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A CLASS OF ISI-FREE AND BANDLIMITED PULSES

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Abstract: *A novel class of ISI-free pluses is presented and investigated. We propose and investigate a class of new Nyquist pulses produced by Nyquist filters characteristics obtained from combining two types of characteristics with odd- symmetry and a linear characteristc. They show comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses.*
Keywords: *intersymbol interference, Nyquist filter, error probability.*

1. Introduction

The most common pulse used in telecommunication is the so-called *raised cosine* (RC) pulse. Recently, some new ISI free pulses [2], [3] were proposed. They show better performance than RC with respect to timing error sensitivity.
This paper presents a class of new Nyquist pulses that perform better than RC and FE pulse [2].

2. A class of new Nyquist pulses

We propose o class of new Nyquist pulses with piecewise characteristics, that are illustrated in Figure1.
The Nyquist filter characteristic is obtained from combining two types of characteristics with odd-symmetry and a linear characteristic. Here $H_i(f)$ and $G_i(f)$ are the family of parabolic and cubic ramps.
For i odd they show odd symmetry around B and their definition is:

$$S_i(f)=\begin{cases}1, & |f|\le B(1-a)\\ L(f) & B(1-a)\le|f|\le B(1-c)\\ G_i((|f|-B(1-a))) & B(1-c)\le|f|\le B(1-b)\\ H_i(f) & B(1-b)\le|f|\le B(1+b)\\ 1-G_i((B(1+a)-|f|)) & B(1+b)\le|f|\le B(1+c)\\ L'(f) & B(1+c)\le|f|\le B(1+a)\\ 0, & B(1+a)\le|f|\end{cases} \quad (1)$$

$$G_i(f)=1+\frac{(f)^i}{2B^i a(a-b)^{(i-1)}} \quad (2)$$

$$H_i(f)=1+\frac{(B-|f|)^i}{2B^i ab^{(i-1)}} \quad (3)$$

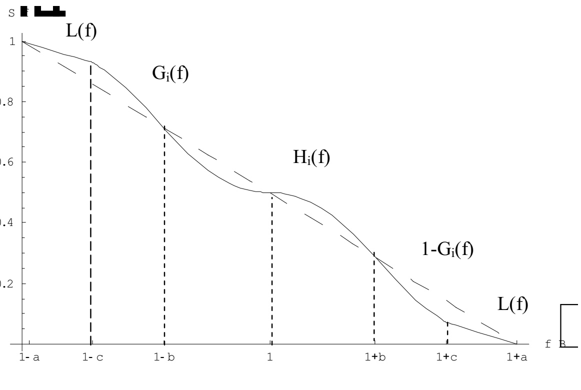

Figure 3. Proposed filter characteristic.

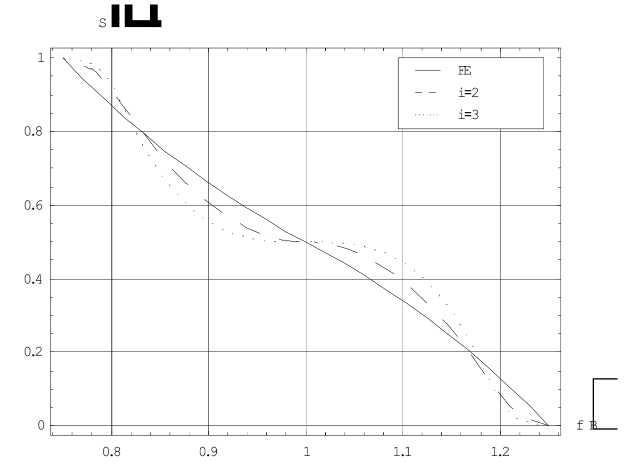

Figure 4. Frequency characteristics for an excess bandwidth a=0.35 (positive frequencies).

