THE DEVELOPMENT OF THE PERSPECTIVE STEREOMETRIC PROJECTION PROCEDURES

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***Abstract**: In the work algorithms of geometric transformations for perspective projection of stereometric points to the screen plane are described. Models with two or three points of projecting beams' tracing are considered. Offered algorithms allow building the images of stereometric objects taking into account angle changing under moving the object from the observation point that corresponds to the natural depiction.*
***Keyword:** parallel projection, perspective projection, Interactive Computer Graphics.*

Task validation

During creation of software products of architectural or constructive nature problems of subproducts visualization are often arisen. It is used frame model or model of invisible lines removing. Most of all methods of parallel projection are used, because these algorithms are simple for realization and they can be done by standard tools of programming languages of general assignment. Demerit of parallel projection is interpretation ambiguity of the obtained "frame" images (fig. 1). Figure a) shows frame model of cube, b) – two interpretations of opaque models.

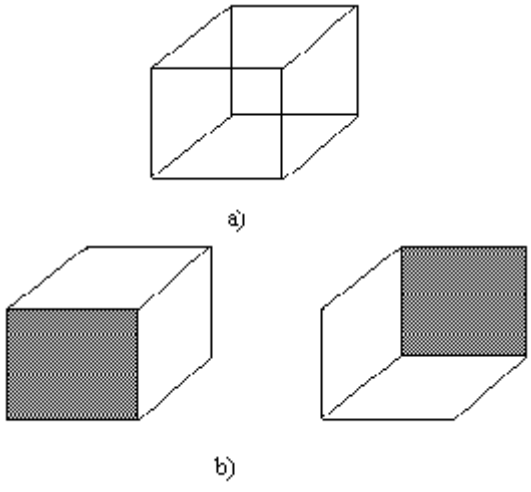


Figure 1.

To model the "depth" algorithms of perspective central projection are used. These algorithms imitate shortage of lines under moving them from the observation point. Thus the method of central perspective projection creates inadequate "natural" images, because of retaining parallelism in front planes (fig.2, a). "Natural" features are typical for two- and tree-points perspective projections (fig. 2, b, c).

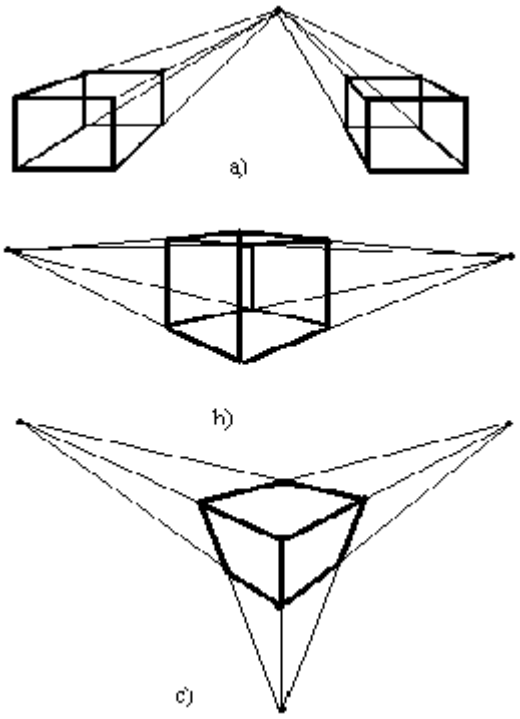


Figure 2.

Naturally, graphical tools such as OpenGL can be used, but they need rather labour-intensive structure of data presentation. Developer should learn extra one more specific programming language. Thus, the idea of intermediate creation was arisen, i.e. simple procedures of stereometric perspective projection in addition to standard graphical tools of programming languages of general assignment[1,2,3].

Algorithm description

Algorithm of two- point perspective projection includes two phases: at first geometric transformation of the object's points taking into account point and sector of observation takes place. Then orthogonal projection of the received points to the computer screen is made. Projection geometric transformation of the points is made according to the algorithm:

- Taking into account the position of point and sector of observation, equations of planes of observation sector (observation direction is perpendicular to the plane of screen) are determined.
- Tracing beams points (left and right focuses) are determined. Vertical position of focus points that corresponds to the observation point are determined (fig. 3).

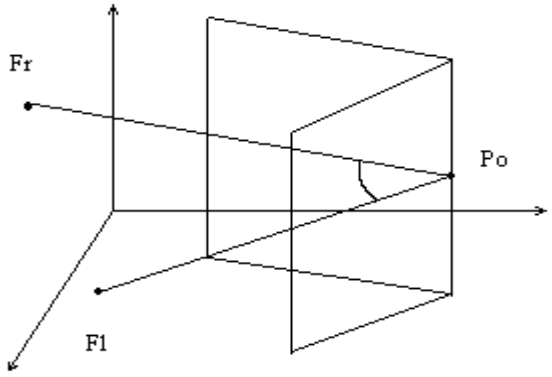


Figure 3.

