Controllability of Linear Systems with a Lag with Change of Phase Space Measurability

Fedir SOPRONIUK[1], Tetyana GABUZA[2]
"Yuriy Fedkovych" Chernivtsi National University
str. Universietetska nr.28, UA-58012, Chernivtsi, Ukraine
[1]*fosopr@chnu.cv.ua*, [2]*GabuzaT@mail.ru*

***Abstract*—This paper is proposing a way of finding the solution of linear control systems with a lag and with change of phase space measurability and the method of investigation the controllability's property for such systems.**

***Index Terms*—controllability, controlled system, determining equation, lag, phase space.**

I. Introduction

The aim of the given scientific work is the finding the solution [2,4] and investigation the controllability of systems [1,3] with a lag and with change of phase space measurability.

The research controllability of control systems with lag is much more difficult, than objects which are described by the usual differential equations because the conditions of such objects are characterized not by a set of final quantity of sizes. Besides in the decision of some control problems one opportunity of the object transition from any condition to another is not enough. Specificity of control systems with lag is more difficult that is why they we enter the concepts of a relative controllability of control systems and entirely controllability of such systems.

II. The Mathematical Model of Control Problem with a Lag with Change of Phase Space Measurability

Let $t_0, t_1, ..., t_N$ is some breakdown of a segment $[T_0, T_1]$, where $T_0 = t_0 < t_1 < t_2 < ... < t_{N-1} < t_N = T_1$.

The mathematical model of a control problem looks like:

$$\frac{dx^{(j)}(t)}{dt} = A_{1j}x^{(j)}(t) + A_{2j}x^{(j)}(t-h_j) + \\ +B_j u^{(j)}(t), t \in [t_{j-1}, t_j), \quad (1)$$

$$x^{(j)}(t_{j-1}+q_j) = C_j x^{(j-1)}(t_{j-1}+q_j), q_j \in [-h_j, 0], \quad (2)$$

$$x^{(0)}(t_0+q_1) = j(t_0+q_1),\ C_1 = E_{n_1},$$

$$t_j - t_{j-1} \geq h_j,\ j = \overline{1,N}, \quad (3)$$

where $x^{(j)} = (x_1^{(j)}, x_2^{(j)}, ..., x_{n_j}^{(j)})^T$ – n_j -measurable vector of phase coordinates, $u^{(j)} = \left(u_1^{(j)}, u_2^{(j)}, ..., u_{m_j}^{(j)}\right)^T$ – m_j - measurable vector of control, A_{1j}, A_{2j} – constant square $n_j \times n_j$ measurable matrixes, B_j, C_j, constant rectangular matrixes of $n_j \times m_j$, $n_j \times n_{j-1}$ sizes accordingly, h_j – lag, $j = \overline{1,N}$, and at we shall count, that if $j=1$ then $C_1 = E_1$ $n_1 \times n_1$ - measurable unit matrix, $j(q)$ - continuous initial function, $x^{(0)}(t_0) = j(t_0)$ – an initial condition of system (1).

Let's enter the base definitions, necessary in the further researches.

Attribute 1. The control system is called controlled, if for any function $j(t_0+q_1) \in X, q_1 \in [-h_1, 0],\ x_1 \in X$, where X - phase space of control system and any parameters $T_0, T_1,\ T_0 = t_0 < t_1 < ... < t_{N-1} < t_N = T_1$ exists such controls $u^{(1)}(t), u^{(2)}(t), ..., u^{(N)}(t)$, which transfer control system from initial $x^{(0)}(t_0)$ to final condition.

We find the solution of control problem (1), (2), (3). Let us assume that exist such matrix functions:

$$\frac{dX_j(t,t)}{dt} = -X_j(t,t)A_{1j} - X_j(t,t+h_j)A_{2j},\ t \leq t, \\ X_j(t,t-) = E,\ X_j(t,t) = 0,\ t \leq t \leq t+h_j,\ j = \overline{1,N}, \quad (5)$$

of t, and

$$\frac{\partial X_j(t,t)}{\partial t} = A_{1j}X_j(t,t) + A_{2j}X_j(t-h_j,t), t \in [t, t+h_j), \\ X_j(t,t-) = E,\ X_j(t,t) = 0, t-h_j \leq t \leq t,\ t \in [t_{j-1}, t_j), \quad (6)$$

of t.

