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OPTIMAL INPUT DESIGN FOR ADAPTIVE CONTROL ALGORITHM AND ITS USE IN STOCHASTIC SYSTEMS IDENTIFICATION

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Abstract: *There have been a lot of investigations on optimal input design for dynamic system identification to obtain maximal information on the control system from the time varying observed input / output data. The restricted total least Squares (RTLS) problem is devised for solving over determined sets of linear equations AX≈ B perturbed by error of the forum E=DED. By strengthening the hypotheses on the disturbance and parameter processes, a Marcovian state process may be constructed, and in this case it is shown that loss function on the joint input-output parameter estimate process converge to their expectation with respect to an invariant probability at a geometric rate. The identification procedure for fuzzy unknown parameters is proposed by using the concept of the moment method, when system is described by one of fuzzy moving average models.*
Keywords: *System identification, input design, adaptive control algorithm, optimization, and auto-covariance sequence.*

Introduction

The main objective in adaptive control theory is to design systems; the primary issue being to mainstreaming closed-loop stability. It consider the stochastic time varying system

$$y(k+1)=a_1(k)y(k)+a_2(k)y(k-1)+\dots$$
$$+a_p(k)y(k-p+1)+u(k)+v(k+1). \qquad (1)$$

Where $y(k)$, $u(k)$ and $v(k)$ are the (scalar) output, input and disturbance processes respectively, and the parameters $a_i(k)$, $I=\overline{1,P}$, $k\geq 0$ are partially observed through the input / output process (u, y). Our goal is to choose a control low which stabilizes this system in a mean square sense. That the system (1) may be written in the regression form

$y(k+1) = \theta^T(k)\varphi(k)+u(k)+v(k+1)$, where

$$\varphi^T(k+1)=[y(k)y(k-1),\dots y(k-p+1)],$$

$$\theta^T(k) = [a_1(k)\ a_2(k)\dots a_p(k)],$$

$$\theta(k+1)=\alpha\ \theta_k+\ \xi(k+1). \qquad (2)$$

A stochastic process z on $\{\Omega, \mathcal{F}, P\}$ taken values in Euclidian space will be said to satisfy condition $B(a, b, c)$, where a, b, c are positive constants, if for each $0\leq m\leq n$

$$log\ M[\prod_{j=m}^{n}\exp\{a|z_j|^2\}]\leq b(n-m+1)+c+0(l). \qquad (3)$$

The term $0(1)$ is assumed to converge to zero as $m\to\infty$, uniformly in n; a, b, c are irrelevant the stochastic process z will just be called *B*-bounded.

The estimates $\hat{\theta}$ of the partially observed parameters the process θ are generated by the gradient algorithm with δ - modification:

$$\hat{\theta}(k+1)=(1-\delta)\{\hat{\theta}(k)+\frac{\varphi(k)}{d|\varphi(k)|^2}(y(k+1)-$$
$$-u(k)-\varphi^T(k)\hat{\theta}(k))\}. \qquad (4)$$

With arbitrary initial condition $\hat{\theta}(0)\in R^P$ is given.

Given the parameter estimates (4), it applies the certainty equivalent minimum variance adaptive control law

$$u_k=-\varphi^T(k)\hat{\theta}(k). \qquad (5)$$

Geometric Ergodicity of a Markovian state process

That it describes how certain stationary assumptions on the disturbance and parameter process imply stronger results.

A1. The parameter and disturbance sequences $\{\hat{\theta}(k),\ a\ (k),\ v\ (k)\}$ are *B* – bounded stochastic processes on the probability space *{Ω, Ƒ, P}*. The parameter difference process Δ defined as $\Delta(k+1)=\theta(k+1)-\theta(k), k\in Z_+$, satisfies condition $B(M_1,\varepsilon_1,M_2)$, where (M_1,ε_1,M_2) are positive constants.

A2. The joint process *w := (s, v)* is an independent and identically distributed process on R^{N+1},and independent of $(\phi(0),\hat{\theta}(0),\psi(0))$. The distribution μ_0 of $\omega(k), k\geq 0$, processes a lower semi continuous density which is positive at the origin.

