Spectral Shaping for Codes with P.S.D. Expressed by Rational Functions

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Abstract— **Based on the fact that spectral shaping of digital data signals is obtained by encoding and that there exist spectrally equivalent digital filters, the method of Justesen was applied to derive digital filters that approximate the power spectral density (p.s.d.) of a code. The (2,2,3) code, which belongs to the class of FAS (Finite Autocorrelation Sequence) codes showing a limited number of values of the autocorrelation function that are not zero, was used as an example. The spectral properties of the (2,2,3) code are thoroughly investigated in terms of autocorrelation function values and p.s.d. both as a function of normalised frequency fn and the probability of a mark p. The prediction coefficients for the digital filter implementations are derived using approximations by Markov processes of the third, fourth and fifth order. The resulting spectra are compared with those of (2,2,3) code.**

Index Terms—**Codes, Digital filters, Linear predictive coding, Markov processes, Spectral analysis**

I. INTRODUCTION

The p.s.d. of a line code can be made equivalent to squared magnitude of the transfer function of a linear causal filter [1]. It is well known that

$$W_x(f) = h \cdot \left|H(f)\right|^2 \tag{1}$$

where h is the unilateral p.s.d. of the white noise process applied at the input of the filter with transfer characteristic $H(f)$ and $W_x(f)$ is the p.s.d. of the output.

A WSS random process $x(k)$ may be represented as the output of a causal and causally invertible linear system excited by a white noise process as shown in figure 1.

Figure 1. Generation of *x(k)* from white noise.

The *z*-transform of the autocorrelation sequence $R_x(k)$ is given by

$$W_x(z) = \sum_{k=-\infty}^{\infty} R_x(k) z^{-k} \tag{2}$$

One can assume [2] that the p.s.d. of the WSS random process $x(k)$ is a rational function expressed as

$$W_x(z) = h\,\frac{B(z)B(z^{-1})}{A(z)A(z^{-1})} \qquad r_1 < |z| < r_2 \tag{3}$$

with $r_1 < 1,\ r_2 > 1$. The polynomials $B(z)$ and $A(z)$ have roots that fall inside the unit circle in the z-plane. The output $x(n)$ can be expressed [2] as

$$x(n) = \sum_{k=0}^{\infty} h(k) w(n-k) \tag{4}$$

Then, the linear filter $H(z)$ that is equivalent to a line coded signal $x(n)$ produced by a random input sequence, if $w(n)$ is rational, can be expressed [2] as

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{j=0}^{N} b_j z^{-j}}{\sum_{j=0}^{N} a_j z^{-j}} = \frac{\sum_{j=0}^{N} b_j z^{-j}}{1 + \sum_{j=1}^{N} a_j z^{-j}} \tag{5}$$

where a_0 was assumed to be 1 and that all zeros of $A(z)$ are outside the unit circle.

The process described by the filter with the transfer function $H(z)$ is termed *Autoregressive Moving Average* (ARMA) and is illustrated in figure 2. The linear system described by the rational system function $H(z)$ satisfies the difference equation:

$$\sum_{j=0}^{N} a_j x(n-j) = \sum_{j=0}^{N} b_j w(n-j) \tag{6}$$

In continuous representation this is written as

$$\sum_{j=0}^{N} a_j X_{t-j} = \sum_{j=0}^{N} b_j W_{t-j} \tag{7}$$

W_t is a random variable in the sequence $\bar{W}$ of uncorrelated random variables

$$\bar{W} = \mathbf{L}, W_{t-1}, W_t, W_{t+1}, \mathbf{L} \tag{8}$$

The term $b_0 W_t$ is uncorrelated with the past and may be identified [2] as the prediction error

$$e_p = X_t - \hat{X}_t = b_0 W_t \tag{9}$$

Then

$$\sum_{j=0}^{N} a_j X_{t-j} = e_p + \sum_{j=1}^{N} b_j (X_{t-j} - \hat{X}_{t-j}) \tag{10}$$

As

$a_0 = 1$,

$$X_t + \sum_{j=1}^{N} a_j X_{t-j} = X_t - \hat{X}_t + \sum_{j=1}^{N} b_j (X_{t-j} - \hat{X}_{t-j}) \qquad (11)$$

and

$$X_t = -\sum_{j=1}^{N} b_j \hat{X}_{t-j} + \sum_{j=1}^{N} (b_j - a_j) X_{t-j} \qquad (12)$$

As $W(z)$ has no real zeros, the numerator of $H(z)$, which is $B(z)$, does not have any zeros on the unit circle.