The mathematical model of a control problem on a segment $[t_0, t_1)$ is:

$$\frac{dx^{(1)}(t)}{dt} = A_{11}x^{(1)}(t) + A_{21}x^{(1)}(t-h_1) + B_1 u^{(1)}(t),\ \ t \in [t_0, t_1),$$

$$x^{(0)}(t_0+q_1) = j(t_0+q_1),\ q_1 \in [-h_1, 0],\ C_1 = E_{n_1}.$$

The left and right parts of equation multiplied by $X_1(t,t)$ and integrated over t :

$$\int_{t_0}^{t} X_1(t,t)\frac{dx^{(1)}(t)}{dt}dt = \int_{t_0}^{t} X_1(t,t)A_{11}x^{(1)}(t)dt + \\ +\int_{t_0}^{t} X_1(t,t)A_{21}x(t-h_1) + \int_{t_0}^{t} X_1(t,t)B_1 u^{(1)}(t)dt.$$

The last equation is integrated by parts:

$$-\int_{t_0}^{t}\frac{\partial X_1(t,t)}{\partial t}x^{(1)}(t)dt = -X_1(t,t)x^{(1)}(t)\Big|_{t_0}^{t} +$$
$$+\int_{t_0}^{t}X_1(t,t)A_{11}x^{(1)}(t)dt+\int_{t_0}^{t}X_1(t,t)A_{21}x(t-h_1)+$$
$$+\int_{t_0}^{t}X_1(t,t)B_1u^{(1)}(t)dt.$$

Or:

$$-\int_{t_0}^{t}\frac{\partial X_1(t,t)}{\partial t}x^{(1)}(t)dt = -x^{(1)}(t)+X_1(t,t_0)x^{(1)}(t_0)+$$
$$+\int_{t_0}^{t}X_1(t,t)A_{11}x^{(1)}(t)dt+\int_{t_0}^{t}X_1(t,t)A_{21}x(t-h_1)+$$
$$+\int_{t_0}^{t}X_1(t,t)B_1u^{(1)}(t)dt.$$

We can see, that:

$$\int_{t_0}^{t}X_1(t,t)A_{21}x^{(1)}(t-h_1)dt = \int_{t_0}^{t}X_1(t,t+h_1)A_{21}x^{(1)}(t)dt+$$
$$+\int_{t_0}^{t_0+h_1}X_1(t,t)A_{21}C_1x^{(0)}(t-h_1)dt.$$

That is why we obtain the equation:

$$\int_{t_0}^{t}X_1(t,t)A_{11}x^{(1)}(t)dt+\int_{t_0}^{t}X_1(t,t+h_1)A_{21}x^{(1)}(t)dt =$$
$$=-x^{(1)}(t)+X_1(t,t_0)x^{(1)}(t_0)+\int_{t_0}^{t}X_1(t,t)A_{11}x^{(1)}(t)dt+$$
$$+\int_{t_0}^{t}X_1(t,t+h_1)A_{21}x(t)+$$
$$+\int_{t_0}^{t_0+h_1}X_1(t,t-h_1)A_{21}j(t-h_1)dt+$$
$$+\int_{t_0}^{t}X_1(t,t)B_1u^{(1)}(t)dt.$$

So:

$$x^{(1)}(t)=X^{(1)}(t,t_0)x_0+\int_{t_0}^{t_0+h_1}X_1(t,t)A_{21}C_1j(t-h_1)dt+$$
$$+\int_{t_0}^{t}X_1(t,t)B_1u^{(1)}(t)dt.$$