A3. The parameter process $\theta(k), k\geq 0$ is the output of the stable minimal linear system

$\psi(k+1)=F\psi(k)+G\xi(k+1);$

$\theta(k)=H\psi(k)+\theta(0)$, where $\theta(0)\in R^P$ is constant. Under this condition it is easy to show that joint process ϕ defined for $k\in Z_+$ as

$$\phi_k^T:=(\varphi^T(k),\hat{\theta}^T(k),\psi^T(k)), \qquad (6)$$

is a Marcovian chain. For the state process ϕ and construct a certain Lyapunov function suppose that conditions *A1-A3* hold that the control law (5) is applied. Then the state process ϕ defined in (6) possesses a unique invariant probability π.

Models and criteria in frequency domain

It considers the optimal input design problem for efficient discrimination of linear stochastic models in frequency domain for long time global optimization. It derive an optimal input by solving a mathematical programming problem which maximized the time – average of the Kullback discrimination information *(KDI)* under the input power constraint. Consider a linear stochastic model with controllable input signal *{u(k)}*. Consider a linear stochastic model with controllable input signal $\{u_i\}$

$$A(Z^{-1})y_i=B(Z^{-1})u_i+\varepsilon_i. \qquad (7)$$

Where Z^{1} is a delay operator, $(Z^{-1})y_i$ means y_{i-1} and $A(Z^{-1})$, $B(Z^{-1})$ are given by

$$A(Z^{-1})=1-\sum_{k=1}^{p}a_kZ^{-k},\ B(Z^{-1})=\sum_{k=1}^{q}b_kZ^{-k}, \qquad (8)$$

ε_i-independently normally distributed with mean zero and variance σ^2. It assume the system (1) is stabile. That consider the problem to find an input u_i, which determine efficiently the order of the model (7) under the input power constraint $M[u_i^2]\leq c,$ (9)

where *c* is a given constant. The order determination problem can be restated as a model discrimination problem, that is, discriminating one model among the following two rival models :

$$M1: A_1(Z^{-1})y_i=B_1(Z^{-1})u_i+\varepsilon_i^{(1)}, \qquad (10)$$

$$M2: A_2(Z^{-1})y_i=B_2(Z^{-1})u_i+\varepsilon_i^{(2)}. \qquad (11)$$

Where $(p_1,q_1)\neq(p_2,q_2)$ and $\{\varepsilon_i^{(j)}\}$, are independently normally distributed random sequences with mean zero and variance $\sigma_j^2\ (j=1,2)$, respectively. The KDI for discrimination in favor of the model M_1 over the model M_2 may be defined by

$$I_t[1:2;y^t,u^{t-1}]=I_0[1:2;y_0]+I_t[1:2;y^t,u^{t-1}]^{(1)}+ \\ +I_t[1:2;y^t,u^{t-1}]^{(2)}+I_t[1:2;y^t,u^{t-1}]^{(3)}. \qquad (12)$$

Where

$$I_t[1:2;y^t,u^{t-1}]^{(1)}=\frac{t}{2}\left(\frac{\sigma_1^2-\sigma_2^2}{\sigma_2^2}-\lg\frac{\sigma_1^2}{\sigma_2^2}\right). \qquad (13)$$

$$I_t[1:2;y^t,u^{t-1}]^{(2)}=\frac{1}{2\sigma_2^2}\sum_{k=1}^{t}(A_2(Z^{-1})(\frac{B_1(Z^{-1})}{A_1(Z^{-1})}- \\ -\frac{B_2(Z^{-1})}{A_2(Z^{-1})})u_k)^2 \qquad (14)$$

$$I_t[1:2;y^t,u^{t-1}]^{(2)} = \frac{t\sigma_1^2}{2\sigma_2^2}\cdot\frac{1}{2\pi i}\oint(\frac{A_2(Z^{-1})}{A_1(Z^{-1})})-1)^2\frac{dz}{z}.$$

Assume that the input sequence *{u_i}* is a stationary ergodic Gaussian process with mean zero. Then, the output sequence *{y_i}* becomes a stationary ergodic process with mean zero. We can evaluate $I_t[1:2;y,u]^{(2)}$ for

$$I_t[1:2;y,u]^{(2)} = \frac{1}{2\sigma_2^2}\lim_{t\to\infty}\frac{1}{t}\sum_{k=1}^{t}(A_2(Z^{-1})(\frac{B_1(Z^{-1})}{A_1(Z^{-1})} - \frac{B_2(Z^{-1})}{A_2(Z^{-1})})u_k)^2 \quad (15)$$