$$L(f)=\frac{(1+a)(a-c)^{i-1}}{2a(a-b)^{i-1}}-\frac{(b-c)(2a-b-c)^{i-2}}{(a-b)^{i-1}}-\frac{f}{\dfrac{2a(a-b)^{i-1}B}{(a-c)^{i-1}}}$$

$$L'(f)=\frac{(1+a)(a-c)^{i-1}}{2a(a-b)^{i-1}}-\frac{f}{\dfrac{2a(a-b)^{i-1}B}{(a-c)^{i-1}}} \qquad (4)$$

For *i* even, the vestigial symmetry is obtained by choosing $H_i(f)$ for the frequency interval $B(1-\alpha)\le f\le B$ and $1-H_i(f)$ for $B\le f\le B(1+\alpha)$.

The expressionss were derived imposing continuity conditions at $f=B(1-c)$, $f=B(1-b)$, $f=B(1+b)$ and $f=B(1+c)$ and a value of 0.5 at $f=B$.

$L(f)$ and $L'(f)$ are two linear ramps which show characteristics like $x-\frac{f}{y}$, $x\in \mathrm{N}$; $y\in \mathrm{N}$.

This technique is illustrated in Figure1.

Figure 2 illustrates this types of new Nyquist filter characteristics for $i=2\,and\,3$, together with the *flipped exponential* (FE) defined in [2], taken as a reference.

Since they are more concave than the FE pulse, a decrease of the first side lobe in time domain is to be expected, as shown in Fig.3, where a time-scaled replica of pulses is represented for $a=0{,}25$. The impulse responses $s_i(t)$ are given in the **Appendix A**.

Their behavior around $t/T=3,4,...$ is more flat, which accounts for their better properties

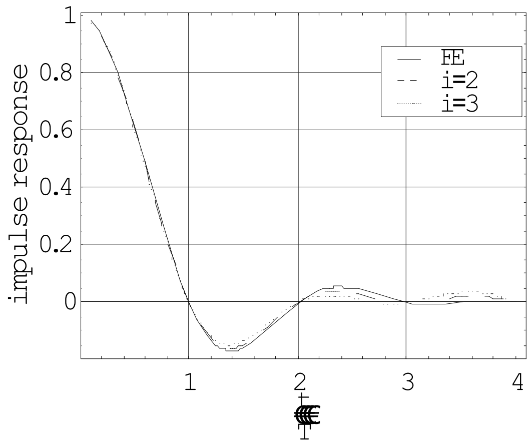

Figure 5. Impulse responses (a=0.25).

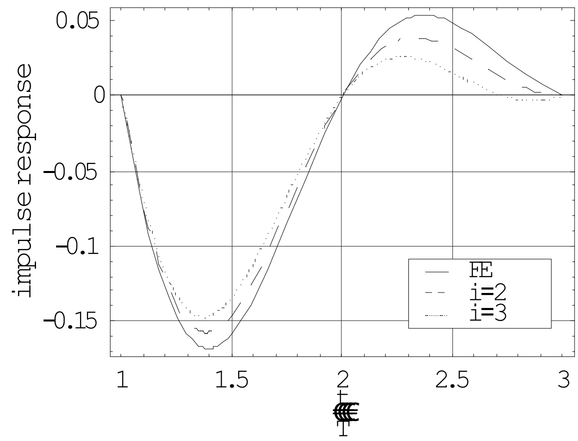

Figure 6. Impulse responses (a=0.25).

regarding the error probability when sampled with a small time offset.

A look at Fig.4 that further illustrates the decay of impulse responses reveals that the new pulse defined by (1), (2) and (3) with $i=2\,and\,3$, follows closely the FE pulse.

3. Error probability

The error probability is calculated using the method of [13] for these proposed pulses are illustrated in Table I, together with those for FE pulse.

The figures 5 and 6 illustrate the probabilities of error for the proposed pulses and different values of parameters *b* and *c*, with different timing offsets.