II. RATIONAL POWER SPECTRA

It was assumed [2] that the data to be transmitted belong to a finite alphabet $B = \{b_1, b_2, \mathbf{K} b_I\}$ and are i.i.d. They are converted into a continuous real-valued function of time as a one-to-one mapping from the input information assumed signal to be a sequence of i.i.d. random variables

$$\overline{X} = \mathbf{L}, X_{k-1}, X_k, X_{k+1}, \mathbf{L} \qquad (13)$$

to the ensemble of output discrete signals that constitute a stationary ergodic random sequence

$$\overline{Y} = \mathbf{L}, Y_{k-1}, Y_k, Y_{k+1}, \mathbf{L} \qquad (14)$$

They satisfy the requirement of exhibiting a certain power spectrum with prescribed shape $W_Y(f)$.

The intermediate conversion is subjected to two restrictions [2] placed on the statistics of the encoded sequence:

1. the probability distribution of each Y_i has a constant value $p_Y(v_k)$.

2. the p.s.d. of coded sequence $\overline{Y}$ equals a given function $W_Y(f)$.

The coded process $\overline{Y}$ is assumed to be zero-mean, i.e.

$$E[Y_n] = 0 \qquad (15)$$

A relation must be found relating the p.s.d. $W_Y(f)$ and the entropy $H(Y)$ of $\overline{Y}$, which may be written as

$$H(Y) = H(Y_n \,|\, Y_{n-1}, Y_{n-2}, \mathbf{L}) \qquad (16)$$

To simplify things, the past in rel. (11.8) is replaced by a function Φ of the past, and we have

$$H(Y) = H(Y_n \,|\, Y_{n-1}, Y_{n-2}, \mathbf{L}) \le H(Y)$$
$$= H[Y_n \,|\, \Phi(Y_{n-1}, Y_{n-2}, \mathbf{L})] \qquad (17)$$

As the entropy of the left-hand side in (17) is smaller, taking into account the fact that a sequence that can be predicted with small prediction error exhibits small entropy in opposition with a sequence that cannot be predicted and has a large entropy, the logical idea is to use as the function Φ a predictor for $\overline{Y}$. In other words, the function Φ should provide an estimate of Y_n with sufficient accuracy.

III. LINEAR PREDICTION CODING

Given a set of past samples of the output signal $Y_{n-1}, Y_{n-2}, \mathbf{L}$ and using linear prediction coding, $\hat{Y}_n$ of Y_n, the predictor for the present sample value of the signal is given by

$$\hat{Y}_n = \sum_{j=1}^{\infty} h_j Y_{n-j} \qquad (18)$$

The prediction error $e(n)$ is defined as

$$e(n) = Y_n - \hat{Y}_n \qquad (19)$$

This form of prediction is referred to as one-step forward linear prediction. The prediction coefficients h_j are usually optimised by minimising the mean-square value of the prediction error $e(n)$. The predictor is then denoted as a LMMSE (linear minimum mean-square error) estimator.

The LMMSE estimation is orthogonal, i.e.

$$E(e_n \cdot x_{n-j}) = 0 \qquad j > 0 \qquad (20)$$

Combining equations (18) and (19) we get

$$E(e_n \cdot Y_{n-k}) = E[(Y_n - \hat{Y}_n) \cdot Y_{n-k}] =$$
$$E[(Y_n - \sum_{j=1}^{\infty} h_j Y_{n-j}) \cdot Y_{n-k}] = \qquad (21)$$
$$E[Y_n \cdot Y_{n-k}] - \sum_{j=1}^{\infty} h_j E[Y_{n-j} \cdot Y_{n-k}] = R(k) - \sum_{j=1}^{\infty} h_j R(k-j)$$

This results in Wiener-Hopf equation

$$\sum_{j=1}^{\infty} h_j R_Y(k-j) = R_Y(k) \qquad k = 1, 2, \mathbf{L} \qquad (22)$$

The LMSSE estimator can either be approximated using rel. (18) and restricting the sum to a finite number of terms or be calculated accurately, if the power spectrum can be expressed in a rational form, which is the case here.

Substituting the function Φ in rel. (17) with the LMMSE predictor $\hat{X}_t$, one has

$$H(X) = H(X \mid \hat{X}_t) \qquad (23)$$

So, Markov shaping represents the equivalent of an ARMA process as opposed to an autoregressive AR (feedback only) or a moving average MA (feedforward only) process.