Let

$$\mathrm{H}(z^{-1}) = \mathrm{A}_2(z^{-1})\mathrm{B}_1(z^{-1}) - \mathrm{A}_1(z^{-1})\mathrm{B}_2(z^{-1}) = \sum_{j=1}^{n}\mathrm{h_j}z^{-j}, \quad (16)$$

where $h_1 = b_1^{(1)} - b_1^{(2)}$ (17)

$$h_j = \sum_{i=1}^{\min(j-1,t)}(a_i^{(1)}b_{j-1}^{(2)} - a_i^{(2)}b_{j-i}^{(1)}) + b_j^{(1)} - b_j^{(2)},\quad j=\overline{1,n} \quad (18)$$

with $t = \max(P_1,P_2),\quad m = \max(P_1,P_2),$ $n = \max(P_1+Q_2,P_2+Q_1)$

It have $I[1:2;y,u] = \frac{1}{2\sigma_2^2}M\left[\left(\sum_{j=1}^{n}h_j\widetilde{U}_{ij}\right)^2\right]$ (19)

and this is completely determined by σ_2^2 and the following auto- covariance sequence of the filtered input $\widetilde{U}_i$; $\widetilde{P}_k = M[(\widetilde{U}_i,\widetilde{U}_{t-k})], k=\overline{0,n-1}$ that is,

$$I[1:2;y,u]^2 = \frac{1}{2\sigma_2^2}\left\{\left(\sum_{j=1}^{n}h_j^2\right)\overline{P_0} + \left(2\sum_{j=1}^{n}h_jh_{j+1}\right)\overline{P_1} + \dots \dots + 2h_1h_n\overline{P_{n-1}}\ \right\}. \quad (20)$$

The following conditions are also requested by the nature of auto- covariance function $\{\overline{P_k}\}$ $\overline{P_0}\succ 0, \overline{R_{n-1}}$ is nonnegative definitely, where

$$R_{n-1} = \begin{bmatrix} \overline{P_0} & \overline{P_1} & \dots & \overline{P_{N-1}} \\ \overline{P_1} & \overline{P_0} & \dots & \overline{P_{N-2}} \\ . & . & . & . \\ \overline{P_{N-1}} & \overline{P_{N-2}} & \dots & \overline{P_0} \end{bmatrix}.$$

The optimal filtered input $\overline{U_i^0}$, having this auto covariance sequence, can be realized in an autoregressive model with a form :

$$\overline{C}(z^{-1})\overline{U_i^0} = \overline{\xi_i} \quad (21)$$

where

$$\overline{C}(z^{-1}) = 1 - \sum_{k=1}^{n-1}\overline{C_k}Z^{-k} \quad (22)$$

With a suitable coefficients $\{C_j\}$ and ξ_i is an independently normally distributed with mean zero and suitable constant variance $\overline{\sigma^2}$. Another realization approach is the Chebyshev systems approach and it choose the optimal input as the following form :

$$\overline{U_i^0} = \sum_{j=1}^{\tau}m_j\cos(w_jt+\varphi_j) \quad (23)$$

where *r=n/2* or *(n+1)/2*, $\omega_p\in[0,\pi], \omega_i\neq\omega_j, m_j\succ 0,\quad \varphi_j$,s are non formality distributed on $(0,2\pi), i\neq j$. Once the optimal filtered input $\overline{U_i^0}$ is easily obtained by

$$U_1^0 = A_i(z^{-1})\overline{U_i^0}. \quad (24)$$

Parameter identification for fuzzy moving average models

Let the fuzzy stochastic systems (FSS) is described formally by

$$\begin{cases} Y(k) = f_k(k-1),\dots,y(k-m);\xi(k),\dots,\xi(k-m);\widetilde{\theta}; \\ Y_0 = y(-1) = \dots = y(-n+1) = 0(non-fuzzy) \end{cases} (25)$$

where $\widetilde{\theta} = (\widetilde{\theta}_1 \dots \widetilde{\theta}_R)^T$. (26)

The level set of Y(k) will be given by the following equation under certain conditions associated with $\widetilde{\theta}$ and f_k:

$L_\alpha Y(k) = f_k(L_\alpha y(k-1),\dots,L_\alpha y(k-n);\xi(k),\dots \dots,\xi(k-m),L_\alpha\widetilde{\theta})$ (27)

where $\alpha\in[0,1]$

$f_n(L_\alpha y(k-1),\dots,L_\alpha y(k-n);\ \xi(k),\dots,\ \xi(k-m);$

$L_\alpha\widetilde{\theta}$) = {y(k) $\in$ R | $y_i \in L_\alpha$(k-i) for i = $\overline{1,n}$, $\widetilde{\theta} \in$ $L_\alpha\widetilde{\theta}$ with y(k) = f_k(y(k-1),…,y(k-n); ξ(k),…, ξ(k-m); θ }. (28)

Hereafter, for simplicity of discussions, the FSS is restricted to the fuzzy moving average model of order m (FMA(m)) described by

Y(k) = ξ(k) + $\widetilde{\theta}_1$ ξ(k-1) + … + $\widetilde{\theta}_1$ ξ(k-m) (29)

$\widetilde{\theta}_i \in F(R)$ and inf{x$\in$ $supp\,\widetilde{\theta}_i$ }≥0 for i = $\overline{1,m}$

{ ξ(k)} is the sequence of bounded and positive valued i.i.d. random variables with mean m_ξ(>0) and the variance $\sigma_\xi^{\ 2}$; F(R) is the set of all fuzzy sets satisfying the following properties:

$L_\alpha U$ is compact and convex subset of R for each $\alpha\in$[0,1], L_1U is non-empty, U is compact and convex fuzzy set in the real line R.

Theorem 1. Let $\widetilde{\theta} \in$ F(R), then at any time step k, the output process {y(k)} of FMA(m) are also elements of F(R). In the case of FMA(m), it can fortunately be shown that the two α – level sets of y(k) given by [1,2].

LαY(k) = {y(k) | y(k) = f_k (y(k-1),…,y(k-n), ξ(k),…, ξ(k-m), $\widetilde{\theta}_0$ } (30)

$\theta_{\alpha,i} \in L_\alpha\widetilde{\theta}$, i = $\overline{1,l}$ coincide with each under the condition of $\widetilde{\theta} \in$ F(R). The fuzzy stochastic process (FSP) is defined by the sequence of fuzzy random variables (FRV), which are the outputs of the FSS. Then the set – valued function F is defined by F: Ω → P(R), P(R) denotes the collection of all subsets of R,

S(F): = {f $\in$ L^1 (Ω,P) | f(ω)$\in$ F, w.p.1} denote the set of all L^1(Ω,P) selection of F. The mean of the FSP is given by [1].

(M[Y(k)])(x)=sup{α$\in$[0,1]|x$\in$ $\int_U F$ =

$\{\int_\Omega fdP / f \in S(F)\}\int_U L_a Y(k)\}$ and the product moments of the FSP are given by $\left(M[Y(k)Y(k+1)]\right)(x) = \sup\{\alpha \in [0,1] \mid x \in \int_U F :=$

$\{\int_{S_2} fdP \mid f \in S(F) \int_U L_\alpha Y(k)Y(k-r)\},$

for k=1,2,3,…, r =$\overline{0,m}$.

Not that is can be shown that the mean M[Y(k)] and the product moment M[Y(k)Y(k-r)], r=$\overline{0,m}$ of the output process of FMA(m) are elements of F(R) and hence, their existence and uniqueness for each k=1,2,3,… guaranteed [4,5]

Fuzzy estimators for M[Y(k) Y(k-r)] are given by: $M_N = \frac{1}{N}\sum_{k=1}^{N} y(k)$ (31)

$$\Psi_N = \frac{1}{N}\sum_{k=1}^{N} y(k)y(k-r); r = \overline{0,m} \quad (32)$$

$$(M_N)(x) = \sup\{\alpha \in [0,1], x \in L_\alpha \hat{M}_N\} \quad (33)$$

$$(\psi_N)(r) = \sup\{\alpha \in [0,1], x \in \alpha\hat{\psi}_N(r)\} \quad (34)$$

$$L_\alpha \hat{M}_N = [\frac{1}{N}\sum_{k=1}^{N} y_a(k), \sum_{k=1}^{N} y_b(k)] \quad (35)$$

$$L_\alpha(\hat{\psi}_N(r)) = [\frac{1}{N}\sum_{k=1}^{N} y_a(k)y_a(k-r), \sum_{k=1}^{N} y_b(k)y_b(k-r)]. \quad (36)$$