4.Conclusions

A new class of ISI-free and bandlimited pulses generated by Nyquist filter characteristics resulted from combining two types of characteristics with odd- symmetry and a linear characteristic was presented. The pulses show decreased symbol error probability in the presence of timing error as compared with the FE pulse [2] with the same roll-off factor *a*. Its transmission properties were thoroughly investigated and show that the pulses have practical importance.

They achieve best performance if the width of the concave shaped region in the frequency range $(B(1-\alpha), B)$ is maximized at the expense of convex and linear regions.

Table 1. ISI error probability of several Nyquist pulses for N=2^{10} interfering symbols and SNR = 15 dB

P_e	*B=1*			$t/T_B = \pm 0.05$	$t/T_B = \pm 0.1$	$t/T_B = \pm 0.2$
	a	*b*	*c*			
FE	0.25			$5.81166*10^{-8}$	$1.29804*10^{-6}$	$3.56785*10^{-4}$
	0.35			$3.92526*10^{-8}$	$5.40211*10^{-7}$	$1.01287*10^{-4}$
	0.5			$2.41342*10^{-8}$	$1.85795*10^{-7}$	$2.08778*10^{-5}$
$s_2(t)$ i=2	0.25	0.24	0.241	$5.41937*10^{-8}$	$1.13714*10^{-6}$	$3.03865*10^{-4}$
			0.249	$5.39275*10^{-8}$	$1.1306*10^{-6}$	$3.02198*10^{-4}$
		0.231	0.24	$5.48578*10^{-8}$	$1.7396*10^{-6}$	$3.17975*10^{-4}$
		0.239		$5.43497*10^{-8}$	$1.4329*10^{-6}$	$3.05961*10^{-4}$
	0.35	0.34	0.341	$3.62412*10^{-8}$	$4.779*10^{-7}$	$9.12275*10^{-5}$
			0.349	$3.61147*10^{-8}$	$4.75944*10^{-7}$	$9.08504*10^{-5}$
		0.331	0.34	$3.66311*10^{-8}$	$4.90692*10^{-7}$	$9.48001*10^{-5}$
		0.339		$3.63241*10^{-8}$	$4.79941*10^{-7}$	$9.17449*10^{-5}$
	0.5	0.49	0.491	$2.23906*10^{-8}$	$1.71805*10^{-7}$	$2.3126*10^{-5}$
			0.499	$2.23363*10^{-8}$	$1.71282*10^{-7}$	$2.30509*10^{-5}$
		0.481	0.49	$2.25801*10^{-8}$	$1.75637*10^{-7}$	$2.37756*10^{-5}$
		0.489		$2.24289*10^{-8}$	$1.72398*10^{-7}$	$2.32228*10^{-5}$
$s_3(t)$ i=3	0.25	0.24	0.241	$5.17281*10^{-8}$	$1.06603*10^{-6}$	$2.87411*10^{-4}$
			0.249	$5.17726*10^{-8}$	$1.06826*10^{-6}$	$2.88272*10^{-4}$
		0.231	0.24	$5.31151*10^{-8}$	$1.2737*10^{-6}$	$3.09256*10^{-4}$
		0.239		$5.18662*10^{-8}$	$1.07197*10^{-6}$	$2.89491*10^{-4}$
	0.35	0.34	0.341	$3.48359*10^{-8}$	$4.70575*10^{-7}$	$9.7711*10^{-5}$
			0.349	$3.48554*10^{-8}$	$4.71236*10^{-7}$	$9.79258*10^{-5}$
		0.331	0.34	$3.55853*10^{-8}$	$4.91119*10^{-7}$	$1.03271*10^{-4}$
		0.339		$3.49121*10^{-8}$	$4.72606*10^{-7}$	$9.82458*10^{-5}$
	0.5	0.49	0.491	$2.20655*10^{-8}$	$1.79094*10^{-7}$	$3.06125*10^{-5}$
			0.499	$2.20732*10^{-8}$	$1.79267*10^{-7}$	$3.06597*10^{-5}$
		0.481	0.49	$2.24053*10^{-8}$	$1.85768*10^{-7}$	$3.18409*10^{-5}$
		0.489		$2.21005*10^{-8}$	$1.79772*10^{-7}$	$3.07321*10^{-5}$

