

## ON THE MEASUREMENT OF THE COMPLEX PERMITTIVITY OF DIELECTRIC RESONATORS

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**Abstract:** Resonant cavity techniques are widely used for determining microwave properties of materials by measuring the shift of resonant frequency and the change in the Q-factor on the cavity when the dielectric resonator is inserted into the cavity. Fundamental principles of the shape – independent and size – independent measurements are discussed and experimental results are presented.

### Introduction

The advent of stable dielectric materials has created much interest in the use of dielectric resonators in both active and passive microwave microstrip circuits. It is well known now, that microwave dielectric resonators materials like Ba<sub>2</sub>Ti<sub>9</sub>O<sub>20</sub> and BaTi<sub>4</sub>O<sub>9</sub> made it possible to produce resonators with unloaded quality factor no more than 3000 – 4000.

The resonant cavity techniques is a nondestructive technique and it is used for determining microwave properties of materials by measuring the shift of resonant frequency and the change in the Q-factor of the cavity when the sample is inserted into the cavity. For a given cavity and material sample of regular shape and well defined dimensions, one can determine the permittivity of the material.

### Theoretical Considerations

A small dielectric object nonmagnetic of volume  $v'$  is placed in a rectangular waveguide resonant cavity having volume  $V$  ( $V' \ll V$ ), which has the dielectric field  $E_0$  and magnetic field  $H_0$  in its unperturbed mode. The fields inside of the object are  $E$  and  $H$ .

The change in the resonant angular frequency may be expressed as [1], [2], [8], [9], [10]:

$$\frac{\omega_s - \omega_0}{\omega_0} = -(\epsilon_r - 1) \frac{\int EE_0^* dv}{2 \int |E_0|^2 dv} \quad (1)$$

where  $\epsilon_r = \epsilon' - j\delta\epsilon''$  is the complex relative permittivity,  $\epsilon'$  is the dielectric constant and  $\epsilon''$  is the loss factor of the material.

In (1) are two approximations, based on the assumption that the fields in the empty part of

the cavity are negligibly changed by insertion of the object and that the fields in the dielectric object are uniform over its volume.

Let  $Q_0$  be the Q-factor of the cavity in the unperturbed condition and  $Q_s$  be the Q-factor of the cavity loaded with the object. Then the complex resonant angular frequency,  $\Omega_0$  may be defined [10] as:

$$\Omega_0 = \omega_0 \left( 1 + \frac{j}{2Q_0} \right) \quad (2)$$

With the dielectric object the resonant frequency is changing from  $\Omega_0$  to  $\Omega_0 + \Delta\Omega$  and the real part becomes  $\omega_0 + \Delta\omega$ .

$$\Omega_0 + \Delta\Omega = (\omega_0 + \Delta\omega) \left( 1 + \frac{j}{2Q_s} \right) \quad (3)$$

$$\frac{\Delta\Omega}{\omega_0} = \frac{f_0 - f_s}{f_0} + j \frac{1}{2Q_0} \left( \frac{Q_0}{Q_s} - 1 \right) \quad (4)$$

With the transmission factor  $\Delta T$  [8] one obtains:

$$\frac{\Delta\Omega}{\omega_0} = \frac{f_0 - f_s}{f_0} + j \frac{\Delta T}{2Q_0} \quad (5)$$

and (1):

$$2 \frac{f_0 - f_s}{f_0} - j \left( \frac{1}{Q_0} - \frac{1}{Q_s} \right) = (\epsilon_r - 1) \frac{\int EE_0^* dv}{\int |E_0|^2 dv} \quad (6)$$

With  $\Delta f = f_0 - f_s$  and comparing (1) with (5) one obtains:

$$-\frac{f_0}{f_0} + j \frac{\Delta T}{2Q_0} = -\frac{1}{2} \left( \frac{1}{A} \right) k (\epsilon_r - 1) \frac{V'}{V} \quad (7)$$

where:

$$A = \frac{1}{V'} \int \frac{|E_0|^2}{|E_{0max}|^2} dv$$

The value of the shape factor  $k$  is dependent upon object shape, orientation and permittivity and such values are in [8], [11].