Note that since $\{y(k), k = \overline{1,N}\}, M_N, \Psi_N(r)$ are elements of F(R) hence their sets (35) (36), the bracket [a , b] denotes the closed interval from α to r L_α and

$$y_a(k) = \inf\{x \in L_\alpha y(k)\}; y_b(k) = \\ = \sup\{x \in L_\alpha y(k)\} \quad k = \overline{1,N}, r = \overline{0,m}. \quad (37)$$

Theorem 2. Assume that conditions $\widetilde{\theta}_i \in F(R)$ and $\inf\{x \in \sup p\widetilde{\theta}\} \ge 0$ for $i = \overline{1,m}$;

{ζ_k}is the sequence of bounded and positive valued i.i.d. random variables with the mean m_ζ(>0) and the variance σ_ζ^2, where F(R) is the set of all furry sets, hold. Then

$$d(\hat{M}_N, M[y(1)]) \to 0 \text{ w.p.1 as } N \to \infty \quad (38)$$

where $d(\hat{M}_N, M[y(1)])$ is the fuzzy metric given by $d(U,v) := \int_0^1 \rho(L_\alpha U, L_\alpha V)d\alpha$ (39)

and $$\rho(L_\alpha U, L_\alpha V) = \max\left\{\begin{matrix}\sup\limits_{u\in L_\alpha U}\inf\limits_{v\in L_\alpha V}|u-v|, \\ \sup\limits_{v\in L_\alpha V}\inf\limits_{u\in L_\alpha U}|u-v|\end{matrix}\right\} \quad (40)$$

Result of simulation

Consider the system described by FMA(2),

$$y(k) = \xi(k) + \widetilde{\theta}_2 \xi(k-2) \quad (41)$$

Where the membership function of unknown fuzzy parameters $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ assumed to be:

$$(\widetilde{\theta}_i)(x) = \frac{\cos(a_i(x-b_i))+1}{2}; i = \overline{1,2} \quad (42)$$

for $b_i - (\frac{\pi}{a_i}) \le x \le b_i + (\frac{\pi}{a_i})$,and zero for other values of x. That $a_1=8\pi$, $a_2=10\pi$, $b_1=0.8$; $b_2=0.5$,$m_\zeta=1s$;σ=1,6

The unobservable random disturbance {ζ(k)}is the sequence o i.i.d Gaussian random variables of $N(m_\zeta,\sigma_\zeta^2)$ generated by a digital computer, and they truncated so as to satisfy the condition $0 \le \zeta_k \le k$, for k=1,2,… where k is a large constant $k=m_s+3\sigma_\zeta$. The unknown parameters $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ are estimated by using the procedure:

$$\widetilde{\theta}(i) = \widetilde{\theta}_{\alpha,1}(i) := \inf\{x \in L_\alpha \widetilde{\theta}_i\} \quad (43)$$

or $\widetilde{\theta}(i) = \widetilde{\theta}_{\alpha,2}(i) := \sup\{x \in L_\alpha \widetilde{\theta}_i\}$ (44)

i=1,2 for all $\alpha \in [0,1]$.

Conclusions

In this paper it have established stability of a gradient based adaptive control algorithm render far mare general conditions then have previously been used. The principal hypothesis, B-bounded ness of the disturbance and parameter processes, allows infrequent large jumps in the values of the parameters of the system and parameter drift. It have consider the optimal input design problem frequency domain for efficient discrimination of linear stochastic models under input power constraint and have abstained auto-covariance sequence of an optimal input by soloing a mathematical programming problem, which maximize that time-average of the KDI.

It is shown that the input can be realized by autoregressive process or the Chebyshev systems approach.

With the help of the moment method for ordinary moving average models, the identification method of unknown fuzzy parameters has been derived when the fuzzy stochastic systems is described by the FMA model. It has also been shown form numerical viewpoints, using fuzzy metric, that proposed estimators have salient convergence features.

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