So far, there is only one practical procedure for devising a source with prescribed power spectrum introduced by Justesen [2]. We shall apply the method of Justesen to find the equivalent filter that produces a spectral shaping similar to the FAS coding.

IV. SPECTRUM SHAPING WITH FAS CODES

The quantity

$$\Delta \stackrel{\Delta}{=} -\sum_{k=1}^{\infty} k^2 \frac{R(k)}{R(0)} \qquad (24)$$

can be used as an indicator of D.C. suppression [3] and moreover, as a design criterion.

To cope with the infinite series involved in (24) a new restriction was added, namely to exclude the code with an infinite number of non-zero $R(k)$. What we are left with are termed FAS (*Finite Autocorrelation Sequence*) codes and they satisfy

$$R(k) = 0 \quad \textit{for } k \ge M \qquad (25)$$

where M is an integer.

Here FAS stands for *Finite Autocorrelation Sequence* and this design method was introduced by Dieuliis and Preparata [3]. The signaling scheme is supposed to be modeled by a

Mealy automaton or FSM (*Finite State Machine*). The state transition probability matrix is a stochastic one

$$\boldsymbol{\Pi} = \left\| p_{ij} \right\| \quad (26)$$

p_{ij} being the transition probability from state S_i to state S_j. In [3] it was assumed the existence of a function $s : S \rightarrow Z$, where Z is the set of integers so that

$$s\,[g(s,b)] = s\,(s) + g\,[h(s,b)] \quad (27)$$

for every $s \in S, b \in B^n$ and $w \in O$. Here w is an output *m*-tuple composed of ternary symbols as

$$w = \{w_1, w_2, \mathbf{L}\, w_m\} \quad (28)$$

This allows the implementation of the encoder based on a digital counter and the resulted code is denoted as *counter encodable*. And can be specified as (n,m,N), where the input digits are coupled into *n*-tuples, the output digits into *m*-tuples and the number of states of the encoder automaton is N. The content of the counter is the value of the current state $s\,(s)$. The states are labeled so that

$$s\,(s_i) = i \quad (29)$$

and the output functions $\{h_i\}$ should satisfy an odd-symmetry law such as

$$w_i \in h_i(B^n) \Leftrightarrow -w_i \in h_{N-i+1}(B^n) \text{, for } i = 1, 2, \mathbf{K}\, N \quad (30)$$

which results in a balanced character of the code and the process has zero mean.

The behavior of the state transition probability matrix $\boldsymbol{\Pi}$ when it is raised to a ***n-th*** power which nears infinity is of importance in describing the p.s.d. One way to calculate [5] $R(k)$ is

$$R(k) = trace\left(\boldsymbol{D} \times \boldsymbol{\Pi}^{\boldsymbol{k}} \times \boldsymbol{Z}\right) \quad (31)$$

where $\boldsymbol{D}$ is the matrix of stationary probabilities in diagonal form and $\boldsymbol{Z}$ is the correlation matrix [5]. Let

$$\boldsymbol{\Pi}^{\infty} = \lim_{n \to \infty} \boldsymbol{\Pi}^{\boldsymbol{n}} \quad (32)$$

As known, [5] $\boldsymbol{\Pi}^{\infty}$ has identical rows containing the stationary probabilities. Choosing $\boldsymbol{\Pi}$ as a stochastic matrix with identical rows results in

$$\boldsymbol{\Pi} = \boldsymbol{\Pi}^{\infty} \quad (33)$$

Since a channel input codeword is $a_i = (a_{i1}, a_{i2}, \mathbf{L}\, a_{in})$ and the attached encoder state is $s^{(i)}$, one can state that the pair $(s^{(0)}, a_0)$ completely determines the next state $s^{(1)}$. As $\boldsymbol{\Pi} = \boldsymbol{\Pi}^{\infty}$, this implies that the transition probability from state *j* to state *i* equals the stationary probability of state *i*, i.e. $p_{ji} = p_i$.

One can conclude that the following state $s^{(2)}$ and the next ones are statistically independent of $s^{(0)}$, i.e. any state $s^{(h)}, h \geq 2$ is statistically independent of $s^{(0)}$ or any states separated by at least one state are statistically independent. Then,

$$E[a_{0i} \cdot a_{hj}] = E[a_{0i}] \cdot E[a_{hj}] = 0 \quad (34)$$

as $E[a_{0i}] = E[a_{hj}] = 0$ (35)

since the code is balanced and counterencodable. This results in

$$R(k) = 0 \text{ for } k \geq 2m \quad (36)$$

taking into account that each state outputs m ternary digits.