For the rectangular cavity operating in the TE<sub>10p</sub> mode, E<sub>0</sub> is given by:

$$E_0 = E_{0max} \sin \frac{\pi}{a} x \sin \frac{p\pi z}{l}$$

where l and a are the length and the width of the cavity and A=1/4.

Form (7) one obtains:

$$\Delta f = 2(\epsilon' - 1)kf_0 \frac{V'}{V}$$

and

$$\Delta T = 4\epsilon''k^2Q_0 \frac{V'}{V} \quad (9)$$

The values of f<sub>0</sub>, k, V', V and Q<sub>0</sub> are known and the value of ΔT is calculated by (9).

For k=1, [8], [11] and (8) one obtain:

$$\epsilon' = 1 + \frac{f_0 - f_s}{f_0} \frac{V}{2V'} \quad (10)$$

### The Experimental Results

In the figure 1 is presented the first experimental installation for microwave parameters measurement.

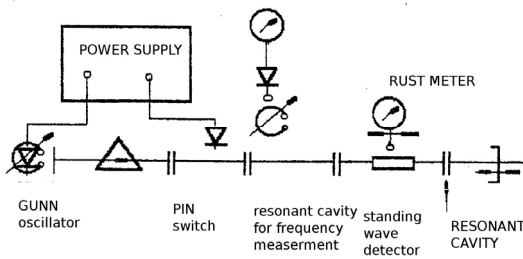


Figure 1.

The dimensions of the resonant cavity are: a=22.86 mm; b=10.16 mm and l=19.87 mm. The Q-factor of the cavity for TE<sub>101</sub> mode is Q<sub>0</sub>=3610.75.

In figure 2 is presented the rectangular waveguide resonant cavity with a B695-A1008 dielectric sample in the center.

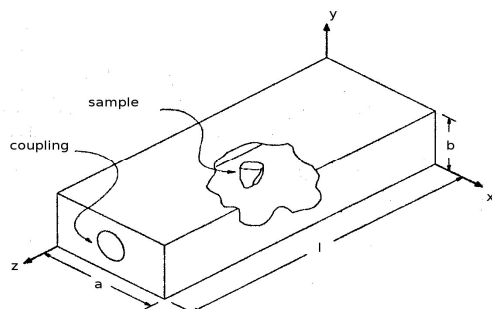


Figure 2.

In figure 3 is presented the trace of WSVR<sub>0</sub> for the cavity in the unperturbed conditions

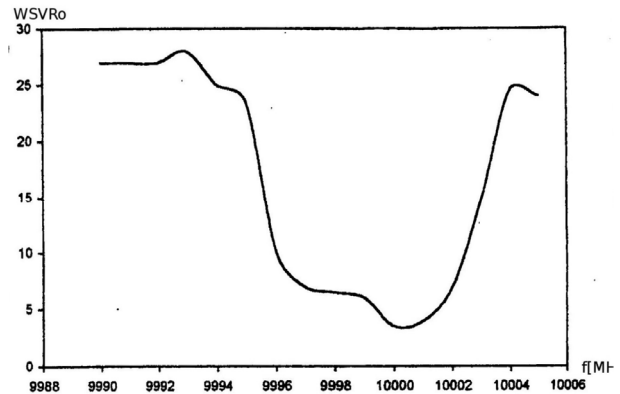


Figure 3.

In figure 4 is shown the trace of WSVR<sub>0</sub> for the cavity with the sample object.

