\| f^{(j)}(t) \right\| = \left(\int_{t_{j-1}}^{t_j} \sum_{i=1}^{n_j} \left(\left| f^{(j)}(\tau) \right|^{q_j} \right)^{\frac{q_{1j}}{q_j}} d\tau \right)^{\frac{1}{q_{1j}}} \leq \overline{R}^{(j)}$$

(11)

$1 \leq q_j \leq \infty$, $1 \leq q_{1j} \leq \infty$, $\overline{R}^{(j)}$ – some added stables, $j = \overline{1, N}$.

Definition 3 Let we call the system (10) on conditions (2) as internally $\left\{ c, B, \Gamma_t^{(1)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$-stable at permanently acting agitations which can be known or unknown and satisfy the condition (11), if $x^{(j)}(t) \in \Gamma_t^{(j)}$ for all $t \in \tau_j$, $j = \overline{1, N}$, as soon as $x^{(1)}(t_0) \in \left\{ x^{(1)} : x^{(1)T} B x^{(1)} < c^2 \right\}$.

Let we consider for the system (10) on conditions (2) also the external $\left\{ c, B, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$ -stability as the generalization of the given task about the practical stability [4] for the systems without the variable measurability of the phase space. Let we suppose that the area of the initial conditions and the multitude $\Gamma_{t_0}^{(1)}$ satisfy the condition

$$\left\{ x^{(1)} : x^{(1)T} B x^{(1)} < c^2 \right\} \cap \overline{\Gamma}_{t_0}^{(1)} \neq \emptyset, \quad (12)$$

where $\overline{\Gamma}_{t_0}^{(1)}$ – is the addition to the multitude $\Gamma_{t_0}^{(1)}$, $\emptyset$ - is the empty multitude.

Definition 4 Let we call the system (10) on conditions (2) as externally $\left\{ c, B, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$ -stable at permanently acting agitations which can be known or unknown and satisfy the condition (11), if for some initial conditions $x^{(1)}(t_0) = x_0^{(1)}$ from the area $x^{(1)}(t) \in G_{10} = \left\{ x^{(1)}(t) : x^{(1)T}(t) B x^{(1)}(t) < c^2 \right\}$ be

found such index $j_0 \in \{1, 2, ..., N\}$ and the signification $\overline{t} \in \tau_{j_0}$, that $x^{(j_0)}(\overline{t}) \in \Gamma_t^{(j_0)}$.

The notion of the external and internal $\left\{ \lambda, \Gamma_t^{(1)}, \Gamma_t^{(2)}, ..., \Gamma_t^{(N)}, T_0, T_1 \right\}$-stability can be enter [4] analogically.

Let $f^{(j)}(t)$ are the stable functions of the parameter t, $t \in \tau_j$, $j = \overline{1, N}$. The solution (10) on conditions (2), which satisfies the initial condition $x^{(1)}(t_0) = x_0^{(1)}$, has the following look:

$$x^{(1)}(t) = W_j(t, t_0) x_0^{(1)} + \sum_{k=1}^{j-1} \int_{t_{k-1}}^{t_k} W_{jk}(t, \tau) f^{(k)}(\tau) d\tau +$$

$$+ \int_{t_{j-1}}^{t_j} X_j(t, \tau) f^{(j)}(\tau) d\tau, \quad (13)$$

where $W_j(t, t_0) = X_j(t, t_{j-1}) C_j ... X_1(t_1, t_0) C_1$,
$W_{jk}(t, \tau) = X_j(t, t_{j-1}) C_j ... C_{k+1} X_k(t_k, \tau)$.

Let we write down the condition that the trajectories of the system (10) on conditions (2) belong to the set $\Gamma_t^{(j)}$, $t \in \tau_j$, as follows:

$$z^{(j)}(t) \in \left\{ z^{(j)}(t) : -1 - l_{s_j}^{(j)T}(t) a^{(j)}(t) \leq l_{s_j}^{(j)T}(t) z^{(j)}(t) \leq \right.$$

$$\leq 1 - l_{s_j}^{(j)T}(t) a^{(j)}(t), s_j = 1, 2, ..., M_j, t \in \tau_j \bigg\}, \quad (14)$$

where $z^{(j)}(t) = W_j(t, t_0) x^{(1)}(t_0)$,

$$a^{(j)}(t) = \sum_{k=1}^{j-1} \int_{t_{k-1}}^{t_k} W_{jk}(t, \tau) f^{(k)}(\tau) d\tau +$$

$$+ \int_{t_{j-1}}^{t_j} X_j(t, \tau) f^{(j)}(\tau) d\tau, \quad t \in \tau_j, \quad j = \overline{1, N}.$$