V. SPECTRAL CHARACTERIZATION OF (2,2,3) CODE

The *(2,2,3)* code was designed by Dieuliis and Preparata and is described by the codebook in Table I.

As the code is balanced, then for every word w, its complement $-w$ must also exist. As the conditions (27), (29) and (30) must be met, it is obvious that if a code word w is met in state *1*, its complement $-w$ must be met in state *3* ($N - i + 1 = 3$ in this case). The code words found in state *2* will include their complements as well, since if $i = 2, N - i + 1 = 2$ for $N = 3$.

TABLE I.CODING RULES OF (2,2,3) CODE

State / **Input**	s_1 **Output/Next state**	s_2 **Output/Next state**	s_3 **Output/Next state**
00	$+-/s_1$	$+-/s_2$	$--/s_1$
01	$++/s_3$	$-+/s_2$	$-+/s_3$
10	$0+/s_2$	$0+/s_3$	$-0/s_2$
11	$+0/s_2$	$0-/s_1$	$0-/s_2$

The codes with zero disparity ($g\,(w) = 0$) will determine a conservation of the state, that is from state *i* to state *i*.

The words with disparity one ($g\,(w) = 1$) will determine a jump into the adjacent state, i.e. from state *i* to state $i+1$, while for disparity two. e.g. $g\,(w) = -2$, a jump into the next adjacent state, i.e. from state i to state $i-2$ is taken.

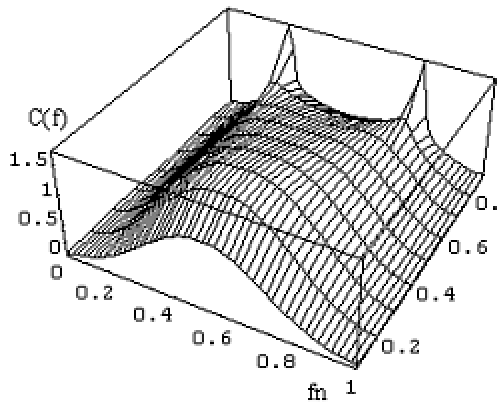

Figure 2. 3D representation of coding factor for (2,2,3) code.

The p.s.d. of the *(2,2,3)* code was determined as

$$\begin{aligned} C(f_n, p) = {} & (p(14 - 66p + 147p^2 - 186p^3 + 148p^4 - 73p^5 + 20p^6 - 4p^7 + \\ & (7p - 41p^2 + 110p^3 - 148p^4 + 104p^5 - 32p^6) \cdot \cos 2p\, f_n + \\ & (-12 + 55p - 98p^2 + 83p^3 - 6p^4 - 46p^5 + 32p^6 - 8p^7)\cos 4p\, f_n + \\ & (2 - 17p + 51p^2 - 72p^3 + 52p^4 - 16p^5)\cos 6p\, f_n + \\ & (p - 7p^2 + 19p^3 - 25p^4 + 16p^5 - 4p^6)\cos[8p\, f]) \cdot \sin^2 p\, f_n)/ \\ & (1 - 2p + 2p^2 + (-1 + 2p)\cos 4p\, f_n) \end{aligned} \quad (37)$$

A 3D representation of the coding factor of (*2,2,3*) code is given in figure 2.

Figure 3 presents the coding factors of (*2,2,3*) and bipolar No.1 code for the equiprobable case ($p = 0.5$).

An increase of the energy of the (*2,2,3*) code can be observed, as the coding elements contribute more energy to the signal (the combination 00 is not used).

