# LIGHT MODULATION IN OPTICAL WAVEGUIDE WITH NEGATIVE INDEX OF REFRACTION

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**Abstract.** Utilizing of materials with a negative index of refraction (NIR) for electro optical modulators and signalizators design was offered. Mathematical model of modulating waveguide was developed and its main characteristics were studied. The possibility of controlled light conducting in optical waveguide while optical properties of surrounding medium are changing was shown. Conditions that provide conducting regime in optical waveguide with NIR were obtained. Bifurcating of high order modes in cutoff region is shown. **Keywords:** electrooptical modulator, optical waveguide, negative index of refraction, guided mode cutoff.

# Introduction

Technologies of optical waveguides manufacturing have been steadily developing and a number of new materials have appeared. As a result waveguide optical elements have become widespread. Such elements are used for information transmission and transducing. Phenomenon of fundamental mode absence is often used for transmission and transducing waveguides information. In where this phenomenon takes place radiation doesn't propagate. Absence of fundamental mode exists even if total internal reflection is provided. Such a regime supports by a special proportion of waveguides parameters. Therefore conducting regime can be switched to regime when radiation doesn't propagate by changing wavelength, thickness or clad refractive index. This phenomenon is often used for modulators and signalizators developing [1].

But in waveguides made of quartz or polymer materials absence of fundamental mode is possible only in antisymmetric case when core is surrounded by media with different indices of refraction. Such waveguide structure can be manufactured only in integral way and additive losses of light's radiation appear when it is connected to fibres connection lines.

V.G. Veselago in 1968 analyzed hypothetical material with simultaneously negative dielectric permittivity and magnetic permeability [2]. NIR is a feature of such materials. But only during

the last few years manufacturing of different materials with such properties became possible and was proved theoretically [3] and experimental [4]. Ways of using and different properties of such materials were considered in a number of works (see references in [5]). In [6] the absence of basic modes in a waveguides made of such materials was predicted.

The aim of this work is to evaluate possibilities of using NIR materials for optical fibre modulators design.

# Theory

Generalized scheme of electrooptical modulator is shown on fig. 1. Light from monochromatical source of radiation 1 is led to modulator 3 via optical fiber 2. Modulated radiation is led to optical connection line 4 and registrated by photodetector 5.



Figure 1. Electro optical modulator diagram.

Modulator 3 structure is shown on fig. 2. Electrical signal I influences optical properties of waveguide cladding 1 that causes changes in wave propagation in core 2. The way that refractive index of cladding is influenced is well-known and the same both for modulators based on conventional waveguides and for waveguides based on NIR materials. That is way it isn't considered in this work. Further only modulator's waveguide part reaction on changes of clad optical properties is considered. Basic characteristics of light conducting are shown for both types of materials: classical and NIR. For simplification all mathematical calculations are shown for symmetric slab waveguide which optical properties are similar to fibre ones.



Figure 2. Waveguide structure.

Symmetric waveguide consists of core and cladding which have dielectric permittivity and magnetic permeability  $\varepsilon_2$ ,  $\mu_2$ and  $\mathcal{E}_1 \ \mu_1$ correspondently. Thickness of waveguide is 2L. Light of wavelength  $\lambda$  is propagating along z axis without losses because of total internal reflection.  $\varepsilon_2$  and  $\mu_2$  can be simultaneously positive in case of classical waveguide  $(\varepsilon_2 > \varepsilon_1)$ or negative in case of waveguide which core is made of material with NIR. Cladding is made of right-handed material ( $\varepsilon_1 > 0$ ) in both cases. In z and y directions waveguide is considered as unlimited. Such a structure can be described using characteristic equation [6]:

$$\pm \frac{\mu_1}{\mu_2} \kappa_2 \tan^{\pm 1}(\kappa_2 L) - k_1 = 0, \qquad (1)$$

$$k_1^2 = h^2 - \left(\frac{2\pi}{\lambda}\right)^2 \varepsilon_1 \mu_1, \qquad (2)$$

$$\kappa_2^2 = \left(\frac{2\pi}{\lambda}\right)^2 \varepsilon_2 \mu_2 - h^2, \qquad (3)$$

where h is wave propagation constant, (+) and (-) correspond to symmetric and antisymmetric guided modes respectively. This equation can be used only for waveguides that have positive index of refraction, when core has higher refracting index than that in cladding:

$$\varepsilon_2 \cdot \mu_2 > \varepsilon_1 \cdot \mu_1. \tag{4}$$

Using (4) and taking to account (2) and (3) the wave propagation h can be valued as:

$$\sqrt{\varepsilon_1 \mu_1} \le h \frac{c}{\omega} \le \sqrt{\varepsilon_2 \mu_2}$$
. (5)

Waveguides made of NIR material are able to conduct light even when (5) is not satisfided. It is obvious that if condition (5) is violated value  $k_2$  in (3) becomes imaginary. To avoid operations with imaginary numbers equation (1) can be presented as:

$$\frac{\mu_1}{\mu_2}k_2\tanh^{\pm 1}(k_2L) + k_1 = 0, \qquad (6)$$

where

$$k_2^2 = h^2 - \left(\frac{2\pi}{\lambda}\right)^2 \varepsilon_2 \mu_2.$$
 (7)