- Each object's point is projected orthogonal to the observation sector plane and Каждая and to dihedral angle rib.
- Equations of supporting planes are determined. "Left" supporting plane goes through the left focus and orthogonal projection to the right sector plan parallel to rib of dihedral angle (fig. 4, b).

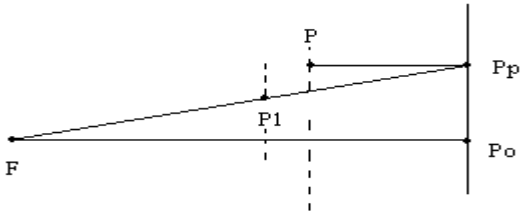
$$\begin{vmatrix} x - x\,Pr & y - y\,Pr & z - z\,Pr \\ xFl - x\,Pr & yFl - y\,Pr & zFl - z\,Pr \\ x\,Pr & y\,Pr & C - z\,Pr \end{vmatrix} = 0 \quad ,$$

$$C \neq z\,\mathrm{Pr}\,.$$

Correspondingly equation of the "right" supporting plane is determined. Third supporting plane goes through two focuses and point projection to the rib of dihedral angle of observation sector.

$$\begin{vmatrix} x - xPp & y - yPp & z - zPp \\ xFl - xPp & yFl - yPp & zFl - zPp \\ xFr - xPp & yFr - yPp & zFr - zPp \end{vmatrix} = 0 \quad .$$

- Transformed coordinates of the point ($P1$) are determined as crossing of supporting planes (fig. 4.).



a) Determination of Z-coordinate.

b) Determination of X, Y coordinates.

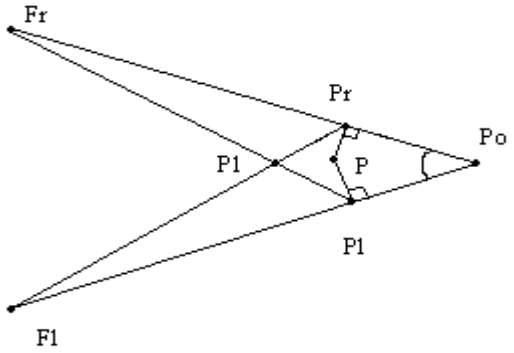


Figure 4.

Thus, position of transformed points is governed by the value of observation sector angle, position of observation point, start distance from the point to observation point, moving focuses from observation point. Notice:

Obviously, when visible angle between "vertical" supporting planes is smaller than 90^0, point is "moving from" observer under transformation, otherwise point is "moving to".

Using three- point model of projection observation sector is changed to the correspondent observation cone, where three point of beam tracing are chosen (fig. 5).

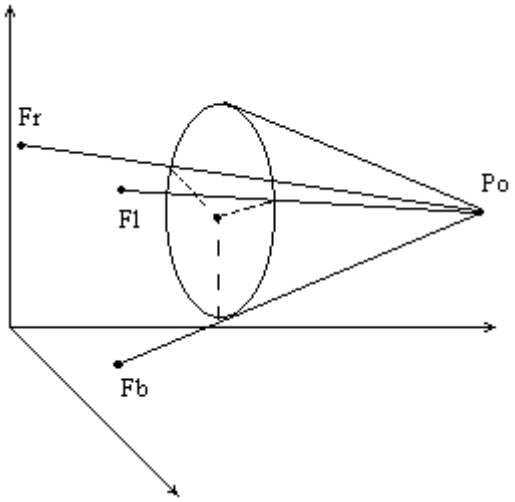


Figure 5.

For geometric perspective transformation following operations are executed.

- Equations of three restrictive planes are determined, each of them is built on two focuses and observation point, e.g. (Fr, Po, Fl).
- Points of stereometric object are projected orthogonal to each rib of dihedral angles. Supporting plane is determined by the point of projection and two opposite focuses.
- Point's coordinates after transformation are determined as crossing of three supporting planes.

Procedure structure of perspective transformation of stereometric points

Obviously, for the usability, algorithm data should be designed in correspondent subprograms i.e. should become additional program tools. For programming language ***Pascal*** subprograms are included in separate program module and are as follows:

```
Type TPoint=Record
  X,Y,Z: Real
            End;
Procedure Proj2P( {2-point projection }
      Po  :TPoint;  {observation point }
     Alfa:Real;     {angle of observation sector }
    Dist :Real;     {distance to the focuses }
Var  P   :TPoint  {transformed point}
                );
```

Procedure of three- point transformation is defined similarly. Usage of given procedures substantially simplify and improve visualization process of not complicated stereometric objects under perspective projection.

Reference

[1] Rogers, David F. and J. Alan Adams, *Mathematical Elements for Computer Graphics*, second edition, McGraw Hill, New York, NY, 1990.

[2] Дж.Фоли, А.ванДем, *Основы интерактивной машинной графики*, тт.1,2, М., "Мир", 1985, 370с, 350с.

[3] Rogers, David F., *Procedural Elements for Computer Graphics*, McGraw Hill, New York, NY, 1985.