VI. DETERMINING THE PREDICTOR VALUE FOR (2,2,3) CODE

Using the MPR program written in MATHEMATICA, the values of the autocorrelation function are obtained as

$$
\begin{aligned}
&R(0) = 2p - p^2\\
&R(1) = p(-4+16p-40p^2+51p^3-28p^4+4p^5)/2\\
&R(2) = p(4-30p+83p^2-109p^3+64p^4-12p^5)/2\\
&R(3) = (1-2p)^2 p(-4+16p-19p^2+7p^3-p^4)/2 \qquad (38)\\
&R(4) = (1-2p)^3 p(2-4p+p^2)\\
&R(5) = (1-2p)^2 p(-4+16p-20p^2+8p^3-2p^4+p^5)/2\\
&R(6) = (1-2p)^2 p(2-8p+9p^2-4p^4+p^5)
\end{aligned}
$$

For p = 0.5 they are

$$R(0) = 3/4 \quad R(1) = -10/32 \quad R(2) = -1/16, \qquad (39)$$
$$R(k) = 0, \quad k \ge 3$$

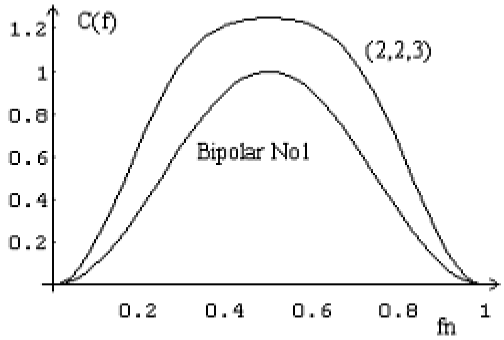

Figure 3. Coding factors of (2,2,3) and AMI code for p=0.5.

We will approximate it by a third-order Markov process [2] that exhibits the same values of the autocorrelation function for $k \le 4$. The Wiener –Hopf equation can be written as

$$
\begin{aligned}
&h_1R(k-1)+h_2R(k-2)+h_3R(k-3) = R(k)\\
&k=1 \to h_1R(0)+h_2R(-1)+h_3R(-2) = R(1)\\
&k=2 \to h_1R(1)+h_2R(0)+h_3R(-1) = R(2) \qquad (40)\\
&k=3 \to h_1R(2)+h_2R(1)+h_3R(0) = R(3)
\end{aligned}
$$

In matrix form

$$[h_1 \;\; h_2 \;\; h_3]\begin{bmatrix} R(0) & R(1) & R(2)\\ R(-1) & R(0) & R(1)\\ R(-2) & R(-1) & R(0)\end{bmatrix} = [R(1) \;\; R(2) \;\; R(3)] \; (41)$$

$$
\text{or}\begin{cases}
\frac{3}{4}h_1 - \frac{5}{16}h_2 - \frac{1}{16}h_3 = -\frac{5}{16}\\
-\frac{5}{16}h_1 + \frac{3}{4}h_2 - \frac{5}{16}h_3 = -\frac{1}{16}\\
-\frac{1}{16}h_1 - \frac{5}{16}h_2 + \frac{3}{4}h_3 = 0
\end{cases}
$$

Solving it results in

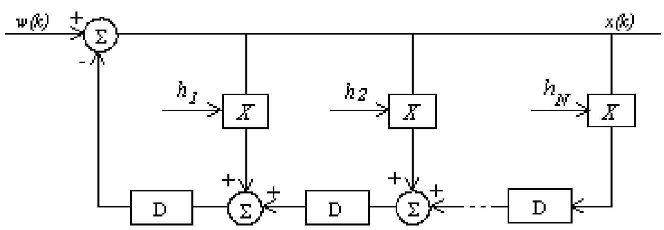

Figure 4. Recursive filter used for prediction.

$$h_1 = -330/533, \; h_2 = -18/41, \; h_3 = -125/533. \qquad (43)$$

The predictor is illustrated in figure 4 and is given by

$$\hat{X}_t = -\frac{330}{533}X_{t-1} - \frac{18}{41}X_{t-2} - \frac{125}{533}X_{t-3} \qquad (44)$$

The spectrum results as

$$W_{x3}(f) = \left|1 + \frac{330}{533}e^{-j2p\,f} + \frac{18}{41}e^{-j4p\,f} + \frac{125}{533}e^{-j6p\,f}\right|^{-2} \qquad (45)$$

An approximation by a fourth-order Markov process results in

$$\hat{X}_t = -\frac{2995}{4512}X_{t-1} - \frac{393}{752}X_{t-2} - \frac{265}{752}X_{t-3} - \frac{859}{4512}X_{t-4} \; (46)$$

and the spectrum is given by

$$W_{x4}(f) = \left|\begin{matrix} 1 + \frac{2995}{4512}e^{-j2p\,f} + \frac{393}{752}e^{-j4p\,f} + \\ + \frac{265}{752}e^{-j6p\,f} + \frac{859}{4512}e^{-j8p\,f} \end{matrix}\right|^{-2} \qquad (47)$$

