# Sensorless AC Driving Systems Based on Adaptive Identification Algorithms and Robust Control Strategies

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Abstract —A control structure is proposed to implement a sensorless speed control driving system with induction machine. The rotor speed is estimated by a model reference adaptive system based algorithm with information from stator variables fundamentals (voltages and currents). Influence of machine parameter sensitivity and low speed estimation errors are discussed. Computer simulated results are presented to validate the proposed sensorless control driving system.

*Index Terms*—AC motor drives, Model reference adaptive control, Identification algorithms, Robust control, Sensorless motion control

#### I. INTRODUCTION

High performance applications of induction motors with vector control technology are widely used [1], [2]. Various concepts for controlled induction motor drives without speed sensor have been developed in the past few years [3], [4], [5]. Ongoing research has focused on providing sustained operation at high dynamic performance in the very low speed range, including zero speed and zero stator frequency [6], [7], [8]. In speed sensorless control two factors are important, namely, wide speed range capability and motor parameter sensitivity [3], [9]. In many existing speed identification algorithms, the rotor speed is estimated based on the rotor flux observer, by forcing the error between the reference model and the adjustable model to be zero [10]. Therefore, these algorithms are, to a certain degree, machine parameter dependent. It is known that the simultaneous estimation of the rotor speed and the rotor resistance is difficult, especially under steady state conditions [11].

The performances of the proposed sensorless control structure of the induction machine (IM), with model reference adaptive system (MRAS) based algorithm for speed estimation, will be demonstrated with the help of the simulated results. The influence of machine parameter sensitivity and low speed estimation errors are discussed.

## II. SENSORLESS CONTROL OF AC DRIVES BASED ON MRAC ALGORITHM FOR SPEED IDENTIFICATION

The MRAS based algorithm is part of the speed sensor-

less vector control structure of the IM, presented in Fig. 1.



Figure 1. Block diagram of speed sensorless vector control structure of the induction machine.

In order to achieve the position sensorless control, the rotor speed estimation has to be indirectly derived based on the measured stator voltages and currents. Therefore, a mathematical model of the induction machine is needed. The model is described in the stationary (stator) reference frame and presented in the Appendix.

The block diagram of the MRAS speed identification is shown in Fig. 2. It contains a reference model, an adjustable model and an adaptive algorithm [10]. Both models have as inputs the stator voltages and currents. The reference model outputs a performance index p and the adjustable model a performance index  $\hat{p}$ . The difference between the two values is used by the adaptive algorithm to converge the estimated speed  $\hat{w}$  to its real value. In order to estimate the rotor speed accurately, the performance index of the reference model has to be robust over the entire speed range and insensitive to the machine parameters [9]. According to the equations of the induction machine described in the Appendix, we can obtain the value of the rotor flux phasor based on stator equations:

$$\frac{d\underline{\Psi}_r}{dt} = \frac{L_r}{L_m} \left( \underline{u}_s - R_s \underline{i}_s - L_s \frac{d\underline{i}_s}{dt} \right) \tag{1}$$

and the same rotor flux phasor based on the rotor equa-

tions:

$$\frac{d\underline{\Psi}_r}{dt} = -\frac{1}{t_r}\underline{\Psi}_r + jw\underline{\Psi}_r + \frac{L_m}{t_r}\underline{i}_s.$$
(2)

Considering the back EMF being:

$$\underline{e} = \frac{L_m}{L_r} \frac{d\Psi_r}{dt}$$
(3)

and decupling (1) on the fix reference frame d-q, we obtain:

$$e_d = u_{sd} - R_s i_{sd} - L_s \frac{di_{sd}}{dt};$$
<sup>(4)</sup>

$$e_q = u_{sq} - R_s i_{sq} - L_s \frac{di_{sq}}{dt}.$$
(5)

Considering a formal magnetizing current

$$\underline{i}_m = \frac{1}{L_m} \underline{\Psi}_r \tag{6}$$

and decupling (2) on the fix reference frame d-q, we have:

$$e_{md} = -\frac{L_m^2}{L_r} \left( \frac{1}{t_r} i_{md} + w i_{mq} - \frac{1}{t_r} i_{sd} \right);$$
(7)

$$e_{mq} = -\frac{L_m^2}{L_r} \left( \frac{1}{t_r} i_{mq} - w i_{md} - \frac{1}{t_r} i_{sq} \right).$$
(8)