For a segment $[t_1,t_2)$ we'll obtain:

$$x^{(2)}(t)=X_2(t,t_1)C_2X_1(t_1,t_0)C_1x_0+$$
$$+X_2(t,t_1)C_2\int_{t_0}^{t_0+h_1}X_1(t_1,t-h_1)A_{21}C_1j(t-h_1)dt+$$
$$+X_2(t,t_1)C_2\int_{t_0}^{t_1}X_1(t_1,t)B_1u^{(1)}(t)dt+$$
$$+\int_{t_1}^{t_1+h_2}X_2(t,t-h_2)A_{22}C_1j(t-h_2)dt+$$
$$\int_{t_1}^{t}X_2(t,t)B_2u^{(2)}(t)dt.$$

With the same ideas and assumptions we can find the solutions for every interval $[t_{j-1},t_j),\ j=1,N$.

The solution of control system with lag (1), with conditions (2), (3) is found:

$$x^{(j)}(t)=W_{j,1}(t,t_1)j(t_0)+\sum_{l=1}^{j-1}W_{j,l+1}(t,t_{l+1})\times$$
$$\times\left[\int_{t_{l-1}}^{t_{l-1}+h_l}X_l(t_l,t)A_{2l}C_lx^{(l-1)}(t-h_l)dt+\right.$$
$$\left.+\int_{t_{l-1}}^{t_l}X_l(t_l,t)B_lu^{(l)}(t)dt\right]+$$
$$+\int_{t_{j-1}}^{t_{j-1}+h_j}X_j(t,t)A_{2j}C_jx^{(j-1)}(t-h_j)dt+$$
$$\int_{t_{j-1}}^{t}X_j(t,t)B_ju^{(j)}(t)dt,$$

where

$$W_{j,k}(t,t_k)=X_j(t,t_{j-1})C_j\prod_{i=j-1}^{k}X_i(t_i,t_{i-1})C_i,\ 1\ge k\ge j.$$

$X_j(t,t)$ – matrix function, which satisfy (5), (6) conditions.

Attribute 2. The determining equation of control system with a lag (1) is called the equation:

$$Q_{k_j}^{(j)}(s)=A_{1j}Q_{k_j-1}^{(j)}(s)+A_{2j}Q_{k_j-1}^{(j)}(s_j-h_j),$$
$$s_j\ge t_{j-1},\ k_j=1,2,...,$$
$$Q_0^{(j)}(t_{j-1})=B_j,s_j=t_{j-1};\ Q_0^{(j)}(s_j)=0,$$
$$s_j\ne t_{j-1},\ j=\overline{1,N}.$$

The solution of the determining equations is a set of matrixes, which are defined for $k_j=1,2,...,\quad s_j=t_{j-1}+h_j,t_{j-1}+2h,....$

And with any fixed k_j, matrixes $Q_{k_j}^{(j)}(s_j)=0$ for $s=t_{j-1}+(k_j+1)h_j,t_{j-1}+(k_j+2)h_j,...$

The solutions of determining equations for any $a_j\in R$ and every interval $[t_{j-1},t_j),\ j=\overline{1,N}$ are those:

$$p_{a_j}^{(j)}=\{Q_{k_j}^{(j)}(s_j),k_j=0,1,2,...,n_j-1,s_j\in[t_{j-1},t_{j-1}+a_jh_j)\}$$

We can represent the next theorem.

The theorem 1. The control system (1) with conditions (2), (3) will be controlled then and only then, when:

$$rank\ p_{a_j}^{(j)}=n_j,\ j=\overline{1,N},$$

where $a_j=\left[\dfrac{t_j-t_{j-1}}{h_j}\right]$.

This theorem can be proved in much the same way as the similar nature theorem in [1] for the case without the change of phase space measurability.

Though, the controllability's problem of control system (1), (2), (3) is reduced to the construction of controllability's matrixes.

When we find the rank of controllability's matrixes we can answer on the controllability question.

For computer realization the results of this investigation work used the mathematical package MathCad and its basic opportunities.

III. CONCLUSION

The controllability's condition of a dynamic system are describes with the help of the determining equation for simplification the result and its practical use. In this case the problem of controllability is reduced to the construction of matrixes with some structure and finding their rank. So, controllability's property of control system with lag can be represented with the help of determining equation of control system.

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