An approximation by a fifth-order Markov process results in

$$
\begin{aligned}
\hat{X}_t = &-\frac{25555}{36811}X_{t-1} - \frac{42623}{73622}X_{t-2} - \frac{245}{562}X_{t-3}\\
&-\frac{21829}{73622}X_{t-4} - \frac{5885}{36811}X_{t-5}
\end{aligned} \qquad (48)
$$

and the spectrum is given by

$$W_{x5}(f) = \left|\begin{matrix} 1 + \frac{25555}{36811}e^{-j2p\,f} + \frac{42623}{73622}e^{-j4p\,f} + \\ \frac{245}{562}e^{-j6p\,f} + \frac{21829}{73622}e^{-j8p\,f} + \frac{5885}{36811}e^{-j10p\,f} \end{matrix}\right|^{-2} \qquad (49)$$

A comparison of the coding factors C(f, 0.5) of the (2,2,3) code of Dieuliis and Preparata, as given by (37) and their substitutes $W_{x3}(f)$, $W_{x4}(f)$ and $W_{x5}(f)$ found above is illustrated in Fig.5.

Increasing the order of the corresponding Markov process results in a better approximation of coding factor.

An exact solution cannot be obtained, as the code is not counterencodable. This explains also the presence of the D.C. component.

TABLE II CODING TABLE OF NEW CODE

State	$X_{t-1}X_{t-2}X_{t-3}$	$z(s_j)$
s_0	(+1,+1,-1)	+191/533
s_1	(+1,-1,+1)	+439/533
s_2	(+1,-1,-1)	-29/533
s_3	(-1,+1,+1)	+29/533
s_4	(-1,+1,-1)	-439/533
s_5	(-1,-1,+1)	-191/533

Based on rel. (11.43) one can see that the combinations of like symbols are excluded as

$$\sum_{j=1}^{3} h_j = \frac{330}{553} + \frac{18}{41} + \frac{125}{533} = \frac{589}{533} > 1$$

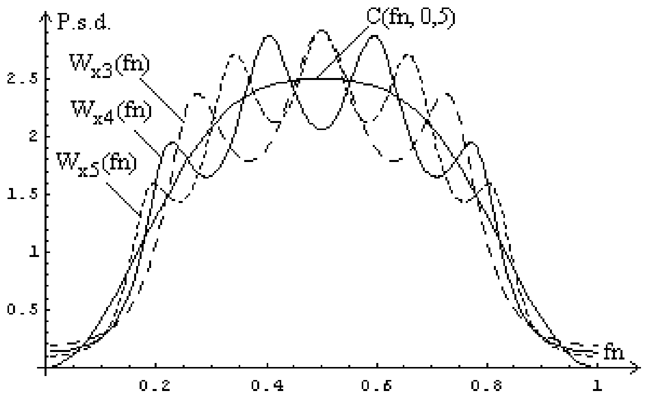

Figure 5. Coding factors of FAS (2,2,3) substitutes.

The states are assigned as shown in Table 2, assuming the linear predictor $\hat{X}_t$ is a function $z\,(S(t))$. One can see that the code is not, as there is no proper relation between the current value of the state and RDS (see rel. (27)).

The same is valid for the arrangements based on fourth-order and fifth-order Markov processes.

VII. CONCLUSIONS

We applied Justesen method to derive digital filters that approximate the power spectral density (p.s.d.) of the counterencodable (2,2,3) code using fourth-order and fifth-order Markov processes.

The obtained substitutes show no proper relation between the current value of the state and RDS (Running Digital Sum), as with (2,2,3) code, but they can be used to shape the spectrum of a digital data string.

APPENDIX

A program was written in MATHEMATICA for solving the Wiener Hopf equation for a 5-th order predictor.

$R0 := 3/4$

$R1 := -5/16$

$R2 :== 1/16$

$R3 := 0$

$R4 := 0$

$R5 := 0$

```
Solve[{h1, h2, h3, h4, h5}.
  {{R0, R1, R2, R3, R4}, {R1, R0, R1, R2, R3},
   {R2, R1, R0, R1, R2}, {R3, R2, R1, R0, R1},
   {R4, R3, R2, R1, R0}} == {R1, R2, R3, R4, R5},
   {h1, h2, h3, h4, h5}]
```

```
{{h1 → - 25555/36811,
  h2 → - 42623/73622,
  h3 → - 245/562,
  h4 → - 21829/73622,
  h5 → - 5885/36811}}
```

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