The reference model is described based on (4) and (5), resulting that it is parameter dependent, namely with the stator resistance  $R_s$  and the equivalent inductance  $L_{\sigma}$ . In the reference model there are no integral operations, so the model can be used also for low speed estimation. To improve the robustness of the reference model one of the two machine parameters can be avoided by choosing an optimal way to define the reference model performance index p. To eliminate the effect of the inductance  $L_{\sigma}$  (4) and (5) are cross multiplied by the derivates of the two stator current components and then subtract, obtaining:

$$p = u_{sd} \frac{di_{sq}}{dt} - u_{sq} \frac{di_{sd}}{dt} - R_s \left( i_{sd} \frac{di_{sq}}{dt} - i_{sq} \frac{di_{sd}}{dt} \right)$$
(9)

Equation (9) describes the performance index of the reference model. To obtain the performance index of the adjustable model, same mathematical operations applied to (7) and (8) give:

Figure 2. Model reference adaptive system algorithm (MRAS) for speed identification.

having the two formal magnetizing current components described by:

$$\frac{di_{md}}{dt} = -\frac{1}{t_r} i_{md} - \hat{w} i_{mq} + \frac{1}{t_r} i_{sd}; \qquad (11)$$

$$\frac{di_{mq}}{dt} = -\frac{1}{t_r}i_{mq} + \hat{w}i_{md} + \frac{1}{t_r}i_{sq}.$$
 (12)

Equation (9) is used for the reference model and (10)-(12) for the adjustable model. The error between the two performance indexes

$$\boldsymbol{e} = \boldsymbol{p} - \hat{\boldsymbol{p}} \tag{13}$$

is the input for the adaptive algorithm, see Fig. 2. This algorithm estimates the w rotor speed in order to converge the performance index of the adjustable model to the performance index of the reference model (converge the error  $\varepsilon$ to zero). In designing the adaptive mechanism of the presented MRAS structure, it is necessary to ensure the stability of the control system and the convergence of the estimated speed to the real one. Based on the hyperstability theory [10], following adaptive algorithm is used:

$$\stackrel{\wedge}{\mathbf{w}} = K_p \mathbf{e} + K_i \int \mathbf{e} dt , \qquad (14)$$

where,  $K_p$  and  $K_i$  are the gain parameters of the adaptive algorithm.

The MRAS algorithm presented above can also be used for on-line identification of some parameter of the induction machine, namely the stator resistance or the equivalent inductance or the rotor time constant [9].

## III. ROBUST CONTROL OF THE SENSORLESS DRIVING SYSTEM

The extended  $H_{\infty}$  control theory is used to design a robust speed-control solution for sensorless induction motor driving system. It satisfies the robust stability characteristics of the control structure as well as the dynamic performances of the driving system. The  $H_{\infty}$  optimal control designing problem in the particular case of applying the small gain problem is to form an augmented plant of the process P(s), like in Fig. 3. with the weighting functions



Figure 3. Structure of speed control system with weighted process.

W<sub>1</sub>(s), W<sub>2</sub>(s), W<sub>3</sub>(s) and to find an optimal stabilizing H<sub>∞</sub> controller, presented in Fig. 4, so that the infinity norm of the cost function T<sub>y-u</sub> is minimized [5], and is less then one:  $||T_{y-u}||_{1} < 1$ .

Considering the robust stability and robust performance criteria, the weighting functions for the optimal  $H_{\infty}$  controller are chosen and then the iterative computing process continues, until the norm condition is full fit. The performance design specifications of the speed control loop with the  $H_{\infty}$  controller are imposed in frequency domain:

robust performance specifications: minimizing the sensitivity function S (reducing it at least 100 times to approximate 0.3333 rad/sec).

robust stability specifications: -40 dB/decade roll-off and at least -20dB at a crossover band of 100 rad/sec.

Considering the robust stability and robust performance criteria, the weighting functions for the optimal  $H_{\infty}$  controller are:

$$\begin{cases} \frac{1}{W_1(s)} = W_1^{-1}(s) = \frac{1}{g} \cdot \frac{(3s+1)^2}{100} \\ \frac{1}{W_3(s)} = W_3^{-1}(s) = \frac{150}{s+145} \end{cases},$$
(15)

where  $\gamma$  represents the actual step value.