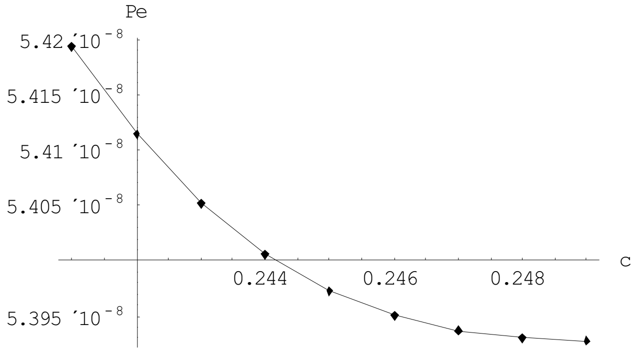

Figure 5a.

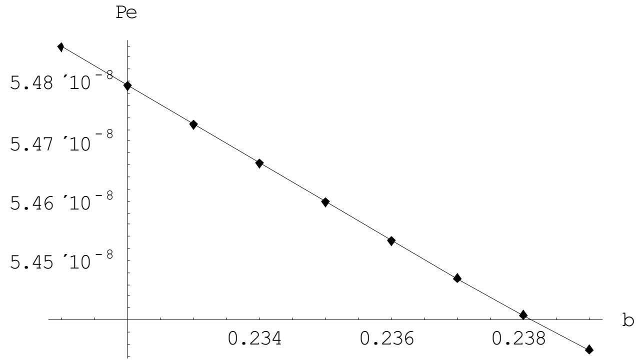

Figure 5b.

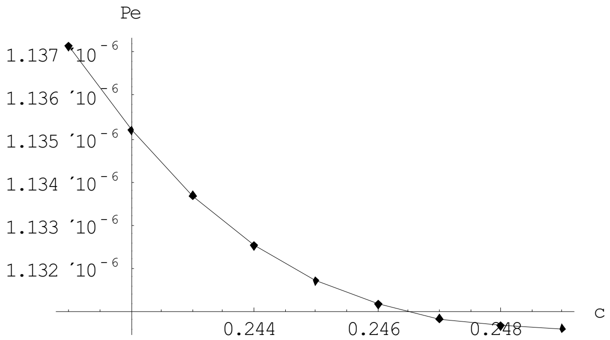

Figure 5c.

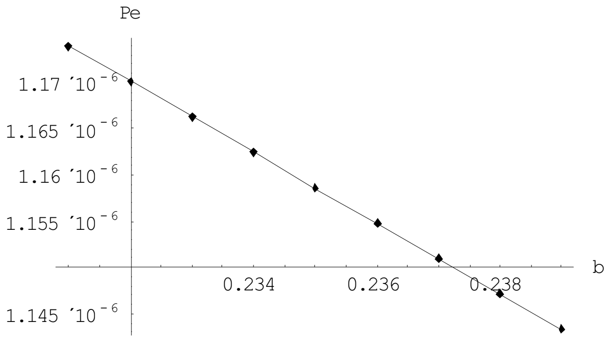

Figure 5d.

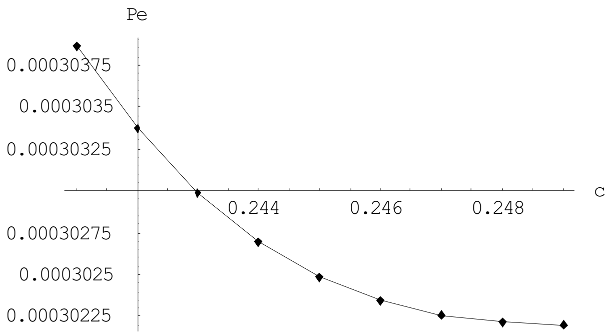

Figure 5e.