Total power that propagates in the waveguide divides in three parts according to three layers of waveguide. System that shows radiation power in each layer of right-handed waveguide is obtained using equations that describe power carried by modes and component of Poynting vector in z axis direction:

$$P_1 = \frac{hA^2}{4\omega\mu_0 k_1}, \ x \in (-\infty; -L)$$
 (8a)

$$P_{2} = h \frac{(\frac{\sin(2k_{2}L)}{2k_{2}} + L)(B^{2} - C^{2}) + 2LC^{2}}{2\omega\mu_{0}}, \quad (8b)$$

$$x \in (-L; L)$$

$$P_{3} = \frac{hD^{2}}{2\omega\mu_{0}k_{1}}e^{-4k_{1}L}, x \in (L; +\infty)$$
(8c)

Power localized in cladding is calculated as:

$$K = \frac{P_1 + P_3}{P_1 + P_2 + P_3} \tag{9}$$

For left-handed waveguide  $\kappa_2$  in (8b) must be changed to  $k_2$ .Values of coefficients A, B, C, Dare found from continual tangential field's components condition.

## Calculations

To explore features of light propagation in slab waveguides, generalized characteristic of wave guide – normalized frequency is considered:

$$V = \frac{4\pi}{\lambda} L \sqrt{\varepsilon_2 \mu_2 - \varepsilon_1 \mu_1} .$$
 (10)

For next unification and to make obtained results convenient for comparison h value is used in normalized ranges:

$$b = \frac{h\frac{\lambda}{2\pi} - \sqrt{\varepsilon_1 \mu_1}}{\sqrt{\varepsilon_2 \mu_2} - \sqrt{\varepsilon_1 \mu_1}}.$$
 (11)

Waveguide's modes properties are studied at the expense of cladding properties, wavelength or waveguide size changing. Dependency between normalized propagation constant and normalized frequency (dashed lines for NIR materials) is shown in fig. 3. Dependency for classical righthanded waveguide is shown on the same figure (solid lines). Comparing these dependencies, unusual properties of left-handed waveguide become obvious. The first one is absence of fundamental mode in symmetric structure, the second is reducing of b value for first-order mode while V is increasing, the third is existing of no conducting regime, the fourth is existing of two wave propagation constants for each high-order mode in cutoff region.



Figure 3. b versus V for conventional (solid lines) and left-handed (dashed lines) waveguides.

This result is confirmed by dependency between power localization coefficient K and V (fig. 4). First-order mode part of light flux in cladding is growing while V is increasing. Each high-order mode in cutoff region consists of two parts. Behavior of one of this part is similar to the first-order mode behavior. The second part behaves similar to corresponding mode of conventional waveguides. Waveguide doesn't localize radiation in a particular rang of Vbetween conditions of first and second order modes cutoff. No conducting region defines as difference between second-order mode and first-order absence appearing mode conditions. First-order mode cutoff condition is found from (8) as:

$$h = \frac{2\pi}{\lambda} \sqrt{\varepsilon_1 \mu_1} \tag{12}$$

Condition of second-order mode appearing can be found as minimum of reverse function:

$$\frac{\mu_2}{\mu_1}ch^2(\kappa_2 L) - \frac{\mu_1}{\mu_2}sh^2(\kappa_2 L) - k_1 = 0 \quad (13)$$

To determine special characteristics of waveguide in mentioned cases, (12) and (13) should be solved in system with (6).



# Figure 4. *K* versus *b* for conventional (solid lines) and left-handed (dashed lines) waveguides.

### Conclusion

Performed analyze shows fundamental mode absence possibility in symmetric left-handed waveguides. Accurate condition that provides propagating of directed modes in such waveguides is obtained. This condition depends on optical parameters of cladding. Using electrooptical materials as a clad makes waveguide switched from conducting regime to regime of light absence. In this manner light can be modulated. Analyzed structure can easily be manufactured as a fiber because of its symmetry. Condition of radiation absence in left-handed waveguides is close to singlemode regime in classical symmetric waveguides. So modulator's core thickness can be close to singlemode fibres This feature allows electro optical size. modulators that easily can be integrated to

communication lines without losses to be designed.

### References

[1] Uchida N., Mikami O., Uehara S., Noda J. Optical field distribution in a waveguide loaded with high refractive-index film; modulation efficiency improvement in a planar-type modulator// Appl. Opt. 15, 455-(1976)

[2] Veselago V.G. The electrodynamics of substances with simultaneously negative values of and// Sov. Phys. Uspekhi. 10 (4), 509-514, 1968.

[3] Markos P., Soukoulis C.M. (2002) *Transmission studies of left-handed materials//* Physical Review B 65, 033401.

[4] Smith D.R., Padilla W.J., Vier D.C., Nemat-Nasser S.C., Schultz S. Composite medium with simultaneously negative permeability and permittivity// Phys. Rev. Lett. Vol. 84, No. 18, pp. 4184-4187, 2000.

[5] Alu A., Engheta N. Guided modes in a waveguide filled with a pair of single-negative, double-negative, and/or double-positive layers// IEEE transactions on Microwave theory and techniques, vol. 52, no. 1. Pp.199-210, 2004.

[6] Shadrivov I.V., Sukhorukov A.A., Kivshar Y.S. Guided modes in negative-refractive-index waveguides// Phys. Rev. E 67, 057602, 2003.

[7] A. W. Snyder, and J. D. Love, *Optical waveguide theory*, London, Chapman & Hall, (1983).