The iterative process continues, until the graphic representation in Bode diagram of cost function  $T_{y-u}$  reach his maximum value in the proximity of 0 dB axis. In our case, for  $\gamma=39,75$  we obtain the infinite norm

$$\left\|T_{y-u}\right\|_{\infty} = 0,9999$$
 (16)

and the corresponding  $H_{\infty}$  speed controller is:

$$H_{\infty}(s) = \frac{2327s^2 + 22211s + 16495}{s^3 + 822951s^2 + 548632s + 91442}.$$
 (17)

The dynamic performances and the robust and stability performance criteria are performed. The sensitivity function S(s) of the close loop for the nominal plant is:



Figure 4. Robust H∞ controller.

$$S(s) = \frac{1,015s^3 + 1,5s^2 + 0,66s + 0,09}{s^3 + 157,2s^2 + 1486,5s + 1103,6}.$$
 (18)

The logarithmic Bode diagram of the direct-loop transfer function of the weighted process is presented in Fig. 7. According to them, we establish the following stability parameters:

crossover band  $\Delta \omega_{\rm B} = 153,7$  rad/sec; stability margins: gain margin = 130,3 dB; phase margin = 86,8°.

For the same performance and robust stability specifications, a great number of weighting functions described by (15) can be chosen, so the solution of designing an optimal  $H_{\infty}$  controller is not unique.

#### IV. SIMULATED RESULTS AND CONCLUSIONS

The control structure will be implemented on a driving system based on a induction machine with following main rated parameters: power P= 2,2kW; speed n=1435 rpm.; stator current  $I_s$ =4.9A at stator voltage  $U_s$ =400V; load torque  $M_{em}$ =14.7Nm.

The simulated results of the MRAS algorithm are presented in figure 3, where the reference model and adjustable model performance indexes (9), (10) and the error (13) are presented for a starting process to the rated speed with no load. The simulated speed of this process, the estimated speed based on the MRAS algorithm and the speed error are presented in figure 5. The influence of a rated load step on the performance indexes and the real and estimated speed are presented in figure 6 (triggered after t=0.3 sec.)



Figure 3. The performance indexes for a starting process with no load.



Figure 4. The rotor speed, the estimated speed based on the MRAS algorithm and the speed error, for a starting process with no load.

The simulated results confirm that the estimated speed based on the presented MRAS algorithm properly follows the rotor speed for different load torque or/and speed steps. The reference and the adjustable model contain only derivates (no integrations) so the estimator can be used also at low speed. Problems that may occur by derivation of the measured stator currents can be avoided using specific digital algorithms. The proposed control structure with MRAS estimator is robust to parameter variations as leakage inductance an sensitive to parameter variations like stator resistance and mutual inductance (the reference model) or rotor time constant (the adjustable model). Most common solution is to use a robust, H-infinite based controller (the C(s) transfer function in Fig. 1) to ensure the stability of the driving system. For applications where high accuracy of speed estimation is needed, recent solutions for on-line estimation of stator resistance or simultaneous estimating of rotor speed and rotor constant time [11] can be considered.

## APPENDIX A

The mathematical model of the induction machine in the fix reference frame d-q, namely the voltage equations:

$$u_{sd} = R_s i_{sd} + \frac{d\Psi_{sd}}{dt};$$
  

$$u_{sq} = R_s i_{sq} + \frac{d\Psi_{sq}}{dt};$$
  

$$0 = R_r i_{rd} + \frac{d\Psi_{rd}}{dt} + w\Psi_{rq}$$
  

$$0 = R_r i_{rq} + \frac{d\Psi_{rq}}{dt} - w\Psi_{rd}$$

the flux equations:

$$\begin{split} \Psi_{sd} &= L_s i_{sd} + L_m i_{rd} ; \\ \Psi_{sq} &= L_s i_{sq} + L_m i_{rq} ; \end{split}$$

$$\begin{split} \Psi_{rd} &= L_m i_{sd} + L_r i_{rd} \; ; \\ \Psi_{rq} &= L_m i_{sq} + L_r i_{rq} \; , \end{split}$$

equivalent inductance and rotor constant time:

$$L_{s} = L_{s} - \frac{L_{m}^{2}}{L_{r}}; \qquad t_{r} = \frac{L_{r}}{R_{r}}$$

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