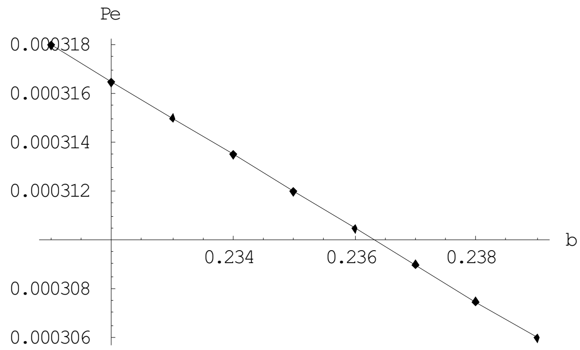

Figure 5f.

Figure 7. Error probability for $s_2(t)$; 5a) a=0.25, b=0.24, $c \in [0.241,0.249]$; $t/T_B=0.05$
5b) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.05$;
5c) a=0.25, b=0.24, $c \in [0.241,0.249]$;$t/T_B=0.1$;
5d) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.1$;
5e) a=0.25, b=0.24, $c \in [0.241,0.249]$;$t/T_B=0.2$;
5f) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.2$;

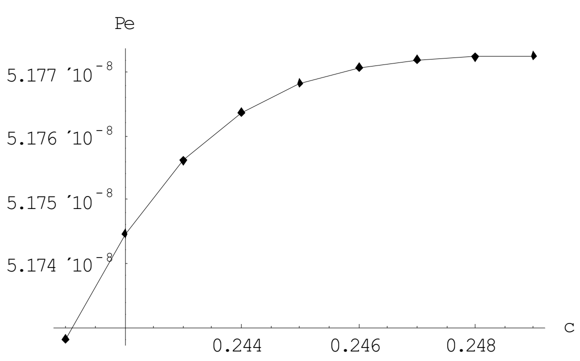

Figure 6a.

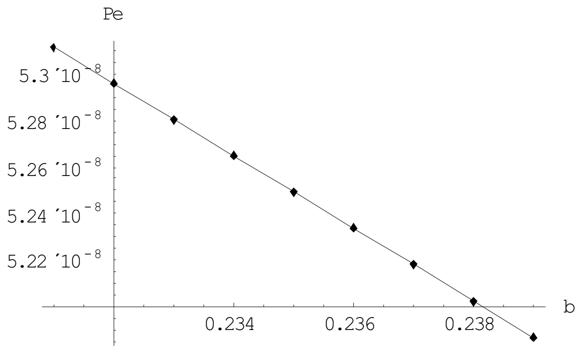

Figure 6b.

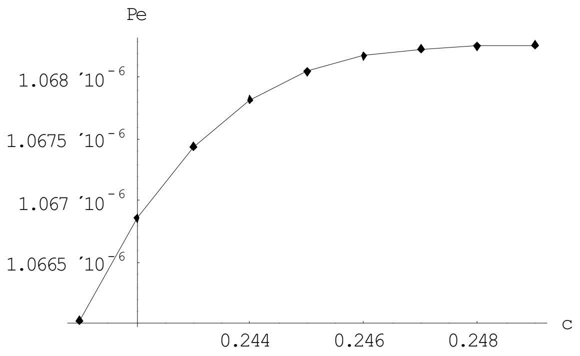

Figure 6c.

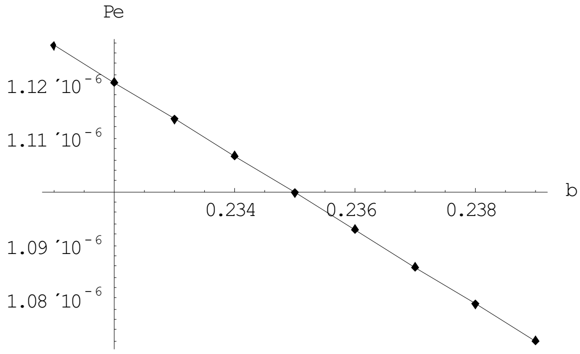

Figure 6d.

