The Methods of the Fast Shading

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Abstract—The problem of increasing of shading productivity according to widespread methods Gourand and Phong is examined. The article summarizes authors’ achievements in increasing shading speed and performance. Bidirectional Reflectance Distribution Function is simplified for a faster computing and hardware implementation.

Index Terms—shading, fast shading, BRDF approximation

I. INTRODUCTION

The Gourand and Phong methods are the most widespread for today. This is caused by their relative easiness, satisfactory quality and hardware supply ability. This article proposes new approaches of their realization that allows considerable increase of the mentioned methods performance.

II. THE METHODS OF THE SHADING PRODUCTIVITY INCREASING

This is theoretically proved [1] that during the shading process according Gourand growth of the color intensities ΔIₓ and ΔIᵧ correspondingly along horizontal and vertical scanlines are constant values and this eliminates necessity of their computing for every rasterization line. Let’s find out interconnection between growth of color intensities along horizontal and vertical rasterization lines. It’s proved that growth of intensities ΔIₓ, ΔIᵧ and ΔIḏ in horizontal, vertical and diagonal directions correspondingly are constants and equal.

\[
\Delta I_x = (I_A - I_C) \cdot \Delta Y_{BC} - (I_B - I_C) \cdot \Delta Y_{AC},
\]
\[
\Delta I_y = (I_B - I_C) \cdot \Delta Y_{BC} - (I_A - I_C) \cdot \Delta Y_{AC},
\]
\[
\Delta I_d = \Delta I_x + \Delta I_y
\]

where \(I_A, I_B, I_C\) - color intensities at the triangle vertexes (Figure 1).

The obtained property allows to propose sequence of approaches Gourand parrelling rendering [2].

In the first method first order Serpinsky triangulation is used. It lies in dividing initial triangle to 4 parts by drawing centerlines. With considering the fact that color intensities growth is permanent value an approach was theoretically founded.

Analogously during detection of color intensities of internal points belonging to the top left rectangular triangle all the actions of code interpolation are to be duplicated for all the other triangles, but according to their vertexes.

The second method of Gourand shading performance increase is based on the independent forming of even and odd points in the scan line. In the preparation cycle color intensities \(I_1, I_2\) are to be found according to the first and
next points in the scan line. The intensities $I_{pk}$ of the first points in scan lines are to be defined by means of code interpolation of color intensities along the basic edges, and $I_2$ - with help of expression: $I_2 = I_{pk} + \Delta I_p$. $I_3 = I_2 + 2\Delta I_p$, $I_6 = I_4 + 2\Delta I_p$, ..., $I_{2w} = I_{2w-2} + 2\Delta I_p$, ..., $I_{j} = I_{j} + 2\Delta I_p$. $I_\gamma = I_5 + 2\Delta I_p$, $I_{2w+j} = I_{2w+j} + 2\Delta I_p$ ..... (2)

One of the possible approach for the Gourand rendering improvement lies in the detection of highlights in triangle borders and their further forming [3].

Let’s assume that normal vectors $\vec{N}_0, \vec{N}_1, \vec{N}_2$ are given in the triangle $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ edges. In the graphic images forming the most common case [2], is when light source and observer are situated in the infinity, h.i. vector $\vec{L}$ has identical direction for all the triangle points. It was proved [4], that in this case maximal values of color intensity are at the such edge points that fits equations.

\[
t_1 = \frac{\cos\gamma \cos\phi - \cos\beta}{(\cos\phi - 1)(\cos\beta + \cos\gamma)},
\]

\[
t_2 = \frac{\cos\gamma \cos\phi - \cos\lambda}{(\cos\phi - 1)(\cos\lambda + \cos\gamma)},
\]

\[
t_3 = \frac{\cos\beta \cos\phi - \cos\lambda}{(\cos\phi - 1)(\cos\lambda + \cos\beta)}. \tag{3}
\]

Conformity of angles between normal vectors is represented in the figure 3.

![Figure 3. Normal vectors of triangle ABC.](image)

Variables $t_1, t_2, t_3$ are parametric ones and has change interval from 0 to 1. An initial triangle can be divided into several depending on the result of threshold valuations comparing with color intensities in the points $t_1, t_2, t_3$. The triangles received are further shaded with Gourand method. Such an approach increases Gourand rendering quality due to highlights forming in case of their intersection by one or several triangle edges. In the classic Gourand rasterization realization this shading is absent and highlight is eliminated, that is essential artifact.

In the computer graphics systems Blinn illumination model is widespread, according to what intensity of specular color constituent is computed with following formula:

\[
I_s = I_s k_s \cos^n \gamma,
\]

where $I_s$ - intensity of external light source; $k_s$ - coefficient of specular reflection; $\gamma$ - angle between normal vector $\vec{N}$ and surface normal vector $\vec{H}$, that is received by the adding of light direction vector $\vec{L}$ and observing vector $\vec{V}$; $n$ - coefficient of surface specularity.

The maximal amount of computing in the process of specular color constituent detection takes place for BRDF computing $\cos^n \gamma$, $n = 1, 1000$. A new BRDF was proposed that has considerably lower power in comparison with such BRDF like $\cos^n \gamma$. It’s expressed with formula $\cos^k(\sqrt[n]{k} \cdot \gamma)$, where $k$ - coefficient, that is detected depending on surface specularity coefficient $n$ ($k << n$).