Let we mark:

$$\overline{\overline{\mu}}_1 = \inf_{t \in \tau_j} \min_{s_1 = 1, ..., M_1} \frac{\left(1 - \left| l_{s_1}^{(1)T}(t) a^{(1)}(t) \right| \right)^2}{l_{s_1}^{(1)T}(t) \overline{Q}_1^{-1}(t_0, t) l_{s_1}^{(1)}(t)}, \quad (15)$$

$$\overline{\overline{\mu}}_j = \inf_{t \in \tau_j} \min_{s_1 = 1, ..., M_j} \frac{\left(1 - \left| l_{s_j}^{(j)T}(t) a^{(j)}(t) \right| \right)^2}{l_{s_j}^{(j)T}(t) \overline{Q}_1^{-1}(t_0, t_1 -) l_{s_j}^{(j)}(t)}, \quad (16)$$

$$V_1\left(z^{(1)},t\right)=\frac{1}{c^2}z^{(1)T}X_1^T(t_0,t)z^{(1)}, \ t\in\tau_1,$$

$$V_j\left(z^{(j)},t\right)=\frac{1}{c^2}z^{(j)T}X_j^T(t_{j-1},t)z^{(j)}, \ t\in\tau_j,$$

$$j=\overline{2,N}.$$

Theorem 3 In order to the system (10) on conditions (2) is internally $\left\{c,B,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$ -stable at the known permanently acting agitations it is enough that the following conditions will be realized:

$$c^2\leq\min\left\{\overline{\overline{\mu}}_1,\overline{\overline{\mu}}_2,...,\overline{\overline{\mu}}_N\right\},$$

$$\left|l_{s_j}^{(j)T}(t)a^{(j)}(t)\right|<1, \ t\in\tau_j, s_j=1,2,...,M_j,$$

$$j=\overline{1,N},$$

are the proper meanings of the matrixes
$$X_j^T\left(t_j-,t_{j-1}\right)C_{j+1}^TC_{j+1}X_j\left(t_j-,t_{j-1}\right),$$
$j=1,...N-1$, smaller that one.

Proof Evidently that the conditions (12) are achieved if the following proportions realize:

$$\max_{z^{(j)}\in\left\{z^{(j)}:z^{(j)T}Q_j(t_{j-1},t)z^{(j)}<c^2\right\}}l_{s_j}^{(j)T}(t)z^{(j)}\leq 1-$$
$$-l_{s_j}^{(j)T}(t)a^{(j)}(t), \quad\quad (17)$$

$$\min_{z^{(j)}\in\left\{z^{(j)}:z^{(j)T}Q_j(t_{j-1},t)z^{(j)}<c^2\right\}}l_{s_j}^{(j)T}(t)z^{(j)}\geq -1-$$
$$-l_{s_j}^{(j)T}(t)a^{(j)}(t), \quad\quad (18)$$

$t\in\tau_j$, $s_j=1,2,...,M_j$, $j=\overline{1,N}$, which can be received easily from the theorem 3. The theorem is proved.

Theorem 4 In order to the system (10) on conditions (2) is internally $\left\{\lambda,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$-stable at the known permanently acting agitations it is enough that the following conditions will be realized:

$$\lambda^2\leq\min\left\{\overline{\overline{\mu}}_1\rho_{1\max},\overline{\overline{\mu}}_2\rho_{2\max},...,\overline{\overline{\mu}}_N\rho_{N\max}\right\},$$

$$\left|l_{s_j}^{(j)T}(t)a^{(j)}(t)\right|<1, \ t\in\tau_j, s_j=1,2,...,M_j,$$

$$j=\overline{1,N},$$

$$Q_1^{-1}(t_0,t)=X_1(t,t_0)X_1^T(t,t_0) \quad ,$$
$$Q_1^{-1}(t_0,t_1-)=X_1(t_1-,t_0)X_1^T(t_1-,t_0), \text{ the proper}$$
meanings of the matrixes
$$X_j^T\left(t_j-,t_{j-1}\right)C_{j+1}^TC_{j+1}X_j\left(t_j-,t_{j-1}\right) \quad ,$$
$j=1,...,N-1$, smaller that one.

Taking into account the results and the methods of the proof of the theorem (3) and (4) it is possible to receive also the conditions of the external $\left\{\lambda,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$ -stability and external $\left\{c,B,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$ -stability.

If the external permanently acting agitations $f^{(j)}(t)$, $j=\overline{1,N}$, are unknown but belong to some area of kind (9), using the constructive proofs above mentioned theorems it is possible to build the valuations of the area $\left\{\lambda,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$ -stability and $\left\{c,B,\Gamma_t^{(1)},\Gamma_t^{(2)},...,\Gamma_t^{(N)},T_0,T_1\right\}$ -stability, which allows to develop the effective digital algorithms and the procedures of investigation of the practical stability of the system with the variable measurability of the phase space.

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