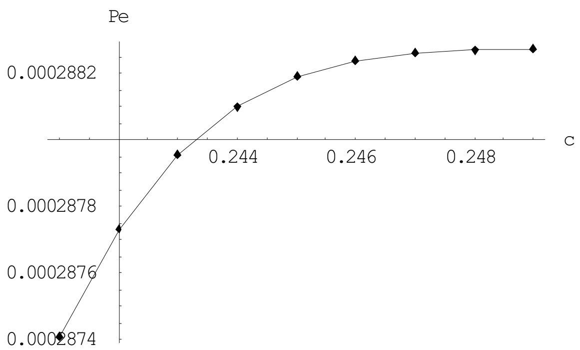

Figure 6e.

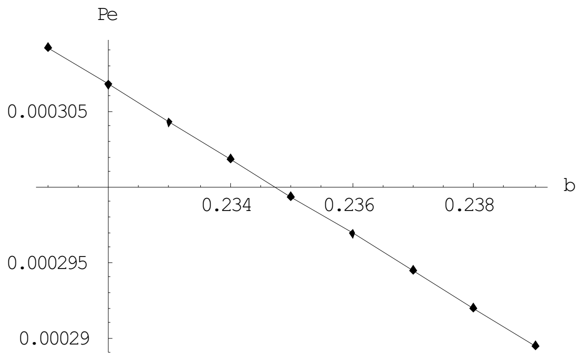

Figure 6f.

Figure 8. Erorror probability for $s_3(t)$;

6a) a=0.25, b=0.24, $c \in [0.241, 0.249]$; $t/T_B=0.05$
6b) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.05$;
6c) a=0.25, b=0.24, $c \in [0.241, 0.249]$;$t/T_B=0.1$;
6d) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.1$;
6e) a=0.25, b=0.24, $c \in [0.241, 0.249]$;$t/T_B=0.2$;
6f) a=0.25, $b \in [0.231, 0.239]$, c=0.24; $t/T_B=0.2$;

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Appendix A

$$s_2(t)=\frac{1}{a(a-b)bB^2\pi^3t^3}\Bigg(\frac{1}{2}bB(a-c)\pi t\cos((-1+a)B\pi t)+(a-b)bB\pi t\cos((1+b)B\pi t)+\frac{1}{2}abB\pi t\cos((-1+c)B\pi t)-$$
$$\frac{1}{2}bBc\pi t\cos((-1+c)B\pi t)+2bB(-a+c)\pi t\cos(B(1+c)\pi t)-2a\sin(B\pi t)+2b\sin(B\pi t)-a\sin((-1+b)B\pi t)+$$
$$a\sin((1+b)B\pi t)+b\sin((1+b)B\pi t)-\frac{1}{2}a^2bB^2\pi^2t^2\sin((1+b)B\pi t)+ab^2B^2\pi^2t^2\sin((1+b)B\pi t)$$
$$-\frac{1}{2}b^3B^2\pi^2t^2\sin((1+b)B\pi t)+b\sin((-1+c)B\pi t)+2b\left(-1+\frac{1}{2}B^2(a-c)^2\pi^2t^2\right)\sin((1+c)B\pi t)\Bigg)$$

$$s_3(t)=\frac{1}{a(a-b)^2b^2B^3\pi^4t^4}\Bigg(\sin(B\pi t)\Big(3a(a-b)bB\pi t\cos(bB\pi t)+3b^2B(-a+c)\pi t\cos(bB\pi t)+$$
$$\frac{1}{2}a^2b^2B^2\pi^2t^2\sin(aB\pi t)-ab^2B^2c\pi^2t^2\sin(aB\pi t)+\frac{1}{2}b^2B^2c^2\pi^2t^2\sin(aB\pi t)-3a^2\sin(bB\pi t)+6ab\sin(bB\pi t)+$$
$$6ab\sin(bB\pi t)+b^2\left(-3+B^2(a-c)^2\pi^2t^2\right)\sin(Bc\pi t)\Big)\Bigg)$$