The usage of various powers $k$ for different diapasons of $n$ values was proposed for guaranteeing of necessary approximation accuracy. For instance in approximation of $\cos^n \gamma$ by function $\cos^k(\sqrt[n]{k} \cdot \gamma)$ with relative error 3% it’s enough to take only to values of $k$: 1, 3. Modification of well known Schlick BRDF was proposed. It lies in using different values of powers depending on specular coefficient. This is the following formula [5]

\[
\cos\left[\left(\frac{\log_2 n - 2}{2}\right)\gamma / (n - n \cos \gamma + \cos \gamma)\right] \tag{5}
\]

A problem of BRDF approximation with function of $2^{nd}$ and $3^{rd}$ degree [4] and its hardware realization was examined.

With the purpose of increasing the shading productivity it is offered to use in one computing process combined and Phong methods. [5] So, for example, it is possible to divide scanline into digital segments of length, divisible by power of a two. The values of color intensities according to the Phong method are determined in endpoints of a segment, and in intermediate – according to the a Gourand method. The length of the digital segment is able for adaptive changing and is determined by curvature of a surface and direction of vector $\vec{H}$.

During the computing specular and diffusive color constituent for points on 3D surface normalization of normal vectors is required. This is explained by the fact that cosines of angles that are used in shading function can easily be found by the inner product of unit length vectors. Taking into consideration that normalization is performed for observer’s vectors, light source’s vector and vector of normal, the problem of computational complexity decreasing for this resource-intensive procedure is urgent. The method of adaptive normal vectors normalization is proposed [5]. The normalization is performed only in cases
of considerable influence upon result of colour intensity determination.

On the figure 4 an example of determination of intermediate normal vectors between vectors $\vec{N}_a$ and $\vec{N}_b$, that are situated at the scanline endpoints is provided. It’s shown, that

$$d_{(1/2)^i} = 4.$$

The following correlation is received for error detection $d_{(1/2)^i}$:

$$d_{(1/2)^i+1} = 1 - \sqrt{\frac{2 + z_{(1/2)^i}}{2}}.$$

![Figure 4. Determination of intermediate vectors of normal.](image)

It is proposed at the error $d_{(1/2)^i}$ value lower than certain threshold valuation to use linear interpolation between vectors of normal.

The normal vectors are received by the bisection of angle between given vectors. So, for example,

$$N_r(1/2) = \frac{N_a + N_b}{\sqrt{2(1 + \cos \psi)}} = \frac{\vec{N}_a + \vec{N}_b}{z_{(1/2)^i}},$$

where

$$z_{(1/2)^i} = \sqrt{2(1 + \cos \psi)}.$$

For the further division following formulas are proposed

$$N_r(1/2^n) = \frac{N_A + N^'}{\sqrt{2 + z_{(1/2)^n}}} , \quad z_{(1/2^n)} = \sqrt{2 + z_{(1/2)^n}}.$$

The problem of spherical angular interpolation for the Phong rendering is examined. Recursive correlations are found, according to which normal vector for current point could be computed with help of normal vectors for two previous points, i.e.

$$N(t + 1) = 2N(t) \cos \varphi - N(t - 1),$$

where $\varphi$ angle between start and final vector in the scanline. Analogous expression takes place for the determination of diffusive color constituent

$$I(t + 1) = 2I(t) \cos \varphi - I(t - 1) \quad (7)$$

The usage of found recursive correlations allows bringing the computation of diffusive color constituent and inner product to two subtraction operations and two shift operations that allows increase productivity to 40%.

The new approach for accelerated diffusive color constituent determination according to the Phong method is proposed in 3D figures shading. The peculiarity of this approach lies in the synchronous hardware determination of two points at the scanline. For this approach new approximation formula is used for normalization of normal vectors, and new properties of Phong rendering.

The adaptive method of shading [5], in which two different illumination models are used depending on availability of highlight within the triangle. The increase of productivity is reached due to using in case of availability of highlight within the triangle the illumination model that considers specular color constituent, and in all the others – simple model, that considerably decreases computation complicity. The method for detecting fact intersection of edge and highlight or its identification within triangle is proposed.

Considering, that highlight on the surface of graphic object occupies a tiny area adaptive approach allows essential increase productivity of the shading.

Not only problems dealt with adaptive usage of illumination models are highlighted but the methods of shading either. At the same time surface curvature is considered and level of specular and diffusive light.

### III. ALTERNATIVE BRDF MODEL

Let’s examine BRDF approximation with

$$W(\gamma, n) = \left( \frac{A}{B} \left( \cos \gamma - 1 \right) + 1 \right)^2$$

in conditions that

$$0 \leq \gamma \leq \frac{\pi}{2}.$$

This function is chosen, because of the following:

a) cosine – is a basic function for both functions;

b) at $\gamma = 0 \quad \cos^n \gamma = \left( \frac{A}{B} \left( \cos \gamma - 1 \right) + 1 \right)^2 = 1$, that fits boundary conditions;

c) both functions are in the $0 \leq \gamma \leq \frac{\pi}{2}$ are positive;

d) the $\left( \frac{A}{B} \left( \cos \gamma - 1 \right) + 1 \right)^2$ function reaches zero value – this provides blooming appearance;

e) the A and B coefficients give a possibility to change highlight.
At the figure 5 graphics of following functions are represented: Schlick function,

\[ \cos^n \gamma \left( \frac{n}{2} (\cos \gamma - 1) + 1 \right)^2 \]

The graphic shows extremely accurate approximation of the highlight zone – and smooth fading after.

IV. CONCLUSIONS

The methods being proposed allow considerable increasing of shading productivity, and can be used for designing software and hardware for computer graphics.

REFERENCES

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