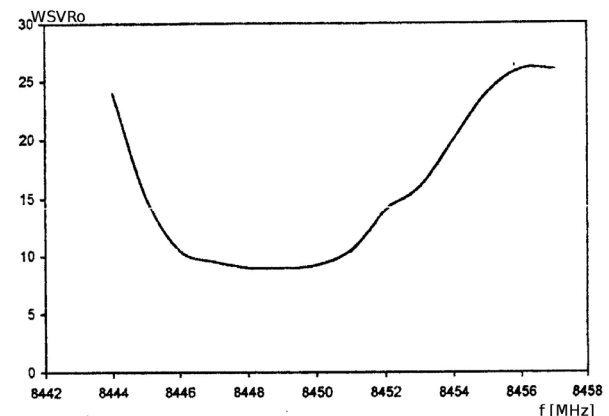


Figure 4.

When the dielectric sample (bar || E<sub>0</sub>) is positioned in the center of the rectangular waveguide resonant cavity one obtains: Δf= 1.552 GHz and ε' ≈35.82.

In figure 5 is shown the trace of ε' when f=9.75 – 10.25 GHz.

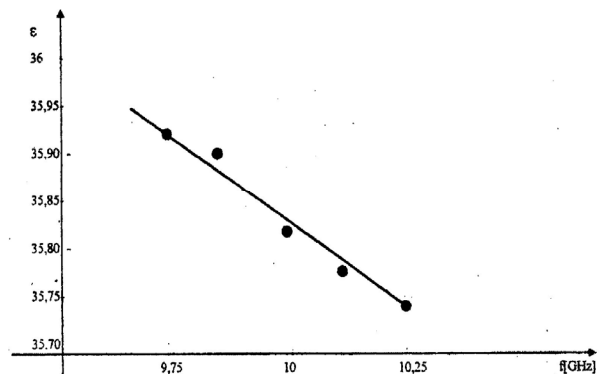


Figure 5.

The second installation is a standard HP sweep test set, consisting of a sweep generator (HP8620C) amplitude analyzer (HP8755) and a reflectometer (HP11692D)

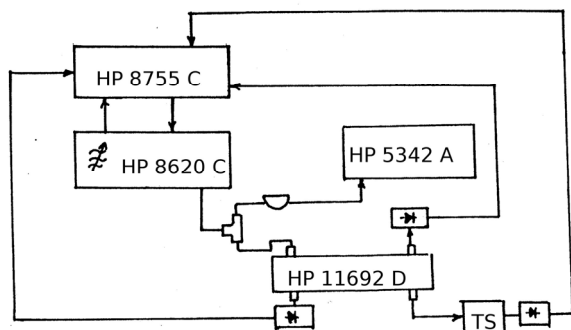


Figure 6.

For the exact resonance and “half power” frequencies measurements a HP digital microwave counter 5342A can be used.

The testing structure TS used for  $\epsilon_r$  and unloaded quality factor measurements are shown in figure 7.

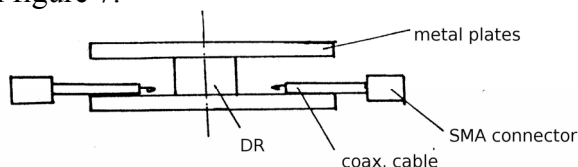


Figure 7.

With the resonance  $TE_{011}$ , frequency measured by the digital counter and the DR diameters known ( $3 < \left(\frac{D}{H}\right)^2 < 11$ ),  $\epsilon_r$  can be calculated by the equations:

$$\epsilon_r = f_0^2 \left( \frac{\lambda_0}{\pi D} \right)^2$$

where

$$f_0^2 = 14.68 \left[ 1 - \frac{1}{\sqrt{14.68 + \left(\frac{\pi D}{2H}\right)^2}} \right]^2 + \left(\frac{\pi D}{2H}\right)^2$$

and  $\lambda_0$  is the resonant mode  $TE_{011}$  wavelength.

## Conclusions

Two different models have been presented in this paper for measuring the permittivity

materials. The results indicate that the models satisfactorily.

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